EDUCATION & TRAINING FOUNDATION

CENTRES FOR EXCELLENCE IN MATHS

No more blank spaces: A toolkit to help students tackle multi-mark measurement questions

Nicola Pearton, Shirley Tonge, Amy West, Amanda Robinson, Refat Shamsi, and Helen Fraser

OUR PARTNERS

FUNDED BY

Department for Education

Working in partnership with the Education and Training Foundation to deliver this programme.

Acknowledgements
We would like to thank the DfE who funded this research, and to all the willing participants who helped us gain insight into this important and interesting area of mathematics education. We are grateful to the teachers and students who participated formally, and those who contributed and supported informally. We would also like to thank our regional lead Holly Connor, the team at Touch Consulting, and our mentor Sheila Evans for their endless positivity, support, and guidance.

About CfEM
Centres for Excellence in Maths (CfEM) is a five-year national improvement programme aimed at delivering sustained improvements in maths outcomes for 16–19-year-olds, up to Level 2, in post-16 settings.
Funded by the Department for Education and delivered by the Education and Training Foundation, the programme is exploring what works for teachers and students, embedding related CPD and good practice, and building networks of maths professionals in colleges.

Contents

Summary

Complex multi-mark problem-solving questions are found to be challenging by many students of mathematics. For GCSE resit students, these questions often seem completely inaccessible and teachers regularly find students do not attempt them at all, thereby missing out on vital marks. Teaching problem-solving is something many teachers find difficult, and given the complexity of the task, and the fact that resit students may appear to have significant topic-knowledge gaps, many GCSE resit curricula do not include explicit problem-solving teaching techniques. We found there to be a lack of resources suitable for teaching problem-solving to resit students, and so set out to attempt to design a resource which allowed resit students to better access these vital questions.

In research cycle one we explored teachers' perceptions and experiences around teaching problem-solving, and students' perceptions and experiences of learning problem-solving, through the use of surveys and analysis of student responses to a problem-solving-related task. We analysed our data by grouping it into themes which corresponded to the key elements of problem-solving. In cycle 2 we used these findings to design an intervention, a Problem-Solving Toolkit, and tested the effectiveness of this teaching resource with GCSE students across three colleges. The toolkit was specific to measurement problem-solving questions, as this topic lends itself well to complex, multi-mark questions, and often includes diagrams and contextual scenarios. We analysed the effectiveness of the toolkit in a primarily qualitative manner and presented findings along the same themes as cycle one, allowing for a generalised comparison of student progress. In cycle 3 we then refined and adapted the toolkit and tested the new version's effectiveness with a group of students on a GCSE programme who were exempt from taking the GCSE examination, and a group of Functional Skills students who had passed their Entry Level 3 examination. Although we were unable to test the refined toolkit with the same sample as the original toolkit due to a fixed revision schedule, we again used the same themes to allow for comparable analysis on how students engaged with the toolkit.

Teachers believed a lack of motivation to be the biggest barrier to students' successfully answering these questions, we found that this was not the case. Students indicated that they were likely to attempt these types of questions in a test or exam situation, and they engaged well with the toolkit. We found that while students were generally willing to attempt these types of questions, it was a lack of conceptual understanding and difficulty making connections between topics which were the primary barrier to students' success. We found that students lacked the experience and resilience to confidently and independently navigate their way through these questions.

Students engaged positively with the toolkit, and we found that it provided them with a starting point which did help them to access these questions. We also found that basic maths knowledge was evident in the majority of students. However, students did not necessarily know which facts to draw on to answer the questions and were inexperienced in using metacognitive processes to problem-solve. Therefore, we would recommend that curricula are designed in such a way that topics are grouped in such a way that links between them are logical and that these links are made explicit to students. We would encourage switching between linked topics in the same lesson, and regularly in the same question. We would also recommend that the practice of metacognition be explicitly discussed and regularly modelled so that students do not find the process abstract and unattainable. We believe the refined toolkit to be an effective tool for teaching problem-solving and plan to use it on a larger scale going forward. We hope other practitioners will use it when teaching measurement or adapt it for other use in different contexts or for other problem-solving topics.

Background

Introduction

The benefits of acquiring good levels of numeracy skills, both in terms of accessing higher levels of education, as well as better employment prospects, are well documented and it is widely known that that adults with basic numeracy skills earn higher wages and are more employable than those who have not acquired such skills (Heckman, 2008; Wolf, 2011; OECD, 2016a; Smith, 2017).

Despite the correlation between acquiring mathematics skills and improved prospects for further study or better paid employment, many students in the United Kingdom leave education without a formal mathematics qualification (Wolf, 2011). In order to address this issue it has, since 2014, been a condition of funding in England that students who are aged 16 to 19 and are yet to achieve a GCSE Mathematics qualification (Grade C/4 or above) must continue to study mathematics as part of their programme of study. In September 2015 the condition was revised to require full-time students with a Grade D/3, or a "near pass" to retake the GCSE Mathematics rather than alternative approved qualifications such as Functional Skills (Education and Skills Funding Agency, 2019). This policy change has resulted in a significant increase in GCSE mathematics entries. However, the outcomes of this increase have not been as successful as hoped.

The national success rates of students resitting their GCSE mathematics qualification have been disappointingly and consistently low since the introduction of the condition of funding change in 2015 (Higton et al 2017; Smith, 2017; Noyes & Dalby, 2020). Despite significantly more students reattempting the GCSE qualification, analysis of retake students' mathematics progress in the 2018/19 academic year highlighted relatively poor progress for those in Further Education colleges with less than a quarter of students without a GCSE grade 4 in mathematics at age 16 achieving this by age 18.

Extensive work has been and continues to be undertaken to try to improve the GCSE Mathematics resit success rates. Greater Brighton Metropolitan College is a Further Education College in the south of England and is one of the Centres for Excellence in Maths. Being a part of the CfEM project has meant that GBMC and our network partners have been afforded the opportunity to engage in formal research and project work to better understand the student experience of the resit context, and to design and implement interventions which aim to increase the number of students who leave compulsory education with a GCSE maths qualification.

Research Context

Greater Brighton Metropolitan College was created by the merger of City College Brighton and Northbrook College Sussex. The college is made up of five sites and has around 750 GCSE mathematics resit students. Chichester College and Crawley College are part of the Chichester College Group and they too have multiple sites. On average, student numbers for 14-16 GCSE resits are similar to GBMET. From August 2022 GBMET will join the Chichester College group, and as part of the planned merger process the maths departments across the three colleges were fairly similarly structured in the year this research took place.

This academic year a major challenge for the colleges involved in this research, and indeed for similar educational institutions across the country, was maintaining pre-pandemic levels of student attendance and engagement. Despite numerous interventions, attendance and engagement remained a barrier to students' learning throughout the academic year. In the maths classroom we found that students were less confident and less resilient than before the pandemic, often giving up after only one attempt. We also found that students were absent more often with mental health issues and that coming back to a classroom environment was very challenging for a large number of students.

As a result of these additional challenges, and in an ongoing attempt to improve GCSE maths resit outcomes, our action research project was designed to help students to rebuild their resilience and to continue our work on the difficulties students experience when answering multi-mark problem solving questions.

Research Aim

The broad aim of our research project was to design and test an intervention to help students to access multi-mark problem solving questions. We chose this topic because both the literature and our own experiences highlighted that although answering these questions in the GCSE exam is vital if students are to achieve a grade 4 or above, it is these questions that students often do not attempt at all. In our experience, as well as the experience of our colleagues, it was often the case that most students had severe mental blocks around these questions and did not attempt them at all, and the few who were willing to attempt them did not have appropriate strategies to draw on in order to solve them correctly. We narrowed our focus down to problem-solving questions which were associated with the topic of measurement such as those relating to perimeter, area, surface area, and volume. It is often the case that the multi-mark questions are set in a context which involves measurement, and traditionally students find these topics challenging.

Given that students had had almost two years of disrupted teaching, with the majority of teaching online, we expected students' problem-solving abilities and strategies to be less robust than they were before the pandemic. Teaching and learning problem solving skills is challenging enough in the physical classroom, and it is likely that many teachers and students found it even more challenging online, and some may have avoided it in favour of shorter, more structured topics.

We were interested in the attitudes and perceptions of both teachers and students, and we planned to design and test an intervention which we hoped would help to provide students with access to these vital questions. We hoped that this research would help us to improve into the teaching and learning of problem-solving skills and strategies in a post pandemic GCSE maths resit classroom.

Literature Review

Introduction

Around half of the young people starting Further Education courses in England have previously been entered for a GCSE mathematics examination in which they experienced failure. Some students can give no examples of good experiences in school mathematics and have experienced exclusion from mathematics since primary school. As a result, self-preservation strategies such as avoidance and passive non-compliance are common in GCSE mathematics resit classrooms, with attendance levels often low and anxiety levels often high (Johnston-Wilder et al. 2015, 2016).

Not only are the number of students who successfully achieve a GCSE mathematics qualification undesirably low, but concerns have been raised that repeated GCSE resits are demotivating and can have a negative impact on students' mental health (Belgutay, 2018). Vidal Rodeiro's (2018) research showed that despite retaking the GCSE multiple times, many students did not achieve the qualification by the time they left compulsory education, and that the probability of improving their grade decreased with the number of resit attempts.

Both research and experience have shown that anxious avoidant students are unable to progress effectively. An area of maths which often causes high levels of anxiety and avoidance are the multi-mark problem solving questions, often found towards the end of the examination papers. Our research aim was to help students to access these types of questions. In order to design our intervention, we looked to the literature to better understand the nature of these types of questions, the processes involved in solving them, and the teaching methods that have helped students access these questions.

Background

The challenges which students and education providers face in relation to the compulsory GCSE resits are numerous and complex. Students retaking mathematics largely lack confidence, are often demotivated, and many have complex learning support needs. Colleges can struggle to recruit mathematics teachers, and teachers may not have the required qualifications and experience. Additionally, the revised GCSE mathematics curriculum which was introduced in 2015 places a strong emphasis on students being able to solve mathematical problems, an area which many find challenging (MEI, 2021).

Another challenge that students face is that, as well as including a higher number of problem-solving questions, the revised curriculum assesses problem-solving skills more rigorously than in the previous curriculum. Students are required to choose the appropriate mathematical techniques, sometimes with multiple approaches possible for the same question, and are given less guidance in the way the question is structured (for example multi-step questions are rarely broken into separate parts).

Given that successfully accessing the multi-step problem solving questions in the GCSE maths curriculum is challenging for students, often incredibly so for resit students, and given that accessing these questions is essential if a student is to achieve a minimum of a grade 4, it is useful to look at the nature of these questions, and the processes and skills required to access them in more detail.

Multi-step Problem-Solving Mathematics Questions

Mevarech and Kramarksi (2014:23) use the term "CUN problems" (complex, unfamiliar and non-routine problems) to refer to the types of multi-mark problem solving questions that this research project is focused on. The authors differentiate between these problems and routine problems by highlighting that when students solve routine problems, they can rely on memorisation, whereas when they solve CUN problems they are required to utilise mathematical skills that include not only logic and deduction, but also intuition, number sense, and inference.

CUN problems are often complex in their layout, may contain large amounts of text, and students may struggle to recognise the mathematics contained within the question, thereby preventing them from being able to see what is required to begin the process of solving the problem. Additionally, as Mevarech and Kramarksi point out, CUN problems may contain mathematical information that is not always explicitly presented, and there may be multiple methods to solve the problem, and potentially even multiple answers.

Mevarech and Kramarski explain that to successfully tackle CUN problems it is necessary not only to draw on routine knowledge and skills, but also to demonstrate higher-order skill sets. These higher-order skills include mathematical reasoning, mathematical creativity, and mathematical communication. A set of skills which, for students for whom applying routine mathematical knowledge can prove challenging, can seem unachievable. In order to provide opportunities for students to successfully access the discourse of mathematical problem-solving, a vast scale of research has been conducted to discover how students learn this skill, and to translate these findings into pedagogical approaches to successfully teach mathematical problem solving.

However, the process by which students learn mathematical problem-solving skills is complex and multifaceted. There are a number of factors involved and these factors are interdependent and therefore complex to design, implement, and measure in the classroom. In order to problem-solve effectively students are required to understand core mathematical concepts, make connections between these concepts, demonstrate metacognitive skills, and possess the confidence and motivation to attempt and solve the question. In order to understand the potential barriers students may face when attempting CUN questions, and to find ways to help students overcome these barriers, it is necessary to look at these factors in more detail.

Conceptual understanding, making connections, and metacognition

The development of conceptual understanding requires "careful negotiation of meaning in which objects are compared and classified, definitions are built, and representations are created, shared, interpreted and compared" (Swan, 2014:11). Swan (2008:8-9) emphasises that teaching for conceptual understanding is more effective when learning experiences involve students working on rich and collaborative tasks, building on knowledge they already have, confronting difficulties rather than seeking to avoid or pre-empt them, include higher order questions, and encourage reasoning.

Swan (2008, 2014) emphasises the importance of collaborative working, encouraging students to verbalise their thought processes and talk through the problem with their partner or group. However, as Evans (2017:3) points out, when solving unstructured problems students often use naïve, inefficient strategies such as 'trial and improvement', rather than considering more powerful methods. Thus, sharing approaches with peers does not necessarily guarantee exposure to a wide variety of strategies. Additionally, students who lack confidence may not know how to, or may not want to, verbalise their process of thinking through the problem to someone else.

As well as being required to demonstrate a solid conceptual understanding of numerous mathematical concepts, in order to answer multi-mark problem-solving questions, it is also necessary that students are able to make effective links between these concepts. According to Evans (2017) if students are able to compare and link different approaches to an unstructured problem their perception of the problem situation will be extended, and they will be supported in developing general heuristics which they will then be able to apply to other problems.

However, as Gick and Holyoak (1983) point out, inexperienced problem solvers are not readily reminded of similar structured problems from the past and can find it difficult to recycle 'old' knowledge in new situations. This difficulty is likely to be exasperated if students have not fully mastered the concepts in the first place as it is widely recognised that insufficient original learning of mathematical concepts can prevent students transferring knowledge to new situations (Evans & Swan, 2014).

A wealth of research into how to enhance students' abilities to solve both routine and CUN tasks has indicated that metacognition plays a vital role in these processes. Metacognition can be defined as "thinking about and regulating thinking" and Mevarech & Kramarksi (2014:15) emphasise the role of metacognition as the "engine" that starts, regulates and evaluates the cognitive processes". They explain that metacognition is recognised as having two main components: "*knowledge of cognition* (declarative, procedural and conditional knowledge), and the more important *regulation of cognition* (planning, monitoring, control and reflection)" (p.35).

However, not all students are able to successfully use this tool in their learning processes and research shows that successfully demonstrating metacognitive skills is a barrier to some students. Desoete's (2007) research found that lower achievers and students with learning difficulties have deficits in monitoring and controlling their learning. Efklides et al. (1999) found that these students are likely to have difficulties in assessing their learning and in using metacognitive knowledge in solving complex problems. One study's conclusion which is of particular relevance to GCSE resit students is the study undertaken by Paris and Newman (1990) where it was found that having negative metacognitive experiences may lead lower achievers to abandon tasks without even trying to attempt them.

Confidence, motivation, and resilience

Hinton and Fischer (2010:119) make a statement all teachers are likely able to relate to based on their own classroom experience: "Particular components of the learning experience can usefully be labelled cognitive or emotional, but the distinction between the two is theoretical since they are integrated and inseparable in the brain". In other words, anxious students cannot learn effectively. And given that throughout their education students' success in mathematics will be measured by their ability to demonstrate the successful application of both cognitive and metacognitive skills to complete mathematical problems, it is not surprising that students who are not able to do so, or not able to do so as quickly as others, become discouraged, lose confidence, and may become so anxious they disengage with mathematics altogether. Unfortunately, this is the case for many students who find themselves in a GCSE mathematics resit classroom in our colleges and in other similar settings.

Research conducted by Johnston-Wilder et al. (2016), as well numerous studies cited in Mevarech & Kramarksi's (2014) research, found that reducing the amount of anxiety an individual may experience while engaging with mathematics, led to more positive attitudes towards mathematics and more sustained engagement with mathematics was noted. Johnston-Wilder and Lee (2010) developed the term "Mathematical Resilience" to describe a positive stance towards mathematics. These researchers characterise resilient learners as learners who develop approaches to mathematical learning which help them to overcome the affective barriers and setbacks that can be part of learning mathematics for many people.

Conclusion

The literature shows that an approach to teaching problem-solving that combines cognitive, metacognitive and motivational support is likely to be the most effective. When designing our research, we drew on the literature, but were mindful of the context in which our research was taking place. While our research was informed by the complex, higher level problem-solving strategies which many other researchers have focused on, we wanted to reframe these into more accessible language and processes which inexperienced problem-solvers would be more likely to be able to make use of.

Therefore, we decided to focus on exploring how students felt about these types of questions, what they were currently doing when faced with answering them, and whether we could design a teaching aid that would help them to better access these questions. We attempted to incorporate cognitive, metacognitive, and motivational aspects into the design of our teaching aid.

Methods

Research Design

Our action research consisted of three cycles. Cycle one was an exploratory cycle designed to gather both qualitative and quantitative information around the attitudes and perceptions of both students and teachers in relation to complex problem-solving measurement questions. Based on the findings from this cycle, as well as information from the literature, we designed the first version of the Problem-Solving Toolkit. Cycle two involved the testing of this toolkit to explore the impact it had on students' experiences of answering these types of questions. We then interpreted the data by identifying key themes, refined the toolkit based on findings from cycle two, and designed cycle three to test the refined version. Finally, we compared cycle three themes to those from cycle one and two and produced a list of recommendations for future research and use of the toolkit.

Data Collection and Analysis Methods

Each of the six teachers in the action research group chose one class to invite to be involved in the research. Involvement was optional and students were told they were able to opt out at any time. In order to comply with research ethics participants were provided with a written overview of the research and asked to sign if consenting to taking part. It was made clear to participants that they could remain anonymous if they so wished, and that even if they chose to put their names on research materials, their names would not be published in the final report. All student and teacher responses were stored safely.

Unfortunately, the pandemic did adversely affect our ability to gather data in the way that we planned. Staff illness and cover requirements affected the timing and scale of our data collection, and the release of the advance information meant we had less time to explore a range of mixed problem-solving measurement topics, given that it was prudent to take advantage of knowing specifically which measurement topics would be asked in each paper.

	Purpose	**Data Collection**
Cycle 1	Explore student and teacher perceptions and practices around problem-solving questions	13 GCSE teachers surveyed 69 GCSE students observed and surveyed
Cycle 2	Test the Problem-Solving Toolkit	86 GCSE students observed
Cycle 3	Test the revised Problem-Solving Toolkit	22 GCSE and FS students observed

Research Aim and Objectives

Our research aim was to design and test a teaching resource created to help students to access multi-mark problem-solving measurement questions.

Our research objectives were as follows:

- (RO1) To discover the nature of the attitudes and perceptions teachers and students had around the teaching and learning of multi-mark problem-solving measurement questions
- (RO2) To look into previous research and literature to identify the key components needed to successfully teach mathematical problem-solving skills to students and to use this information, as well as our own experience to develop a Problem-Solving Toolkit
- (R03) To test the usability of the toolkit and to gather data on whether students were able to better access the problem-solving questions with the toolkit than they were without it, and to recommend best practices when teaching problem solving based on these findings

Results and Discussion

Research Framework

As the purpose of our research project was to try to help students to access multi-mark measurement questions, and problem solving skills are required in order to do so, we designed our research framework around the skills which the literature had highlighted as being central to effective mathematical problem-solving, namely understanding of core mathematical concepts and making connections between these concepts, demonstrating metacognitive skills, and possessing the confidence and motivation to attempt and solve the question.

Our research explored the levels at which students were able to demonstrate these key skills before an intervention (cycle one), with an intervention (cycle two), and with a revised intervention (cycle three). We also wanted to gain insight into teachers' perceptions of the levels at which students might be able to demonstrate these key skills, as well as to understand more about teacher confidence around teaching problem solving.

We expected, based on our experiences in the classroom, that student confidence and motivation were likely to be the biggest barrier to effective problem-solving, so we made this the main focus of cycle one. We used our findings from cycle one to design an intervention tool to help students to access these types of questions. Based on the literature and our own experiences, we designed a tool to guide students through the problem-solving process. Based on our findings from cycle two we redesigned to the toolkit and tested its effectiveness in cycle three. Although our cycles did not repeat the exact same process, and did not necessarily involve the same students, we wanted to describe how students engaged with the toolkit and whether it helped them to access these questions. As the majority of our data was qualitative, we grouped our analysis into themes which corresponded with key problem-solving skills and kept these themes consistent across the three cycles.

Participants

Our student sample for cycle one consisted of 69 GCSE resit students across three colleges. These students were predominantly those who had achieved/been awarded a grade 3, with a smaller percentage having achieved/ been awarded a grade 2. Cycle one also had 13 teacher respondents. Cycle two consisted of 86 GCSE resit students. These groups were the same as cycle one however due to differences in attendance some students may have been present for one cycle and not cycle two. Unfortunately, we were not able to use GCSE exam students in cycle three due to the fact that this cycle needed to take place at a time when students were on a very targeted revision programme.

Therefore, cycle three consisted of 14 students who were on a GCSE programme but were exempted from taking the GCSE exam, and eight Functional Skills students who had passed Entry 3 and were working on level 1 work. Most of these students typically had high learning needs, and many had extreme exam anxiety. On average the GCSE students were working at a grade 1 to 2 level.

Perceptions, mindsets, and motivation

Given that the aim of the first cycle of the research was to gain a better understanding of the challenge we were researching, we designed a questionnaire to gather data on both teachers' and students' attitudes and perceptions around multi-mark problem-solving questions.

Of the 13 teacher respondents across three colleges all 13 agreed that GCSE resit students generally encounter difficulty with these types of questions. Teacher responses to what difficulties students might experience when answering these types of questions were grouped, and the most commonly mentioned can be seen in the following chart:

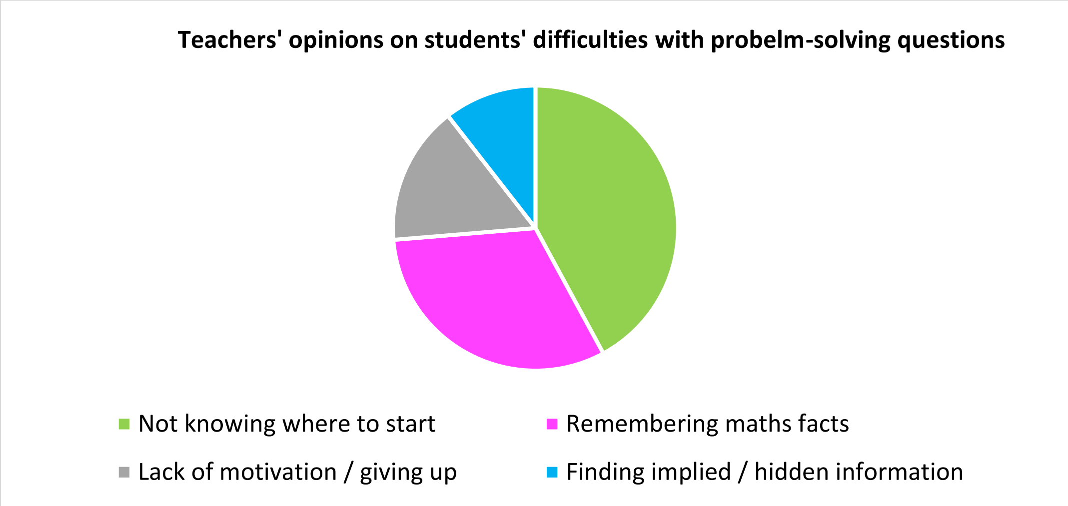


Figure 1: Graph to show the most commonly identified barriers which teachers believe students face when answering problem-solving questions

Almost all (ten out of 13) teachers said that they found it difficult to teach problem-solving skills. Given that problem-solving is a complex and multi-faceted area, the fact that so many of the teachers found it challenging to teach is not surprising, but rather supports our belief that this in an important area which requires further research and focus.

When shown a typical measurement problem-solving question and asked whether they would attempt a question like it in a test or exam situation, and how they felt about this type of question, student responses were as follows:

Likelihood of answering question in test/exam			**Feelings when seeing this type of question**	
Would not attempt it	22%		Anxious	57%
Might attempt it	54%		Neutral	32%
Would attempt it	24%		Fine	11%

Figure 2: Table to show student responses when asked to reflect on their perceptions of a problem-solving question

Cycle two saw students revisiting these types of questions, but this time with the Problem-Solving Toolkit as a resource. We designed the toolkit based on findings from Cycle one. We were encouraged by the fact that 78% of students said they would or might attempt these problem-solving questions in a test or exam situation. However, student responses to the Cycle one task showed us that while more students than we had expected were willing to attempt these questions, the vast majority did not attempt them, indicating that they did not do so because they did not have the skills or knowledge to do so.

The toolkit was trialled by five teachers with a sample of 86 students across the three colleges. After students had seen the use of the Problem-Solving Toolkit modelled by their teacher with the aid of a Powerpoint presentation, they attempted to use it to solve a set of past paper exam questions. It was hoped that at the end of the lesson students would be able to answer a particularly challenging exam question independently.

Teacher reflection data indicated that the response to the toolkit was on the whole favourable, with almost all students engaging with it positively. Teachers found that students generally liked having a step-by-step guide, responded well to the design and format of the toolkit, and were very willing to try to follow the steps. Most students were engaged in discussions if working with a partner, and students regularly called their teacher over to check if their work was correct or to clarify how to apply the steps in the toolkit to a particular question.

The exempt GCSE students and the Functional Skills students who formed the sample for cycle three engaged even more positively than the GCSE students in cycle two. They fed back that they liked the layout of the toolkit, and they enjoyed the game-like structure of the “which measure?” tool. These students were highly enthusiastic and motivated to use the toolkit to answer the problem-solving questions.

From a motivation and engagement perspective findings from all three cycles were very encouraging. However, students still had a range of difficulties using the toolkit to successfully answer problem-solving questions independently and this indicated difficulties with cognition, metacognition, or more likely both.

Metacognition

In cycle one, as well as being designed to gain insight into students’ cognitive processes when considering the multi-mark question, the second two questions were also included to gain insight into students’ metacognitive processes. Analysis of student responses showed that they were not able to describe their process well, and/ or did not have a range of problem-solving strategies to draw on.

When asked what they were trying to find out, an overwhelming majority of students stated the final step to the multi-step answer. This is understandable, they were not incorrect, and that may well be what many students thought we were asking. But when asked what steps they would follow, around half of the students in each set left the question blank.

As the aim of this question was to gauge presence of explicit metacognition, we looked for steps that could be categorised as a metacognitive process, or general problem-solving strategy. We tallied all the steps students listed (including blanks). For students who considered the ratio question “draw a triangle” was listed 12 times (out of a total of 60 steps). At GBMC five of the total 22 steps listed could be described as metacognitive, with “break the question down” being cited four times, and “highlight important information” listed once. While it may appear that these students showed promising beginnings of problem-solving strategies, the presence of these alone did not help the students, as none of those who mentioned them were able to gain further access to the question.

In cycle two the “Don’t Panic Prepare Tool” was designed to help students to use metacognition processes more overtly. As was previously discussed, the tool provided students with a problem-solving structure which teachers reported helped students to engage with problems many of them would not have engaged with otherwise. However, while students engaged with the metacognitive steps, and a great deal of discussion was had around these steps, difficulties with conceptual understanding and making connections meant that many students were not able to answer the questions independently or fully.

For many students listing the topics and trying to describe the bigger picture, or what was “going on” in the question did not significantly improve their ability to successfully and independently answer these types of questions. Observation analysis and analysis of student responses in both cycle one and two showed that students were not able to articulate what they were trying to find out.

In order to try and mitigate this issue in cycle three, we revised the “Don’t Panic – Prepare Tool”. We found that the first step (skim-get the big picture), which is a typical metacognitive step suggested for problem-solving, was much too vague for students. We incorrectly assumed that students were often not making the link between the term such as “cover” and the concept of area, or the term “fill” and the concept of volume. The data indicated that students were, on the whole, overwhelmed by the questions to the point where they were not able to apply even the most basic understanding of the concepts of perimeter, area, and volume in order to articulate what the question was about. One such example was in a question where students were given a drawing of a cylinder and asked to calculate the volume, observational data indicated that very few students seemed to be cognizant of the fact that they were dealing with the space inside a specific shape.

A revised step one required students to identify the measure, and the addition of the “Which Measure Tool” was included in order to help them to do so via a set of metacognitive questions. The changes to the toolkit in cycle three appeared to provide an entry point to guide students’ metacognition when faced with a complex measurement question. By following a clear pathway, students were able to cut out a lot of the “noise” and see more clearly about what the question was about. This provided them with much more effective access to the question and allowed them to consult the formula sheet to find the required formula.

While data from all three cycles indicated that students were motivated to attempt these questions, and data from cycle three indicated that revisions to the toolkit provided opportunities for students to access metacognitive strategies which would help them to answer these types of questions, students were still not able to answer the questions fully or independently. In order to understand the nature of the difficulties it was necessary to explore the role of cognition, in particular students’ abilities to apply conceptual understanding and to make links to identify the less obvious topics in the questions.

Conceptual understanding and making connections between concepts

As part of the cycle one task students were given a problem-solving question and asked to answer questions designed to explore their conceptual understanding and ability to make connections between topics and concepts. Two different problem-solving questions were used because being different colleges, we were at different points in our scheme of work. Although all students were resit students, and should have covered all the GCSE topics before, we wanted to use questions which related to work which we ourselves had covered with students.

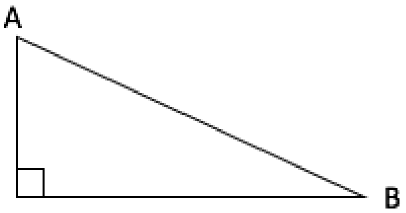
We wanted to reduce the chances of subject knowledge preventing students from accessing any of the questions as we felt this would demotivate them if they felt they were being given work which they had not yet been taught by us. Students at Greater Brighton Metropolitan College were given a question which linked to algebra, and students at Crawley and Chichester College were given a question which linked to ratio. The questions can be seen below.

In a **right-angled** triangle

smallest angle : largest angle = 2 : 5

Work out the three angles of the triangle.

Figure 3a: Cycle 1 question used by Crawley and Chichester College students



In the above right-angled triangle, angle A is 30 degrees less than twice the size of angle B. What is the size of angle A?

Figure 3b: Cycle 1 question used by Greater Brighton Metropolitan College students

When asked to identify which topics/parts of maths were in the question our data indicated that although the majority of students were able to identify some surface level information, most did not identify all the topics/parts of maths that were given in the question, and the majority were unable to make the necessary links to "hidden" topics which would be needed to answer the questions successfully. Student responses are presented in the graphs below.

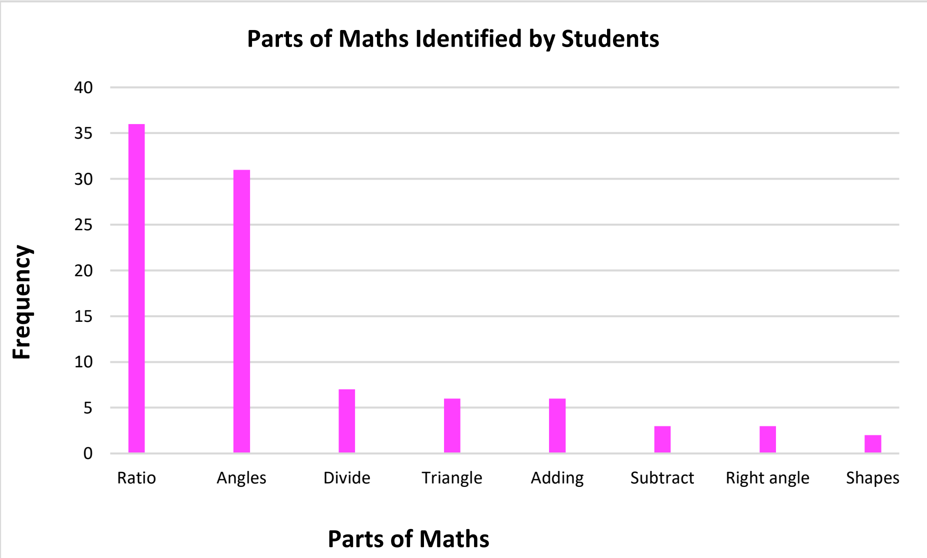


Figure 4a: Graph to show frequency of parts of maths identified by students at Crawley and Chichester College

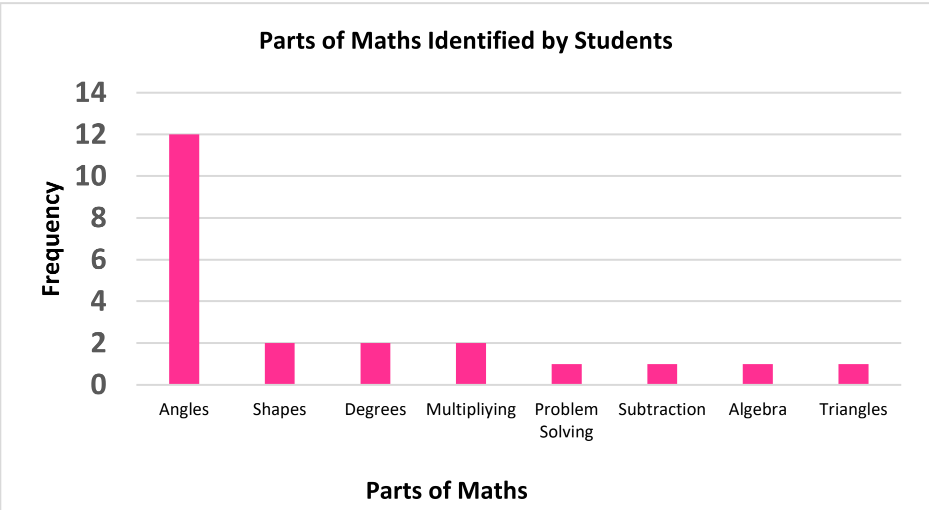


Figure 4b: Graph to show parts of maths identified by students at Greater Brighton Metropolitan College

As can be seen in the above graphs, students at Chichester and Crawley College were able to identify the most obvious topics - ratio and angles. However, two key topics which were both fairly clear from the question were triangles, which was only mentioned six times, and right angle, which was only mentioned three times. Many students at GBMC were able to identify were able to identify angles, the most obvious topic. Concerningly triangle was only mentioned once.

As is typical with problem-solving questions, both questions contained “hidden” information and required students to make links between topics. The ratio question required students to infer that the largest angle in the triangle was the 90-degree angle. When asked what information they knew after reading the question no students explicitly mentioned this fact. Both questions required students to make use of the theorem stating that the sum of the three angles in a triangle is 180 degrees, and this was not mentioned by any students.

GBMC students’ question required students to make the link between measurement and algebra. Forming an equation, substituting into the equation, and solving the equation is a process which is needed to solve a large proportion of the GCSE exam questions and was required to successfully answer the question. Only one student at GBMC was able to make the link between measurement and algebra.

In cycle two the “Don’t Panic, Prepare Tool” was designed to help students to make connections between topics by requiring that they explicitly list the topics/parts of maths (step two) in each question. Teacher reflection data indicated that, as in cycle one, students found this challenging, and were still unable to identify most of the information needed to answer the questions without help from a classmate or their teacher. When attempting a question about covering a cylindrical tank in paint, only two out of the 12 students who attempted this question identified “surface area” and “cylinder”, despite the fact that the word cylinder was in the question and there was a diagram of the tank.

However, observation data and analysis of students’ answers to the questions showed that most students were able to access step three (label-add info to the diagram) independently, and this did help many students to access step four, inferring or adding “hidden” information. One such example can be seen below. By adding in the 12cm the student was able to deduce that each of the radii would be 6cm and was then able to substitute the value into the formula for area.

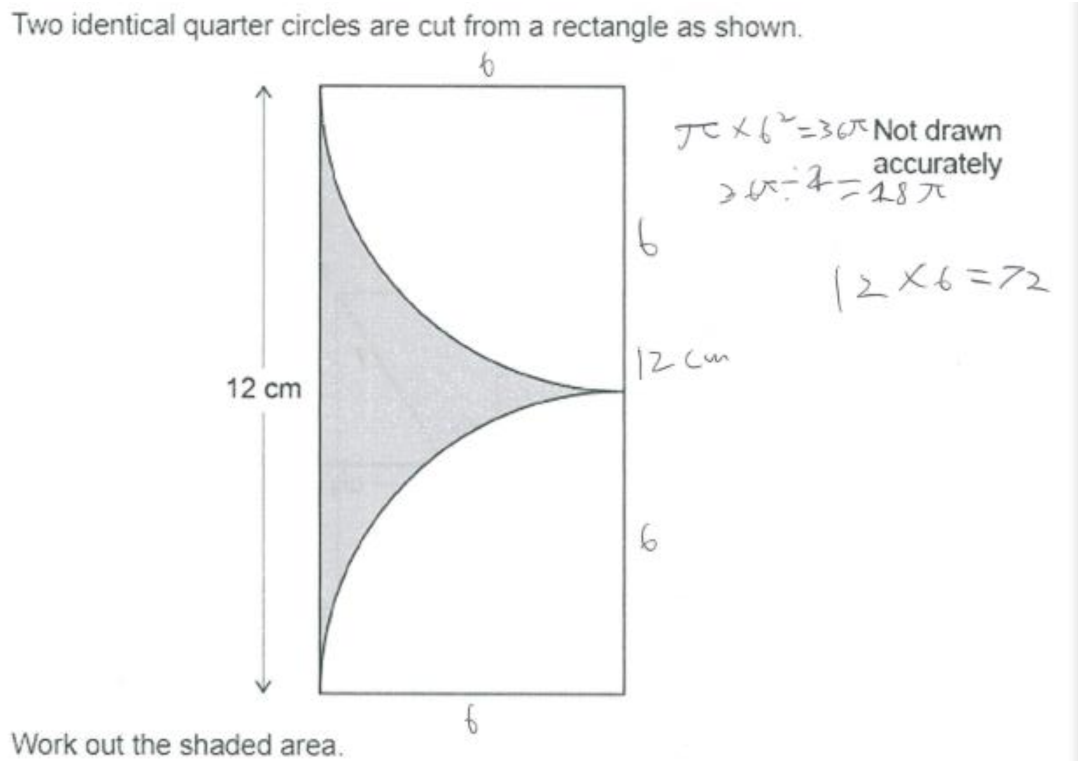


Figure 5: Chichester College student's working demonstrating successful application of step 3 and 4.

Given the small sample size in cycle three, and the fact that these students were different to those in cycle two, it is difficult to compare whether the revised toolkit helped students to make improve conceptual understanding or make connections between topics. While the redesigned thetoolkit gave students a more accessible starting point, some of the Functional Skills students struggled to substitute into the more complicated formulae correctly. This was not surprising as these students were working at Level 1 and were not experienced in this skill. We took the fact that students were able to identify the measure and write down the formula correctly as very positive progress. However, in order to fully assess the revised toolkit's usefulness in helping overcome difficulties relating to conceptual understanding and making connections, it would need to be trialed with students who had undergone the same introduction to the toolkit as the students who took part in cycle one and two.

Conclusions and Recommendations

Conclusions

The researchers and teacher respondents initially believed that motivation was the main barrier to students successfully answering measurement problem-solving questions. It would be reasonable to assume that students who do not make attempts on these questions or refuse to engage with them at all lack the motivation to do so. However, we found that the students who took part in this research were, on the whole, very motivated to tackle these types of questions. We concluded that it was not motivation that was students' main barrier, but rather a lack of problem-solving experience and low levels of resilience.

The main aim of our research was to help students to engage with these types questions rather than to leave them blank. Our research showed that the problem-solving toolkit did provide a means for students to access these questions. In cycle two most students were able to add information to the diagram and elicit further information as a result of this process. In cycle three students were able to correctly identify the measure and make an attempt on finding information to substitute into the measure's formula. And while this is very promising as it shows significant progress from cycle one, before any form of intervention, our data indicated that students displayed poor conceptual understanding, as well as difficulties making connections between concepts. However, teacher observation discovered that when students were given some assistance by their teachers, it was clear that they had some content knowledge about the concepts, and they could see the links between topics, but they were unable to do so independently.

It is understandable that teachers may conclude that students' mathematical understanding is poor when students are given a problem which involves a right-angled triangle and asked to list the parts of maths and the information they know from the problem and "right angle" and "90 degrees" are not mentioned. But our research showed that students understood what a right-angled triangle was, but often did not transfer this knowledge to the question. Similarly, most students knew what a cylinder was, and most knew what volume was, but they did not recognise that applying this knowledge was a fundamental step in the problem-solving process. Students, on the whole, did possess enough content knowledge, but were not able to elicit these maths facts and successfully apply them in a problem-solving strategy. We conclude that it is a lack of experience and resilience, rather than a lack of motivation and subject knowledge, that was the biggest barrier to students successfully answering these questions.

Recommendations

- Focus less on facts, and more on conceptual understanding and making connections

  While it can be tempting to try to help and support less confident students by spending class time reteaching topics from the beginning, our research showed that for most students the main issue was not a lack of maths content knowledge but rather in knowing which knowledge to draw on and apply.

- Curriculum design

  Many students find it difficult to make links between topics and this should be taken into consideration when structuring how topics will be grouped and arranged on the GCSE or Functional Skills course. However, as both the GCSE and Functional Skills curricula contain a large number of topics, and multi-mark questions often require students to link multiple topics together, it is not feasible to cover all of the possible topic combinations. Instead, making the fact that topics will be linked very explicit, and designing teaching materials to reflect this, could allow students to be more accustomed to the process of linking topics, even if they do not always make the correct links.

- Metacognition as a problem-solving strategy

  Many problem-solving teaching resources focus on the importance of metacognition as a step in the problem-solving process. While metacognition is important, it should be a focus in all lessons, not as a standalone problem-solving strategy for multi-mark questions. Students need to get to the point where they use this skill naturally and if this is not a central part of teaching and learning it is unlikely students will be able to demonstrate the skill when faced with questions which require so many other skills to be used simultaneously.

- Using the Problem-Solving Toolkit

  We would highly recommend the use of the toolkit, or something similar. Confidence is key to building resilience, and blank pages do not inspire self-confidence. The toolkit provided students with a starting point to access these types of questions and broke the process down into manageable steps. However, in order for it to be effective we would strongly recommend it be used regularly, and from the beginning of the academic year. The steps in the toolkit need to become second nature for students, and in order for this to be possible they need to become very familiar and confident with applying these skills. For the purpose of this research we focused on measurement questions, but toolkits could be created for a range of different topics.

References

Belgutay, J. (2017). Most disadvantaged students fall behind peers in resit success. TES [online]. Available at: https://www.tes.com/news/mostdisadvantaged-students-fall-behind-peers-resit-success

Belgutay, J. (2018). GCSE Resits: Students forced to sit exams 9 times. TES [online]. Available at: https://www.tes.com/news/gcse-resits-students-forcedsit-exams-9-times

Dalby, D. & Noyes, A. (2020). Mathematics in Further Education Colleges [online]. Available at: https://www.nottingham.ac.uk/research/groups/crme/documents/mifec/final-report.pdf

Desoete, A. (2007). Evaluating and improving the mathematics teaching-learning process through metacognition, Electronic Journal of Research in Educational Psychology, 5(3), pp. 705-730

Education and Skills Funding Agency. (2019). 16 to 19 funding: maths and English condition of funding [online]. Available at: https://www.gov.uk/guidance/16-to-19- funding-maths-and-english-condition-of-funding

Efklides, A., Samara, A, and Petropoulou, M. (1999), Feeling of difficulty: An aspect of monitoring that influences control, European Journal of Psychology of Education, Vol. 14(4), pp. 461-476

Evans, S. (2017). Making links between solutions to an unstructured problem: The role of designed student responses' In: Handbook of Research on Driving STEM Learning with Educational Technologies. IGI Global.

Evans, S., & Swan, M. (2014). Developing students' strategies for problem solving in mathematics: the role of pre-designed "Sample Student Work". Education Designer, 7.

Gick, M. L., & Holyoak, K. J. (1983). Schema Induction and Analogical Transfer. Cognitive Psychology, 15, pp. 1-38.

Heckman, J. (2008) Schools, Skills, and Synapses, Economic Inquiry 46(3), pp. 289-324,

Hinton, C. & Fischer, K.W. (2010). Learning from the developmental and biological perspective, in H. Dumont, D. Istance, and F. Benavides (eds), The Nature of Learning: Using Research to Inspire Practice, Educational Research and Innovation, OECD Publishing, Paris

Higton et al. (2017). Effective practice in the delivery and teaching of English and Mathematics to 16-18 year olds: case studies. DFE Publication [online]. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/662470/English_and_Mathematics_to_16-18_year_olds.pdf

Ireland. J. (2019). Studying English and Mathematics at Level 2 Post-16: Issues and Challenges, Research Matters, 28 (Autumn), pp. 26-33

Johnston-Wilder, S. and Lee, C. (2010). Mathematical Resilience. Mathematics Teaching, 218, 38–41.

Johnston-Wilder et al. (2015). Developing Mathematical Resilience in School Students who have experienced repeated failure. In: Proceedings of 8th International Conference of Education, Research and Innovation, Seville, Spain.

Johnston-Wilder et al. (2016). Developing teaching for mathematical resilience in further education. In: 9th International Conference of Education, Research and Innovation, ICERI2016, Seville (SPAIN), 14-16 of Nov 2016. Published in: ICERI2016 Proceedings pp. 3019-3028

MEI. (2021). Mathematical Problem Solving – A Guide for Teachers [online]. Available at: https://mei.org.uk/resource/5be10b4d-a7c3-4377-630d-08d952a14600/

Mevarech, Z. and B. Kramarski (2014). Critical Maths for Innovative Societies: The Role of Metacognitive Pedagogies, OECD Publishing [online]. Available at: http://dx.doi.org/10.1787/9789264223561-en

OECD. (2016a). Skills matter: further results from the survey of adult skills. [online]. Available at: https://www.oecd.org/skills/piaac/Skills_Matter_Further_Results_from_the_Survey_of_Adult_Skills.pdf

Paris, S.G. & Newman, R.S. (1990), Developmental aspects of self-regulated learning, Educational Psychologist, 25(1), pp. 87-102

Smith, A. (2017). Report of Professor Sir Adrian Smith's Review of Post 16 Mathematics [online]. Available at: https://www.gov.uk/government/publications/smith-review-of-post-16-maths-report-and-government-response

Swan, M. (2008). A Designer Speaks: Malcolm Swan. Designing a Multiple Representation Learning Experience in Secondary Algebra. Journal of the International Society for the Design and Development of Education

Swan, M. (2014). Designing tasks and lessons that develop conceptual understanding, strategic competence, and critical awareness. Tarefas matemáticas: Livro de Atas do Encontro de Investigação em Educação Matemática, pp. 9-28.

Vidal Rodeiro, C. (2018). Which students benefit from retaking mathematics and English GCSEs post-16? Research Matters: A Cambridge Assessment Publication, 25, pp. 20–28.

Wallace, A. (2014). When you're smiling: exploring how teachers motivate and engage learners in the further education sector. Journal of Further and Higher Education, 38(3), pp.346-360

Wolf, A. (2011). Review of Vocational Education – The Wolf Report [online]. Available at: https://www.gov.uk/government/publications/review-of-vocational-education-the-wolf-report

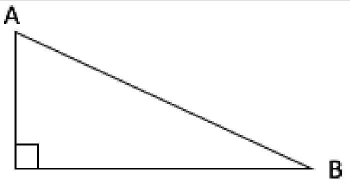
Appendices

Appendix 1: Questions used in cycle 1 student activity

In a **right-angled** triangle

smallest angle : largest angle = 2 : 5

Work out the three angles of the triangle.



In the above right-angled triangle, angle A is 30 degrees less than twice the size of angle B.
What is the size of angle A?

Appendix 2: Student questionnaire - cycle 1

GCSE MATHS PROBLEM SOLVING QUESTIONS STUDENT SURVEY Individual Self Reflection

Please answer the following questions about the maths problem below:

1. Please tick the response which best matches how likely are you to attempt to answer this question in a test or exam

	I would not attempt it
	I might attempt it
	I would attempt it

2. Please tick the response which best matches how you feel when you see these types of questions in a test or exam?

	I feel anxious
	I don't feel anything
	I feel fine

Appendix 3: Student activity - cycle 1

GCSE MATHS PROBLEM SOLVING QUESTIONS STUDENT SURVEY
Paired Discussion Task

In your pairs please consider the following maths problem and answer the questions which follow.

1. What parts of maths / maths topics do you think are in this question?

2. What information do you know from reading the question?

3. In your own words, what are you trying to find out?

4. How will you find this out? What steps will you follow?

Appendix 3: Teacher questionnaire - cycle 1

GCSE MATHS PROBLEM SOLVING QUESTIONS TEACHER SURVEY

1. In your experience do GCSE resit students generally encounter difficulties in answering problem-solving multi-step questions in tests and exams?

Yes	No

2. If you answered yes to question 1, what difficulties might students experience in answering questions such as this exam question?

3. Do you generally find teaching students to successfully answer these types of questions is challenging?

Yes	No

4. If you answered yes to question 3 please explain the types of challenges you have faced in teaching students to successfully answer these types of questions?

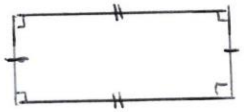
5. What teaching strategies might you use to support a student answering this type of question?

Appendix 4: Problem-Solving Toolkit Version 1 – Facts and Formulae Tool

FACTS AND FORMULAE TOOL
-MEASUREMENT-

SHAPE FACTS

RECTANGLES:



width / height

length / base

- Opposite sides are equal
- sum of angles = 360°
- Perimeter = add all sides
- Area = length × width / base × height
- Surface area = add all areas of each of the 6 surfaces

3D:

- Volume = area rectangle × Height = l × w × h

TRIANGLES

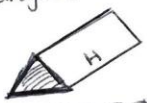
Equilateral = 3 equal sides and angles

isosceles = 2 equal sides base angles equal

Scalene = no equal sides / angles

- Sum of angles = 180°
- Perimeter = add all sides
- Area = $\frac{\text{base} \times \text{height}}{2}$

3D:



- Surface area = add all areas of 5 surfaces
- Volume = area triangle × Height = $\frac{b \times h}{2} \times H$

DEFINITIONS

- Perimeter = Distance around edge of 2-D shape = Add all sides
- Area = Space inside a 2-D shape = multiply using shape's formula
- Volume = space inside a 3-D Shape = area of cross-section × Height of shape

CONVERTING UNITS

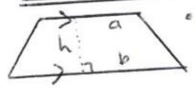
$1cm^3 = 1ml$

km $\xrightarrow{\times 1000}$ m $\xrightarrow{\times 100}$ cm $\xrightarrow{\times 10}$ mm

mm $\xrightarrow{\div 10}$ cm $\xrightarrow{\div 100}$ m $\xrightarrow{\div 1000}$ km

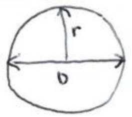
kilo, centi, milli

TRAPEZIA:



- one pair parallel sides
- Area = $\frac{a+b}{2} \times h$

CIRCLES



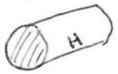
D = Diameter

r = radius

r + r = D

- Perimeter (circumference) = $2 \times \pi \times r$ or $\pi \times D$
- Area = [redacted] $\pi \times r^2$

3D:

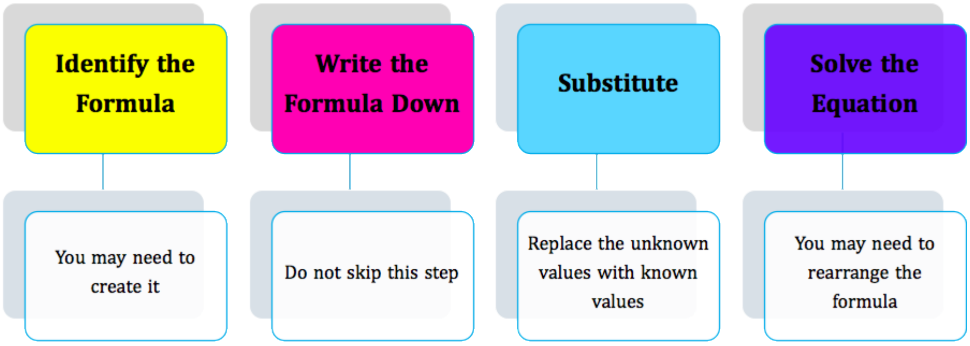


O + O + □ ← cylinder

- Surface area = add areas of 3 surfaces = $2\pi r^2 + 2\pi r \times H$
- Volume = area circle × Height = $\pi r^2 \times H$

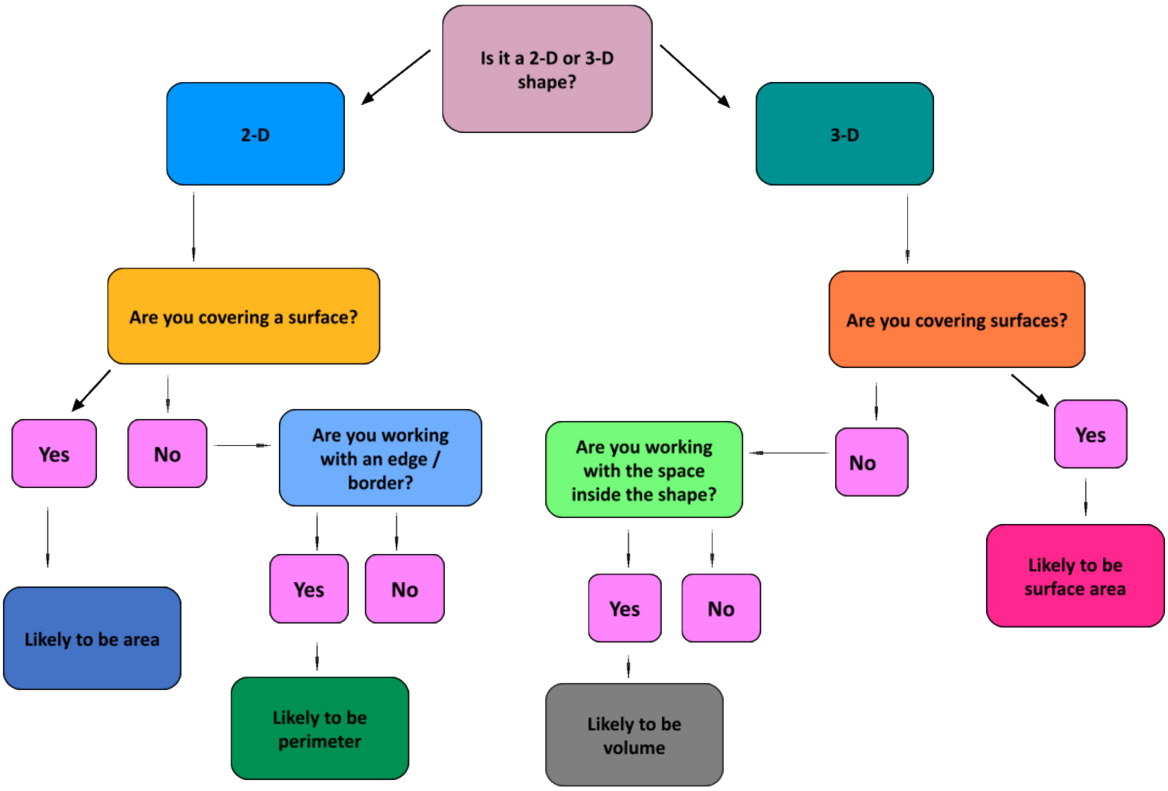
Appendix 5: Problem-Solving Toolkit Version 1 – The Formula for a Formula Tool

THE FORMULA FOR A FORMULA TOOL



Appendix 6: Problem-Solving Toolkit Version 1 – The Don't Panic- Prepare Tool

Appendix 7: Problem-Solving Toolkit Version 2 – The Choose the Measure Tool



Appendix 8: Problem-Solving Toolkit Version 2 – The Don't Panic- Prepare Tool