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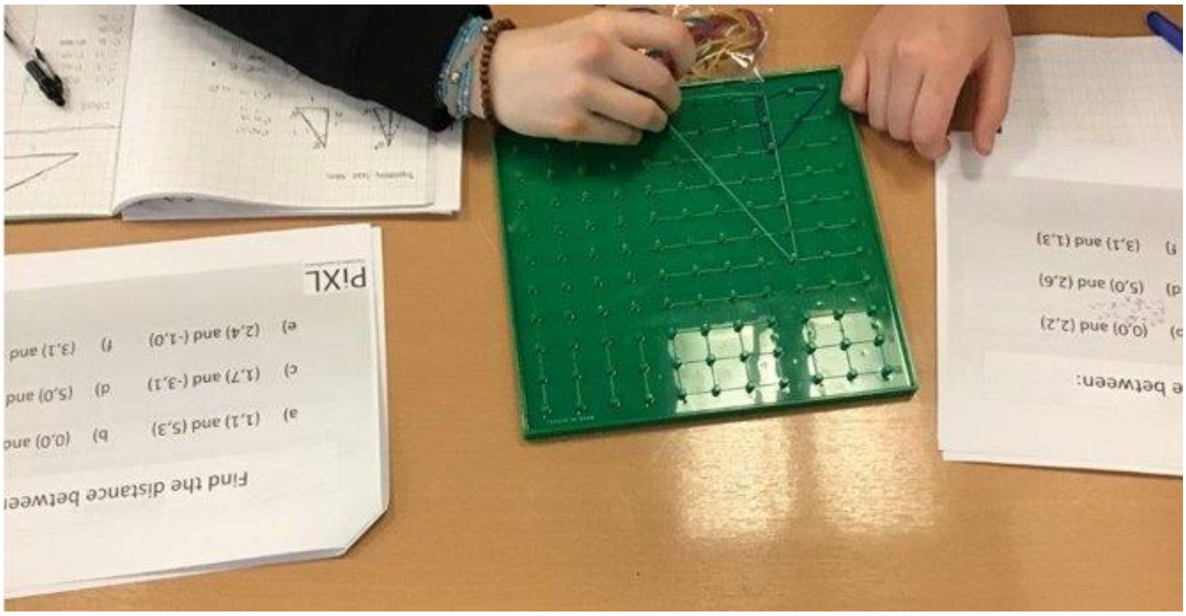

Using manipulatives to boost student engagement in post-16 GCSE mathematics: Professional challenges and student perceptions

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About CfEM

Centres for Excellence in Maths (CfEM) is a five-year national improvement programme aimed at delivering sustained improvements in maths outcomes for 16–19-year-olds, up to Level 2, in post-16 settings.

Funded by the Department for Education and delivered by the Education and Training Foundation, the programme is exploring what works for teachers and students, embedding related CPD and good practice, and building networks of maths professionals in colleges.

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Summary

Manipulative resources are often thought as only appropriate for younger students however there has been a vast amount of attention on their ability to support with learning mathematics and for their role in helping students to move between concrete, pictoral and abstract ways of working. This research looks into whether manipulative resources are appropriate for post-16 GCSE maths learners and whether they can have a positive impact on the engagement of an often-disenfranchised cohort. Two series of lessons were co-planned and trialled by four teachers with ten classes using first algebra tiles and then geoboards. Both qualitative and quantitative data was collected from both teachers and students in three research cycles including surveys, interviews and class discussions. Manipulative resources are often more praised than used and lack of teacher skill and confidence can create barriers to the deployment of these resources in classrooms. Barriers and ways to overcome these barriers are identified. Overall post-16 GCSE maths students from four colleges responded well in terms of their engagement to using algebra tiles and geoboards, preferring geoboards. Reasons for this positive response include enjoying the physical and tactile nature of the resource, feeling more able to concentrate and becoming familiar with using resources over a series of lessons. Open ended tasks were also shown to support student engagement. Some students with existing working methods or those who could not access activities were reluctant to use the manipulatives. Bridging activities and low threshold starting activities with possibilities for extension are discussed as ways to boost student engagement further.

Contents

Background

Introduction

Chirst the King (CTK) Sixth Forms are a hub for the Centres for Excellence in Mathematics (CfEM) project. This action research project was lead from CTK: Emmanuel campus in collaboration with teachers from our partner colleges. The action research was supported with action research training materials from CfEM. It builds on a projects which have run over the past two years looking into the use of virtual manipulative resources and fraction bars and how the concrete-pictoral-abstract model can be applied to enhancing student conceptual understanding.

Outcomes for post-16 GCSE Maths “re-sit” students are worryingly poor with a national pass rate stubbornly sitting below 25% each year. Students in this cohort are among those referred to as “struggling”, and who have “fallen behind” their peers (OFSTED, 2021). It is therefore correct that attention is given to this diverse (ranging from GCSE Grade U to upper Grade 3) cohort to improve their educational outcomes and close the attainment gap.

Proposals to restrict student loans to university applicants without a GCSE grade 4 in maths and English add to the urgency of improving the accessibility of mathematics to our students. Our colleges are based in the South-east of England, both in and out of London, and cater for a wide demographic of post-16 and adult learners. Many of our students are already at an educational disadvantage due to contemporary social and economic issues, compounding their chances of achieving a grade 4 in GCSE maths. We believe our students deserve the best and we believe they can achieve. This research is directed to improving our understanding of how to facilitate this.

Research Aim

Our main research aim was to see if we could boost student engagement using physical manipulatives in our lessons. As the research project developed it became clear that a prerequisite for this was the confidence, skill and willingness of the teacher in using these resources as well as crucially what pedagogic actions these manipulatives afforded us.

Engagement here is defined to mean: *active attention to the mathematics at hand and concentration on a mathematical task which may be set by the teacher or generated by the student*. Our premise is that engagement is vital to learning, enjoying, and making progress in mathematics, as well as to the completion of specific mathematical tasks and problems. We also feel engagement is a particularly important focus with our post-16 “re-sit” learners as so many present as “disengaged” from maths for a variety of reasons.

Physical manipulatives are objects we can use in the classroom which have the potential to make concrete some abstract mathematical concepts and/or aid as problem solving tools.

Literature Review

"I hear and I forget. I see and I remember. I do and I understand."
Confucious 551-479 BC

It has been submitted that manipulatives, by virtue of the fact that they appeal to multiple senses and can be reordered, make abstract concepts more accessible to learners (Heddens, 1997). The usage of manipulatives in classrooms, especially in the younger years, have long been recommended by educators (The National Council of Teachers of Mathematics, 1989). They also seem to have found a relatively happy home in some currently popular ideas in Primary and Secondary schools, for example: mastery approaches; “Concrete-Pictoral-Abstract” frameworks; and Singapore method. It is possible then that they have a place in improving post-16 GCSE maths education despite some perceptions of our students being too old to engage with them. We had to dig quite deep to find research and resources specific to our cohort which could guide us on specifics of which manipulatives might work with our students, or how best to use them.

In this literature review we will: give a summary of what we mean by manipulative, what is available “on-the-market", and what we might look for in a good manipulative; describe some research findings on the effectiveness of manipulatives for improving students educational experience and outcomes and which cohorts have been researched; draw upon literature which addresses student and teacher perceptions of using manipulatives; and discuss pedagogical considerations when deploying them in classrooms.

Overview of available resources

“A mathematical manipulative is defined as any material or object from the real world that children move around to show a mathematics concept.” (Scheweyer, 2000). The very first manipulative that most children encounter is their fingers. Manipulatives are physical objects that can be used as representations or models of mathematical concepts to develop understanding in the user, allowing them to solve problems and gain access to abstract ways of thinking previously unavailable.

Modern examples include Dienes (base-ten) blocks, algebra tiles, Unifix Cubes, Cuisenaire rods, number lines, fraction pieces, pattern blocks, Numicon, and geometric solids. However, manipulatives have been around for a while: Plato refers to Egyptians using manipulatives with their student would-be scribes. The history of manipulatives back to ancient time using by various civilisations including the Asian, Ancient Romans, Chinese and Mayans. “Since the 1900s, manipulatives have come to be considered essential in teaching mathematics at the elementary school level” (*Benefits of Manipulatives*, 2016).

There is a plethora of manipulatives on the market that can be purchased to be utilised in the classroom. They are very varied, from simple objects like beads and cubes to more complex sets such as Deluxe Rainbow Fraction Squares or Geometric shapes. They can be designed for more procedural and specific tasks, for example Algebra Tiles, or more malleable to a range of investigations, exercises and concepts, for example Cuisenaire rods and Geoboards. (See APPENDIX 1 for a table of available manipulatives and their suggested topic areas). Teachers can also create their own resources with lollipop sticks, beans and/or beads, string, paper and more (IMP Attendees, 2018).

Manufacturers of these tools tend to make general and assured statements regarding the involvement of these tools in lesson and the positive impact towards a student's learning. For example, Hand2Mind.com states on their website:

> *"Manipulatives provide concrete ways for students to bring meaning to abstract mathematical ideas. They help students learn new concepts and relate new concepts to what they have already learned. They assist students with solving problems. When students explore with manipulatives, they have the opportunity to see mathematical relationships. They have tactile and visual models that help develop their understanding. Without these concrete references, students are too often lost in a morass of abstract symbols for which they have no concrete connection or comprehension."* (Hand2Mind.com, Accessed 2022)

These must be taken with a pinch of salt, for whilst the benefits of manipulatives have been corroborated by some research, as we will discuss in the following sections, their effectiveness also relies on how they are used by the student and the pedagogy the teacher employs (Ball, 1992; D. Clements, 2000).

Manipulative design

To be of use to a student, manipulatives must allow the user to extract the mathematical structure (Johnston-Wilder & Mason, 2004). The design of manipulatives is however contentious with Durmus & Karakirik (2006) recommending that a physical manipulative need to be "*simplistic [in] design"*, enabling easy manipulation. Laski, Jordan, Daoust, & Murray (2015) add to this that manipulatives should not have distracting or irrelevant features. However, seemingly in direct contrast, Mason & Watson (2019) note that *"materials that create some confusions to be resolved seemed to be more effective for learning than materials that present no problems"* (p. 23).

The majority of concrete manipulatives tend to be geared towards early years pupils, but there is no reason why manipulative tools cannot be used to engage in maths regardless of a student's age. A high level of mathematical sophistication can be achieved, for example with Geoboards, and certainly enough for GCSE mathematics (Clements, 2000; Faux, 2014; Hoggard & Ollerton, 2019).

Cohorts, learning styles and effectiveness in teaching mathematics

There is a body of research suggesting that students learn more by playing an active part in maths lessons and manipulatives, physical objects, playing a great role in supporting this. "In order to have opportunities to learn math, children need first-hand experiences related to math, interaction with other children and adults concerning these experiences and time to reflect on the experiences" (Seefeldt & Wasik, 2006, p. 250).

A meta-analysis of 60 studies into the use of manipulatives from kindergarten to college age students was conducted by Evelyn J. Sowell (1989) (Sowell, 1989):

> *"Results showed that mathematics achievement is increased through the long-term use of concrete instructional materials and that students' attitudes toward mathematics are improved when they have instruction with concrete materials provided by teachers knowledgeable about their use." (Abstract).*

Clements (2000) also provides a number of references to research showing students who regularly use manipulatives outperform those who do not. However, Clements also notes that in some cases the benefit of using manipulatives is very slight or non-existent. This was exemplified by Luke (2013) in his study of 20 adults with numeracy difficulties, 20 children with learning difficulties and 23 typically developing students - no one performed better whilst using manipulatives than not (Luke, 2013).

Differing responses from different groups of students

Research by Zeynel Kablan on 101 7th Grade students (12-13 years of age) showed that students with different "learning styles" may react to using manipulatives in different ways:

> *"Abstract learners showed higher academic performance compared with concrete learners in the environment where only traditional methods were used. For the other two environments, which utilised varying combinations of manipulative tools and traditional methods, the differences in the mathematics achievement levels among students of varying learning styles were not statistically significant. The study also showed that concrete learners demonstrated higher performance in mathematics when manipulatives were used than did their counterparts in the environment where only abstract activities were used; however, in the third learning environment, increasing the amount of manipulative use did not provide an extra benefit to concrete learners."* (pg. 277) (Kablan, 2016)

Luke's work also suggested different reactions between different groups of students, showing that children with learning difficulties performed significantly worse than any other group when working with "perceptually bland" manipulatives (Luke, 2013). Comparing Kablan's and Luke's findings shows that care must be taken when thinking about which groups of students to introduce manipulatives to and which particular manipulatives to choose.

Virtual manipulatives

Clements also found that physical manipulatives often have drawbacks which can be overcome by virtual manipulatives. In his work with groups of children (unspecified age) playing with pattern making and shapes he found the virtual manipulatives were preferable as they enabled the children to store and retrieve their work; were more precise and more flexible that physical manipulatives; they were unlimited in the number of shapes the children had to work with; and the children could record their work (via print outs) and therefore extend it more easily. However, in a study of 115 4th and 5th grade students in Finland in which six 45-minute lessons were taught with either concrete or virtual manipulatives, the concrete manipulative group outperformed the virtual manipulative group in fraction skills (Vessonen et al., 2021).

How and when do I use manipulatives & what do I use them for?

Teacher skill and confidence

Although a lot of research concludes that physical manipulatives are beneficial for teaching and learning mathematical concepts, Clements & McMillen (1996) proposed that using manipulatives does not always guarantee conceptual understanding. Teachers' knowledge, experience and the way manipulatives are structured into lessons are important to achieve these benefits (Clements, 2000; Back 2019; Sowell, 1989) as well as the length of time learners have been exposed to them (Hartshorn, 1990).

“Manipulatives - and the underlying notion that understanding comes through the fingertips - have become part of educational dogma: Using them helps students; not using them hinders students.” (pg. 17) (Ball, 1992). However, manipulatives have often been more praised than used (Johnston-Wilder & Mason, 2004). One reason why teachers may shy away from using manipulatives is that they are not confident in “the most effective and creative way to carry out such lessons” (Hurdle, 2020, pg. 36). Therefore, hands-on training for teachers is required to help them feel more confident and improve their efficacy in the classroom (Hurdle, 2020; Vizzi, 2016).

Appropriate use – just for fun?

Kath Hart, who promoted a research-based approach to the design of textbooks and tasks, often called into question the appropriateness of using manipulatives (Johnston-Wilder & Mason, 2004). Perhaps due to the perception of maths as an unpopular subject (Wolfram, 2014), there is a keen desire among teachers to find “fun” and “engaging” activities and it has been shown that teachers may view manipulatives as such (Moyer, 2001). Hart also showed that teachers often use manipulatives for “fun lessons” but through this subvert the value of them as aids to serious mathematical thinking (Hart, 1993). Moyer (2001) concluded that teachers may use manipulatives as a diversion in classrooms when they were not able to represent concepts themselves.

Teachers may choose to use manipulatives with certain groups of students and not others. Moyer (2001) showed teacher “*beliefs about how students learn mathematics may influence how and why they use manipulatives as they do*”. She found that some teachers had decided whether to use physical manipulatives in the classroom based on the behaviour of the group, with some teachers indicating they were concerned about maintaining ‘control’ of their groups.

Lucy Browne, a lecturer at Reading University, reflects on the use of manipulatives in mathematics lessons to see the differences between teachers’ perception of using manipulatives to that of a school pupil (Browne, 2018). Given her personal positive experience using manipulatives at the Institute of Mathematics Pedagogy she questions why older students may see manipulatives as childish. She used her experience to compare roles of manipulatives in Support, Enlightenment and Aesthetics while working on a mathematical task. She concluded that usually we use manipulatives to help students at the beginning stages of a concept and that may create an assumption about the roles of manipulatives as facilitators to help first grasp a concept or support for students who find a concept difficult to grasp. She proposes that perceiving manipulatives as merely support puts off the older pupils from using them, while having the opportunity to use manipulatives to gain enlightenment and enjoying the aesthetics of the objects may bring older learners more satisfaction and appeal.

Pedagogy and using manipulatives

As discussed above, it is important that teachers gain experience in choosing manipulatives and supporting students with their use, as well as understanding some of the theory behind this. Whilst we have already touched upon manipulatives being suitable for a large age and ability range, “the complexity of the materials provided will increase as children’s thinking and understanding of mathematical concepts increase” (Seefeldt & Wasik, 2006, p. 93).

“Concrete-Pictoral-Abstract”

There are many elements to be considered in planning and delivering lessons with manipulatives including: using them correctly; considering the purpose of using them; as well as the levels of learners. The presence of the manipulatives alone is not enough to convey mathematical meaning to the student. As Clements wrote in *'Concrete' Manipulatives, Concrete Ideas*, "although manipulatives have an important place in learning, their physicality does not carry the meaning of the mathematical idea." (Clements, 2000).

The argument for the use of physical manipulatives to access mathematical concepts appears to have been influenced by Piaget's study (1952) where he advocates the concrete, pictorial, abstract (CPA) approach in the delivery of mathematics to children. For example, algebra has been characterized as the most important "gatekeeper" in mathematics" (Cai, 2005). However, algebra is perceived to be one of the least accessible of topics in maths education by learners because it deals with values that can be simultaneously unknown and variable. Hence the potential introduction of algebra tiles to aid instruction, as an attempt to concretise algebra as a topic.

This idea of manipulatives as a concrete support in a one-way development towards abstraction, in Bruner's enactive-iconic-symbolic model or the CPA/CRA model, is recommended to aid the grasping of mathematical concepts which are "abstract mental constructs" (Gallo-Toong, 2020). Viewing this as their only place, as Browne (2018) argues against above, may have an unsound pedagogical basis: Mason and Watson (2019) promote caution with the CPA "mantra" which may lead teachers to only use manipulatives when in the "concrete" or beginning stages of a topic. In fact, mathematical structure can be extracted and developed by an iterative process between using the manipulative and working abstractly or symbolically (Laski et al., 2015). It is proposed that learning occurs specifically in the multidirectional movement *between* the stages, rather than a one-way directive process towards abstraction (Mason & Watson, 2019).

Pre-requisite conceptual knowledge

Choosing when and how to introduce manipulatives to a class requires some consideration. Firstly, as mentioned above, manipulatives on their own do not automatically teach students mathematical concepts (Ball, 1992; Clements & McMillen, 1996). In fact, students require a certain level of conceptual (all be it informal) understanding if they are to access what it is the manipulative is being used to teach – even though the manipulative is often considered a concrete representation. In 1964, John Holt (cited in Johnston-Wilder & Mason, 2004 and Clements & McMillan, 2000) showed that only students who already understood base and place value could effectively use blocks to solve problems. Clements & Samara write:

> *Manipulatives do not "carry" mathematical ideas. If kindergartners cannot use simple cubes to help them solve addition and subtraction problems, they likely have not learned a strategy to use the cubes to solve the problems. In this case, using a number line would be even more difficult. Without concepts and strategies for how to use manipulatives, manipulatives alone are no help.* (pg. 3) (Clements & Sarama, 2018)

Student fluency & familiarity

Secondly, students must become fluent and comfortable in using a manipulative so that they use it naturally and automatically as a problem-solving tool (Moyer, 2001). Durmus & Karakirik argue that students "*should be given an opportunity to play with manipulatives*" and that just a "*demonstration by a teacher is not sufficient to realize their full potential"* (pg. 4) (Durmus & Karakirik, 2006). "Play" may take a different form for older students and be more in the form of a mathematical exploration, for example finding all the triangles in a 9-point Geoboard (Ollerton, 2020).

How much didactic instruction to give students while using manipulatives is worth considering. Confusingly, the distinction between what may be regarded as concrete or abstract is not as clear cut as one would hope and maybe related to didactic instruction: Wheatly, writing in 1992, (cited in Johnston-Wilder & Mason, 2004) notes that to "show" a student a mathematical concept using manipulatives is still based on the abstract first concept of learning. Clements adds to this that concrete thinking is not facilitated alone by an object one can hold in the hand but by an "interconnected structure of knowledge" which must already be present In the learner (D. Clements, 2000).

Structure of activities within the classroom

Laski et al (2015) argue, from a Montessori perspective, that links between the manipulative and the mathematical concept should be clearly explained. Improving conceptual understanding requires students to link their action with manipulatives to describing the actions (Clements & McMillen, 1996). Hart (1993), in a study in with 8–13-year-olds, found that the process of formalisation through concrete experiences often failed and suggests that this failure of manipulatives to improve students conceptual understanding may be due to lack or ineffectiveness of "bridging activities" linking concrete and formalisation stages (Hart, 1993).

Exploratory and inductive work with manipulatives is recommended by Suydam & Higgins (1976), who conducted a meta-analysis of studies and give the following suggestions on appropriate use of manipulatives:

1. Manipulative materials should be used frequently in a total mathematics program in a way consistent with the goals of the program.
2. Manipulative materials should be used in conjunction with other aids, including pictures, diagrams, textbooks, films, and similar materials.
3. Manipulative materials should be used in ways appropriate to mathematics content, and mathematics content should be adjusted to capitalize on manipulative approaches.
4. Manipulative materials should be used in conjunction with exploratory and inductive approaches.
5. The simplest possible materials should be employed.
6. Manipulative materials should be used with programs that encourage results to be recorded symbolically

Manipulatives must be structured carefully into the lesson and learners given the opportunity of modelling their own processes based on their interpretation the ideas in questions (Back, 2019). Judith McCullouch conducted a study on the effect of giving different amounts of instructions to students using Meccano manipulatives (Mccullouch, 2016). Three groups of teacher trainers explored the differences in both the process of learning and the outcome by providing the same type of manipulative but in different structures. The following points are some of the findings highlighted by this study:

1. Learners are varied in the need of receiving (firm) instructions at different stages of the task, even at the beginning.
2. Providing less instructions gives participants more freedom and creativity in their own learning.
3. Learners use different methods to achieve the same result.
4. Receiving less instruction needs learners to do more of decision making, which is more challenging than following instructions, therefore the learners are more deeply engaged and will learn more.

Conclusion

What becomes apparent from the literature above is that although the potential benefits of using manipulatives may be tempting, the variability in possible outcomes both positive and negative must be guided by a teacher sensitive to both and willing to adapt to student reactions. On top of this the multitude of choices as to which groups to use manipulatives with, which manipulative to choose and how to deploy them within a classroom clearly requires a teacher experienced and confident not only with the manipulative but also a range of pedagogical approaches. Therefore, if manipulatives are to be used effectively to promote mathematical development, this will require training and will inevitably take time. It will also rely on a teacher's willingness to forgo some apprehensions about how classes may respond, or in fact notions that manipulatives are just there for fun!

Methods

We conducted three research cycles over the year all focused on student engagement, but with the latter two also trying to gauge teacher skills and confidence. Both AR cycle 1 and 2 included interventions in the form of teachers using manipulatives in their lessons, algebra tiles and geoboards respectively. None of the teachers had previously used these resources.

All students were taking GCSE maths, most were in the age range 16 – 19 years with one class of adult learners. Prior attainment in GCSE maths ranged from U to grade 3 and averaged just above a 2. Ancillary data about the classes is included in table Y.

Table Y: Details of classes involved in the intervention

Class	Teacher	Number of students on register	Class attendance	Number of students with special educational needs (SEN)	Average GCSE maths score from the start of the year [min - max]	Age range (years)	Interventions
1	A	19	76%	5	1.74 [U – 2]	16 - 19	Algebra tiles & geoboards
2	A	21	72%	3	1.76 [U – 2]	16 - 19	Algebra tiles & geoboards
3	B	10	86%	1	2.1 [1 – 3]	16 - 19	Algebra tiles
4	B	17	80%	1	2.18 [1 – 3]	16 - 19	Algebra tiles
5	C	19	72%	5	2.11 [1 – 3]	19 - 54	Algebra tiles
6	D	16	72%	4	2.3 [1 – 3]	16 - 19	Algebra tiles (mathsbot.com)
7	B	12	84%	2	2.2 [1 – 3]	16 - 19	Geoboards
8	B	15	81%	1	2.7 [1 – 3]	16 - 19	Geoboards
9	C	3	30%	1	1 [1 – 1]	16 - 19	Geoboards
10	D	24	71%	4	2.5 [1 – 3]	16 - 19	Geoboards

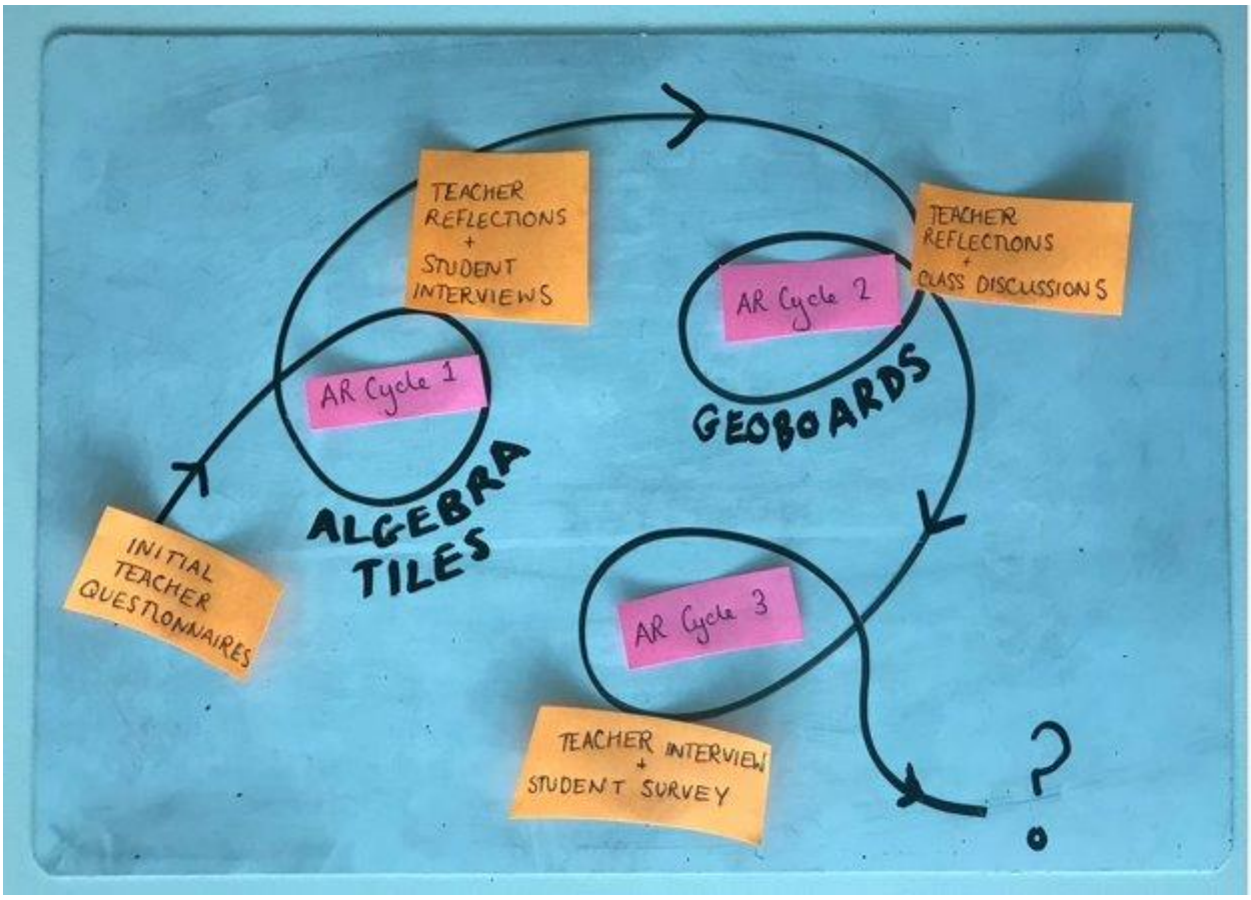
AR cycle 2 interventions were intended to be longer and more numerous to give students and teachers more time with the resources and cover a wider range of topics. Interventions were delivered separately by four teachers in four different colleges to one or two classes each. In AR cycle 3 we aimed to collect more detailed data on teachers and students and expand on emerging themes from cycle 1 and 2.

As a group beginning this research we were invested in the following values:

- Improving student attainment
- Improving student engagement
- Improving student confidence
- Making mathematics/the students feel that mathematics is more accessible
- Removing barriers to learning maths
- Eradicating phobias of learning maths
- Boosting student progress
- Developing conceptual understanding of difficult topics

We believe that student engagement is a key to all these values and so made that our focus. As the research developed it became clear that teacher skill and confidence as well as student perceptions of the manipulatives were important. Our objective therefore became to assess how well manipulatives promoted student engagement, how students perceived the use of the manipulatives as beneficial to their learning and how teachers found using manipulatives for the first time.

A variety of both quantitative and qualitative data was collected via teacher journals, student interviews, class discussions, student surveys and teacher interviews. Qualitative data was coded for themes. All student and teacher data has been anonymized and consent was gained before obtaining it. Data is stored on a secure shared folder for the research group. The group found that across all their colleges engagement and attendance in GCSE maths has been a challenge for students exacerbated by the disruption of COVID-19. The increased workload caused by this made it difficult for some teachers to fully participate in all interventions along with concern about covering departmental schemes of work.



Results and Discussion

Action Research Cycle 1

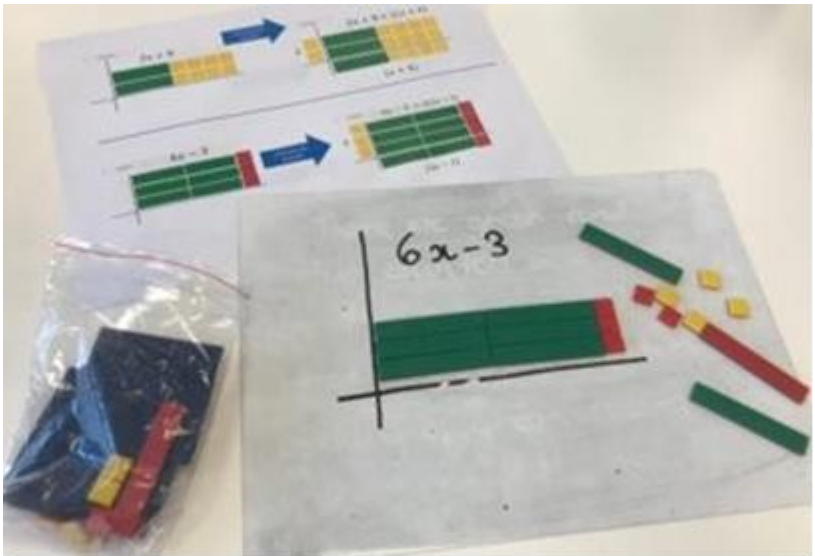
The aim of AR cycle 1 was broadly to understand where we are all at in terms of using manipulatives, what barriers there may be to using manipulatives in the classroom and to start to understand if manipulatives would have any effect on student engagement. Our research question was:

- *How do students respond, in terms of their engagement, to using algebra tiles in 3 short algebra activities?*

We designed three 20-30min activities to use with Algebra Tiles on directed numbers & linear equations; expanding and factorizing single brackets; and expanding and factorizing quadratics (see APPENDIX 2). Materials are a combination of our own and adaptions of some fantastic slides by *pbrucemarths* that we found on the TES website (https://www.tes.com/teaching-resource/algebra-tiles-an-introduction-to-12123410). Each teacher conducted one or more of the three activities with one or two groups (teacher A - 2 groups, 3 activities each; B - 2 groups, 2 activities each; C - 1 group,1 activity; D 2 groups, 1 activity).

Teachers first completed a short reflective questionnaire on their current experience of manipulatives (see APPENDIX 3). After each lesson teachers were encouraged to write a journal entry based on some reflective questions, although this was only collected for teacher A (see APPENDIX 4). Teachers also offered annotations on student interview responses retrospectively as well as feeding back to each other in a group meeting.

Two students were interviewed after each activity (see APPENDIX 5). Eleven interviews were conducted in total. The same students were interviewed each time and they were selected from the class by sub-setting each class for students with over 90% attendance followed by a random sample. Data was processed by coding for themes and reference to the literature. One class completed a survey (n = 11) on Desmos after all the interventions were complete (see APPENDIX 6).



Example of an activity with algebra tiles, AR Cycle 1.

Action Research Cycle 2

The research question for AR cycle 2 was deliberately multi-pronged. We were specifically interested in student engagement; however, we are also aware from the literature, and from feedback from AR Cycle 1 that teacher confidence and skill are important variables for the success of manipulatives within a classroom. Given that we were all new to using manipulatives we wanted to monitor our own confidence and how it may be affecting our classes:

- *Does using 121-pin Geoboards in lessons enhance or limit Post-16 GCSE "re-sit" student engagement when used across a variety of topics?*

- *How does teachers' confidence in using geoboards change over a series of lessons?*

- *Does teacher confidence and their use of open or closed tasks affect student engagement?*

In AR cycle 1 we felt students did not have enough time to get used to the algebra tiles so we designed activities on a range of topics including straight line graphs, Pythagoras' theorem, area and perimeter, and transformations so that students could trial the resources over several lessons whilst still covering breadth in the curriculum (see APPENDIX 7).

Manipulatives can offer a break from closed question and answer type activities, offering potential for exploration and investigation which can be formalized and extended as students are ready; however, they also offer potential as a problem-solving tool for students and a scaffold for specific questions and problems. Both open and closed activities were included in our lesson plans.

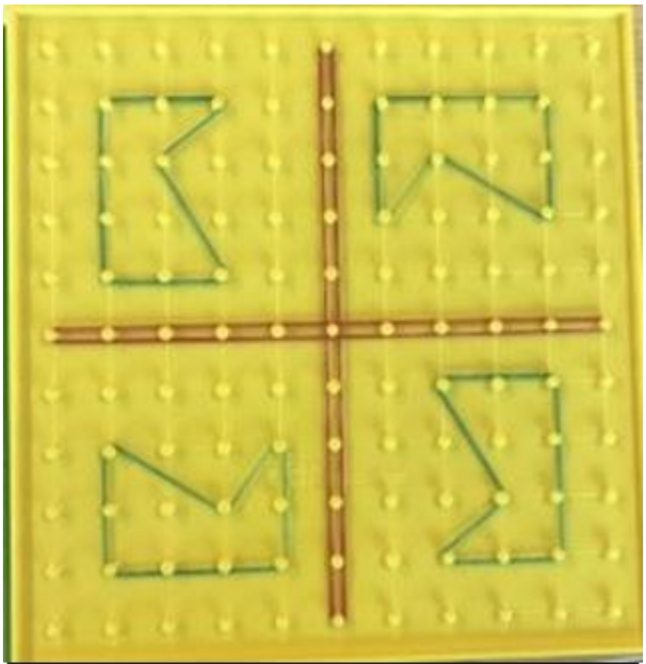
Each teacher aimed to teach two or more topics of their choice (either "lesson" (first time teaching this year: 1-1.5hr) or "review" (recap of a topic already taught: 30mins)) with two or more classes. All teachers managed to use the geoboards once with two teachers managing to use them multiple times across several classes (see table Z). Ancillary data was recorded on classes (see APPENDIX 8).

Table Z: Lessons taught in AR Cycle 2

Teacher	Topics covered	Number of classes taught	Attendance (students per class)
A	Straight Line Graphs	2	8 – 9
	Translations	1	15
	Reflections	2	10 – 11
	Rotations	2	7 – 12
	Pythagoras' Theorem	2	6 – 12
B	Transformations	2	10
	Enlargement	2	5 – 11
	Perimeter and Area	2	5 – 11

	Pythagoras’ Theorem	3	5 – 10
C	Pythagoras’ Theorem	1	6
D	Perimeter and Area	1	1
		Total = 20	Average = 7.8 Range = [1, 15]

Teachers did a pre- and post-lesson confidence check (1 to 5 quantitative) and answered a post lesson qualitative survey (see APPENDIX 9). A whole-class discussion was held with students at the end of each lesson based on a prompt question, some teachers facilitated these discussions whilst others left students alone to write down their feedback to avoid bias (see APPENDIX 9 Q12). Quantitative data was analyzed through diagrams and qualitative data was processed by coding for themes and reference to the literature.



Example of a students work on rotation (can you correct it?!) AR Cycle 2

Action Research Cycle 3

A third research cycle was conducted to improve the quality and quantity of data with slightly refined research questions:

- *How do post-16 GCSE maths students perceive the use of manipulatives in lessons in terms of their engagement in activities?*

- *How do post-16 GCSE maths teachers perceive and respond to the benefits and challenges of using manipulatives in lessons?*

A short group administered student survey was conducted during lessons for all students who participated in AR cycle 1 and 2 to get a wider view of the student body rather than just those who were selected for interviews or chose to speak up in class discussions (see APPENDIX 10). Students could complete the survey on paper or on Microsoft forms.

The survey was designed with advice in mind from Check & Schutt (2011). Data was quantitative and most of the questions were Likert-Type asking students how much they agree or disagree with a series of statements. To reduce agreement-bias some statements were positive about the use of manipulatives whereas some were negative. Negative words such as “don’t” and “not” were avoided to make the survey clearer and illicit a more reliable response. Some potential drivers of positive student experience with manipulatives identified from the literature and AR Cycles 1 and 2 were used to design the questions:

- Student familiarity with manipulative resources (facilitated in part by using them across a range of topics)

- Teacher ability to facilitate use of resource.

- Students' perception of manipulatives being relevant for them and their stage of learning

Teachers conducted short interviews (see APPENDIX 11) with each other in pairs, recording answers through notes and video recordings which could later be cross-referenced for accuracy. The interview included two main themes including scripted follow ups: considerations of teacher professional development; teacher perceptions of student engagement and relevance to the curriculum. Interviewers were asked to stick to the script to standardize the interviews and increase the reliability of the results. Data was processed by coding for themes and reference to the literature.

AR Cycle 1 Results

Initial teacher survey. All teachers had either not used manipulatives before or not as a routine. All teachers were keen to develop their confidence, try new teaching methods and find ways to increase student understanding.

Teacher reflections from post intervention meeting. There were many positive comments from teachers about the potential for manipulatives within their lessons, however there were more negative comments about student engagement than positive. Reasons for this, given by the teachers, were that they did not feel they had adequate time to introduce students to the algebra tiles and the topics and that also students who already had a functional method for the topic at hand rejected the introduction of the algebra tiles. Teachers were also concerned about their lack of experience.

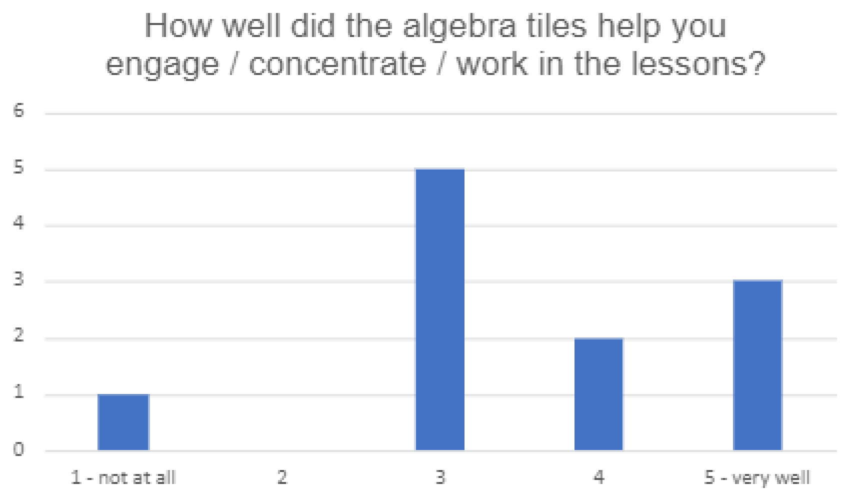
Teacher journal. The most mentioned themes (six occurrences each) were that teacher A felt they had to intervene a lot to support students; that they would have benefitted from more formative assessment of the students before diving into algebra tiles task; and students struggled to work out how to use the algebra tiles to solve the problems set. Overall, there were more negative comments on student engagement (15) than positive (7) or neutral (2). As in the meeting feedback, teacher A noted several times that students with established methods were less willing to try the algebra tiles. There were four occurrences where teacher A commented on feeling a lack of confidence in their own skills.

Student interviews. Out of the 75 coded comments the most common themes from students are as follows:

Count	Theme
8	Felt an improvement in understanding over mental, pen-and-paper, or “abstract” - work.
8	Prefers or can already do “usual” method.
7	Comments on the visual nature of the manipulative.
6	Noted an improvement in concentration.
5	More fun and enjoyable than usual work.

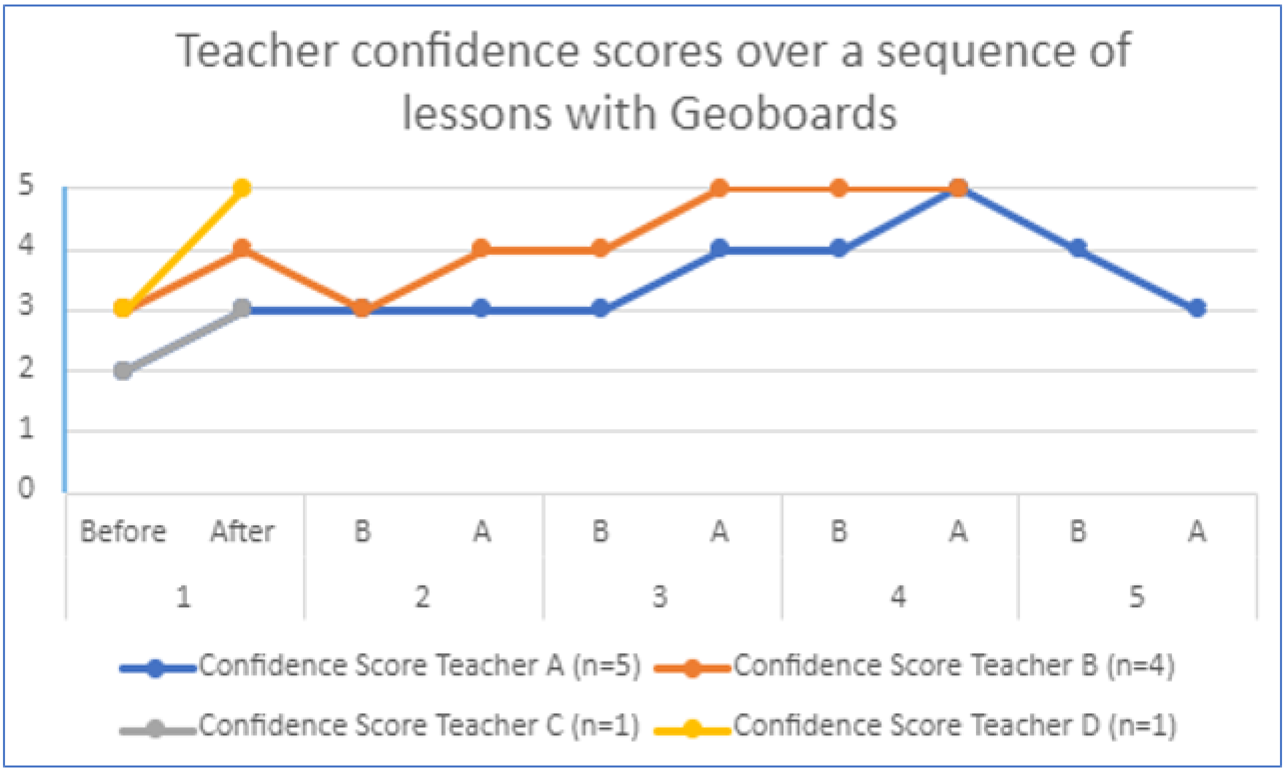
In total there were 42 comments that indicated a positive effect on student engagement and 18 which indicated a negative effect on student engagement. The main reason for students having a negative response to the manipulatives was that they already had or preferred their existing method. Students also commented on the benefits of the visual and tactile nature of the resource 11 times. One person commented on an improvement in understanding after consistent use.

Student survey. Seven students said they would like to use algebra tiles again, one said they would not and three said they would maybe like to use them again. The chart below shows the students' perceptions of their engagement:



AR Cycle 2 Results

Teacher survey. Confidence generally improved after each lesson and over time although teacher A experienced a dip in confidence after the 5th lesson. The average difference between before and after each lesson was 0.73.



There were 88 coded comments from the teacher surveys. Below are some examples of teachers giving reasons for both gaining and not gaining confidence:

Count	Theme	
3	Teacher felt like a lesson did not go well	Teachers **not** gaining confidence
3	Teacher felt like they had to intervene a lot	
2	Teacher felt like students tired of the manipulatives	
2	Teacher feeling like they need more practice	
4	Lesson felt interactive	Teachers gaining confidence
3	Lesson started well / activity went well	
2	Previous lesson going well	
2	Teacher getting confidence from student engagement	

22 comments were classified as teachers observing positive effects on student engagement with the two most common reasons cited being students enjoying the challenges set for them (5 comments) and that they were engaging in deeper mathematical activity than usual (4). For example, one teacher commented: *"there was an increase in the amount of counting and attending to specific distances as the lesson went on".*

Only four comments were coded for concerns about lack of student engagement, although there were a few mentions of students taking time to familiarize themselves with the resources (3); students engaging with the manipulatives in different ways depending on their ability level (3); and students using the manipulatives for something other than the task prescribed by the teacher (2).

In terms of pedagogy and task design there were six comments where teachers referred to successfully starting with an open task and four comments where teachers felt like their starting task was too high-threshold. Some topics were harder to teach than others (2). Teachers felt that geoboards were useful for students drafting and re-drafting their work (2).

Class Discussion. Out of 80 coded student comments 48 indicated a positive impact on their engagement with 12 generally positive comments additionally. This compares to 14 comments indicating a negative impact on their engagement which cited most that they found the tasks confusing (5) or that they would rather use pen and paper methods (5). The top 5 themes were as follows:

Count	Theme
14	Students felt they made progress with the work, found it more accessible and learned something
10	Students felt engaged / that they were concentrating / interested
8	Students found the lesson enjoyable
8	Students felt more confident, calm & relaxed
6	Students found the boards useful to help with visualizing, counting, measuring distance

Students also commented about enjoying having something physical or visual to move around (3) and that they felt the activities were helping them prepare for the exam (3).

AR Cycle 3 Results

Teacher interviews: Out of the 91 coded comments from the teacher interviews, where teachers were asked to reflect on the interventions with both manipulatives, the most common four themes were:

Count	Theme
11	Concern about skill base generally or how to teach with manipulatives
8	Learning to use a manipulative and planning for their use is a challenge and takes time
7	Students seemed to enjoy something different
7	Students found work engaging

When commenting on student engagement teachers mentioned students getting stuck or tied up with the manipulatives (3) but also them becoming more familiar and able to access more work (2). For example, one teacher noted:

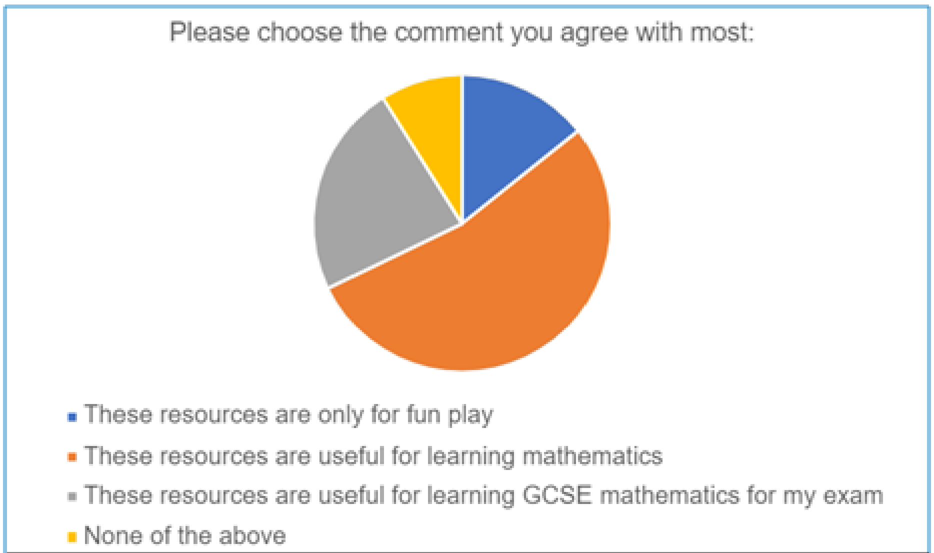
> *"Students who quickly saw the link between the previous lesson were able to use the Geoboard for extension tasks. Some students spent too much time trying to get their heads around constructing triangles on the geoboard."*

Concerns around teacher skills and barriers to getting started included learning to use a different resource or teaching approach (4); concerns around student behavior (3); and concerns around being able to engage and communicate with students (2). Teachers also gave some practical ways they built up their confidence, the most common of which were spending time planning and practicing with the manipulatives (4); using YouTube to skill up (3); and getting positive feedback from students (3).

There was a balanced perspective on whether teachers felt the manipulatives supported with exam preparation with four negative and four positive comments. Some teachers also felt that the manipulatives supported differentiation within the classroom (2).

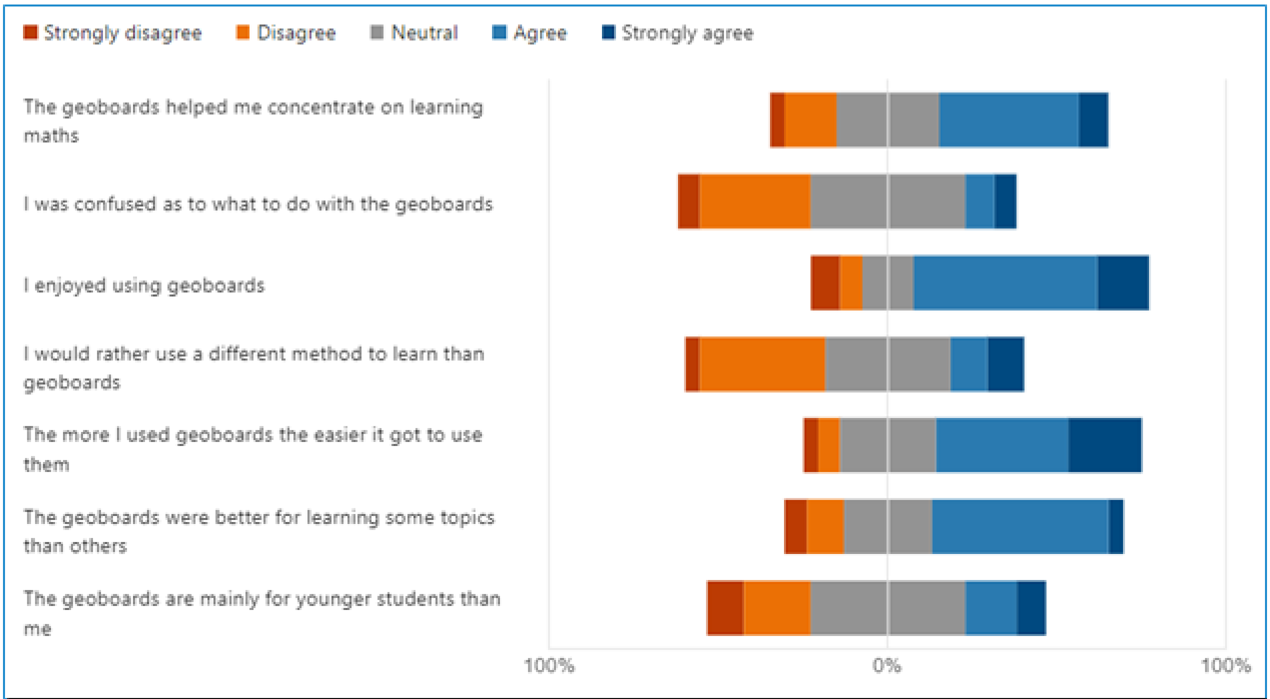
Student Surveys. In total 58 students completed the survey, from classes taught by teachers A (28 students), B (13) and C (17). This uneven response may have caused some biases in the data due to different teacher approaches - these are not accounted for in our analysis.

Over 63% of the students said they would like to use manipulatives again, whilst under 16% said they would not (~17% maybe; ~3% NA). We also asked students to indicate which statement they agreed with most in terms of how useful they found the manipulatives to learn maths, or prepare themselves for the exam. 22% of students agreed most with the statement *"these resources are useful for learning GCSE mathematics for my exam"* whilst another 52% agreed most with the statement *"these resources are useful for learning mathematics".* Only 14% of students indicated they agreed most with the statement "*these resources are for fun play only"* (see diagram below).

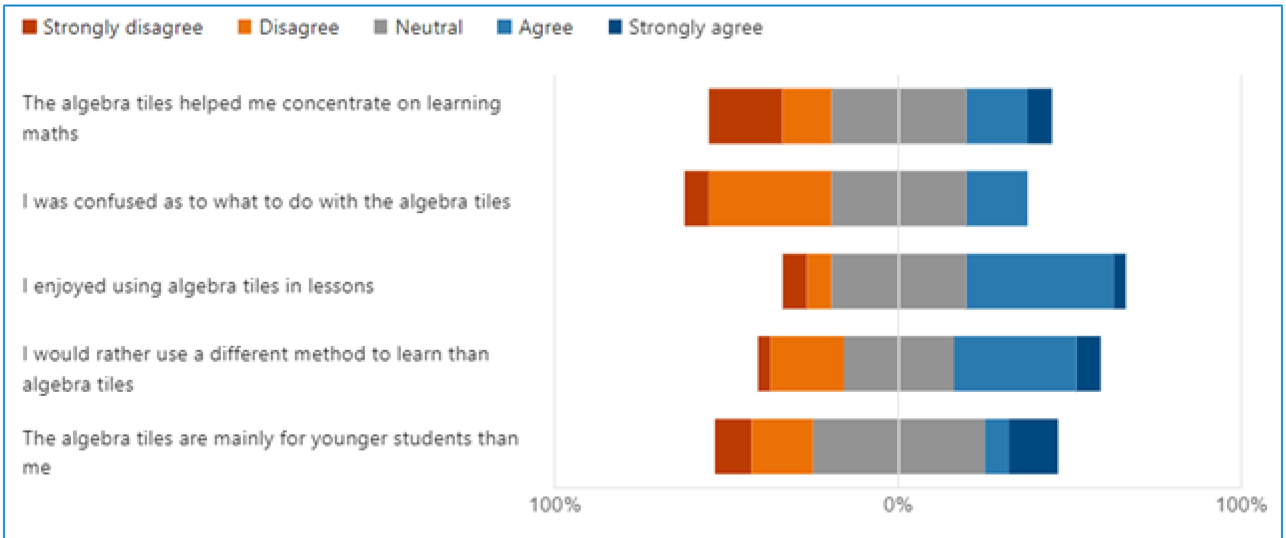


Pie chart showing students view of the appropriateness of the manipulatives to their learning (student survey, AR Cycle 3)

The results of the Likert-Type questions showed that in general students were positive about the manipulatives and were more positive about the geoboards. Students tended to disagree with negative statements about the manipulatives and agree with positive statements, thus showing that our results are robust in terms of agreement-bias (see diagrams below).



Likert-Type statements from the student survey (AR Cycle 3) asking students about geoboards. Both positive and negative statemnets included to test for aggreement-bias (n=46).



Likert-Type statements from the student survey (AR Cycle 3) asking students about algebra tiles. Both positive and negative statemnets included to test for aggreement-bias (n=28).

Discussion

Drawing the results together as a whole we pulled out six main themes: barriers to getting started with manipulatives; teacher confidence and building skills; student engagement; student and teacher perceptions of appropriateness; task design; and bridging activities. Our main focus of this research was on student engagement, however getting teachers into classrooms using manipulatives in productive ways became a clear pre-requisite for understanding how students respond and so we start there.

Barriers to getting started with manipulatives. All teachers were clearly concerned about their skill set and these concerns persisted throughout the year even as their experience deepened and this affected teacher uptake of the intervention (Hurdle, 2020). Teachers felt that their skill facilitating lessons with manipulatives would influence students' progress (Clements, 2000; Back 2019; Sowell, 1989). To support teachers an online training session was organized with Mike Ollerton. This had a positive effect for three of the teachers, however for teacher D there were still significant barriers to being confident in using geoboards in the classroom:

> *"Discomfort stemmed from the fact I felt I should already be competent using these resources. I was comparing myself to the person who ran our CPD session. The research exacerbated this, which meant I felt rushed. Still not a competent user."*

From initial data in AR Cycle 1 time was a concern for teachers both in terms of planning and taking up lesson time to teach students how to use manipulatives. This was still widely felt in the teacher interviews in AR Cycle 3 although one teacher commented that they noticed a marked shortening in planning time as they became more experienced. The quote below from teacher C's interview in AR Cycle 3 exemplifies why planning with manipulatives may take longer when teachers are not used to using them:

> *"I had to spend time on Youtube I was constantly thinking, "What if this happened?" "What if that happened?" So that if "any" question arose would be able to stand there confidently, thinking." Yeah, I can answer all the different scenarios". (…) And I think that's what took quite a lot of time. Of course, it wasn't necessarily just planning the lesson. It was my own deeper understanding of what could happen with using these manipulatives."*

As Moyer et al (2001) found, some teachers were concerned about using manipulatives due to student behavior and larger class sizes. However, this was not true across all teachers.

Teacher confidence and building skills. Confidence generally improved over time through planning & practicing. Teachers gained confidence from student feedback and their own reflections on how things had gone. Sometimes things didn't go so well – some topics were harder than others, sometimes ways of working were different and new.

The main ways teachers-built confidence was to practice, be it in the classroom, whilst planning or whilst watching YouTube videos; and by gathering positive feedback from students. The formal student data collection process for AR Cycles 1 and 2 were especially helpful in boosting teacher confidence and this was supported by informal classroom observations of students at work. For example, in AR Cycle 1 teachers were much more negative about student engagement than the students themselves. By the end of AR Cycle 2 and in AR Cycle 3 teachers were much more in line with the positive nature of students' feedback.

Another potential reason for more positive teacher feedback later was that it seemed geoboards were more popular with students, as can be seen from the student survey. This may be a feedback mechanism with teacher skill and confidence, but it may also be that students had more time to get used to using them with longer lessons and covering a wider range of topics (Hartshorn, 1990).

Student engagement. Students gave overwhelmingly positive feedback about their engagement, progress and enjoyment in both interviews and class discussions. With comments about feeling more relaxed and confident. A student in a class discussion offered: *“it kept my mind more at ease making it more clear for me as I was able to use objects instead of just my head and figures.”* and another commented: *“feels easier to use than pen and paper - the pegs are spaced out evenly.”*

These could well be examples of the manipulatives helping to concretize mathematics for the students, making it more accessible (Gallo-Toong, 2020). Many students also commented on the visual and aesthetic nature of the resources and how that helped them focus (Browne, 2018).

There was evidence that students engaged in serious mathematical activities through using manipulatives, and not just for fun or enjoyment (Moyer, 2001; Hart 1993). This was exemplified in the AR Cycle 3 student survey but also through student comments in the interviews and class discussions, for example these two quotes from AR Cycle 2 and one from AR Cycle 1:

> *"[They are] fun, and they help with understanding, helped visualize the shapes and distance."*
>
> *"I found the geoboard very useful and productive. It helped me to understand and make sure I am measuring correctly."*
>
> *“The tiles made me more confident as I see what I’m doing and you can work out the question yourself! Actually, you do see your mistakes!”*

Not all students took to using the manipulatives and feedback from both teacher and students indicated that this was often due to students already having established methods. This perspective was much more prevalent in the feedback on algebra tiles, perhaps because we were more prescriptive in their usage in our lessons. It is possible that we were not able to provide an extension activity suitable to challenge these pupils and prompt them to engage in a valuable iterative process of moving between concrete, symbolic and abstract experiences (Laski et al, 2015; Mason & Watson, 2019). Conversely some students who may be classed as more concrete learners responded better to the support of the manipulatives (Kablan, 2016).

On the other side of the spectrum some students found understanding what to do with both manipulatives very difficult and confusing. It is possible that the algebra tiles themselves have a higher threshold to understand how to use them however problems may also have arisen due to a lack of conceptual understanding from students in the topics at hand or their prerequisites (Johnston-Wilder & Mason, 2004; Clements & McMillan, 2000; Clements & Sarama, 2018). Both teacher A and B commented on starting with tasks at too high a threshold in both interventions referring to negative effects on student engagement.

There is much more work still to be done in investigating how different demographics of students respond to manipulatives so that we don’t fall into traps of thinking we know who will benefit. This includes students with additional learning needs as we found that they both benefitted (see case studies below) and found activities inaccessible.

Case study: Student with ADHD, AR Cycle 2, Teacher A

"One of my students with quite extreme ADHD who says he hates maths and is very rarely in maths and even when he does it's incredibly difficult to persuade him to do work – took part in a lesson on reflections, he excelled he did more than anybody else – I would have never been pushing him to do the extension work but every time I came back to him he would have done it and would be asking what he could do next and I would move the corner of his shape or challenge him in another way."

Case Study: SEN Student, AR Cycle 2, Teacher B

"CW is a SEN learner with some medical and learning difficulties and a Leaner support colleague assists her in all the lessons. CW had achieved a grade 1 in the diagnostic assessment and a grade 2 in the final mock paper. CW was one of the learners who grabbed my attention from the first lesson that we started using Algebra tiles in class. CW usually needs more guidance and prompt to start answering questions in class but to my surprise she grasped how to use Algebra tiles quickly and started using them confidently. Her ability in Algebra was limited to collect the like terms and was needed to be reminded about the steps to solve simple linear equations. However, she used algebra tiles on her own to solve all the given equations in class.

CW was engaged and saw herself capable of solving equations even quicker than some more able learners in class. She was very good in using Algebra tiles to represent an equation and moving the tiles around to solve it.

CW left the lesson with a smile!"

As Hartshorn (1990), among others, discussed student familiarization with manipulative resources is important for their success in the classroom. We believe that this is one of the reasons for the students' preference to geoboards over algebra tiles. There were examples in the qualitative data from both students and teachers to support the process of familiarization with students overcoming initial challenges and accessing more mathematics.

In the student survey in AR Cycle 3 more students than not agreed with the statement "*The more I used geoboards the easier it got to use them" (see results above).* Right from the start (AR Cycle 1) students were clear on their preference for consistency:

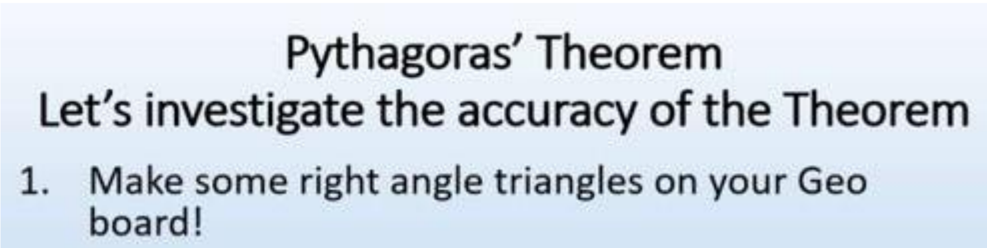
> *"I didn't get it.... I prefer the usual method and the way you explain things... I feel like if we are going to use the algebra tiles to solve equations we should use them all the time, if we are going to use the usual method we should use that"*

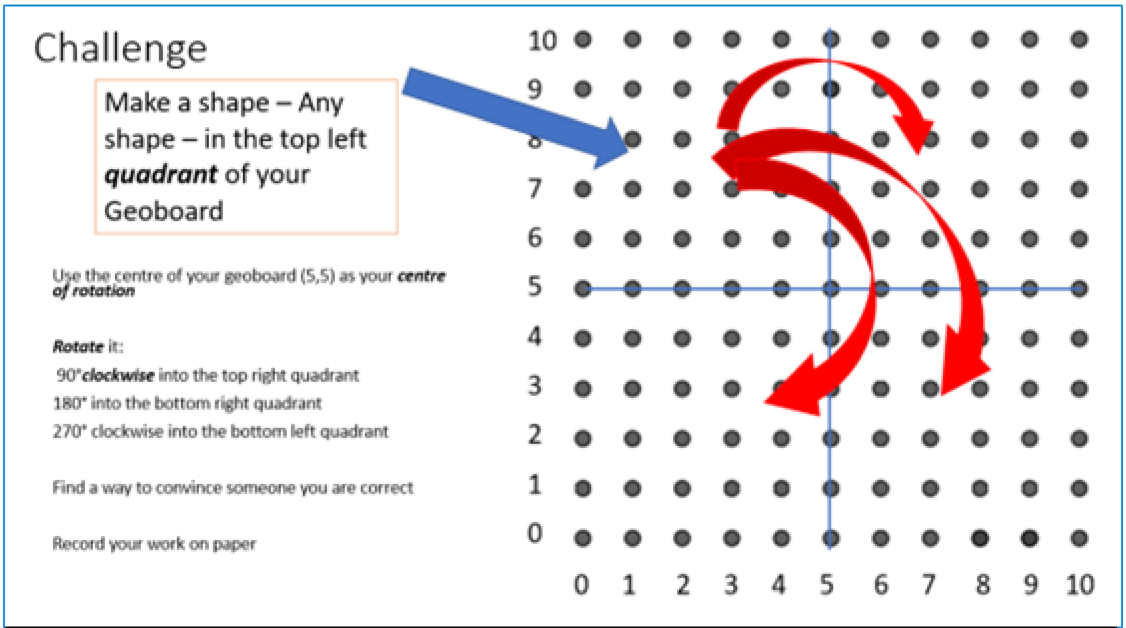
Student and teacher perceptions of appropriateness. The data from the student survey shows that the majority of students felt that these resources were useful for their learning of mathematics but less felt that they would support them in their exam preparation. There were positive comments from class discussions such as: *"I'm more confident to work out perimeter and area questions in the exam;"* but also contentions that these resources would not be available to use these resources in the exam. Teachers had a balanced view with an equal number of comments indicating they believed students had been supported in their exam preparation and not.

The point about manipulatives not being available in exams perhaps indicates a need for communication about possible roles manipulatives can play in a classroom: as facilitators for abstract concepts but also as tools to gain further enlightenment or aesthetic enjoyment (Browne, 2018).

Students seemed quite neutral about the age appropriateness of the resources – a key finding for post-16 GCSE maths as manipulatives are often more geared towards younger students.

Task design. Ideas for more "open" tasks came from the CPD teachers participated in. Starting with "open" tasks, for example where students created their own shapes (shown in the boxes below), were reported to be more beneficial to student engagement by teachers. This may be due to the differentiation it naturally allowed for in the classroom. Students' preference for geoboards may also be down to them being less rigid in the way they can be used, allowing us to plan more open and varied activities.



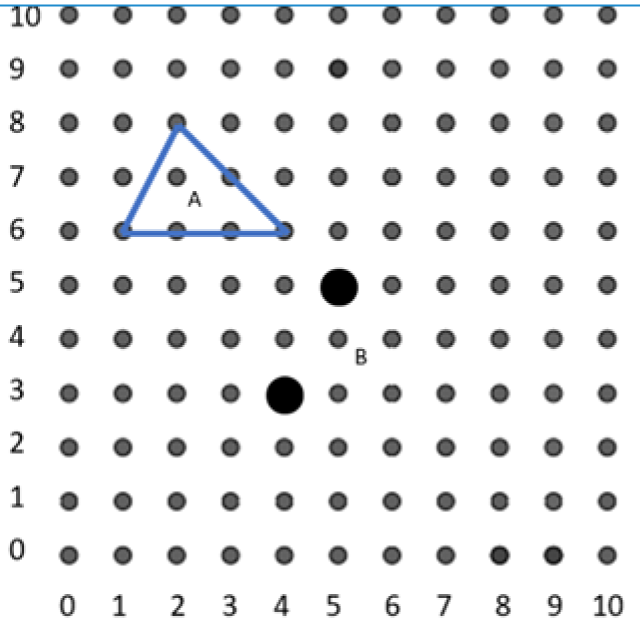


Two activities offering an "open" start to students in AR Cycle 2.

In contrast to the "open" task design some teachers found that certain activities were too high threshold to successfully engage the majority of students:

> *"The task was to fill in gaps in a table which included coordinates and translation vectors. I felt I had to compensate quite a lot for confusion this sheet caused, trying to get students to attend to how the vector notation worked and moving the shape accurately without distorting it. In the end I felt like I might have done better asking them to start with a shape and then move*

it somewhere else, then work on describing the movement: something a bit more open and lower threshold."



Starting with Triangle A as shown to the left.. Complete the transformations on the worksheet:

Example:

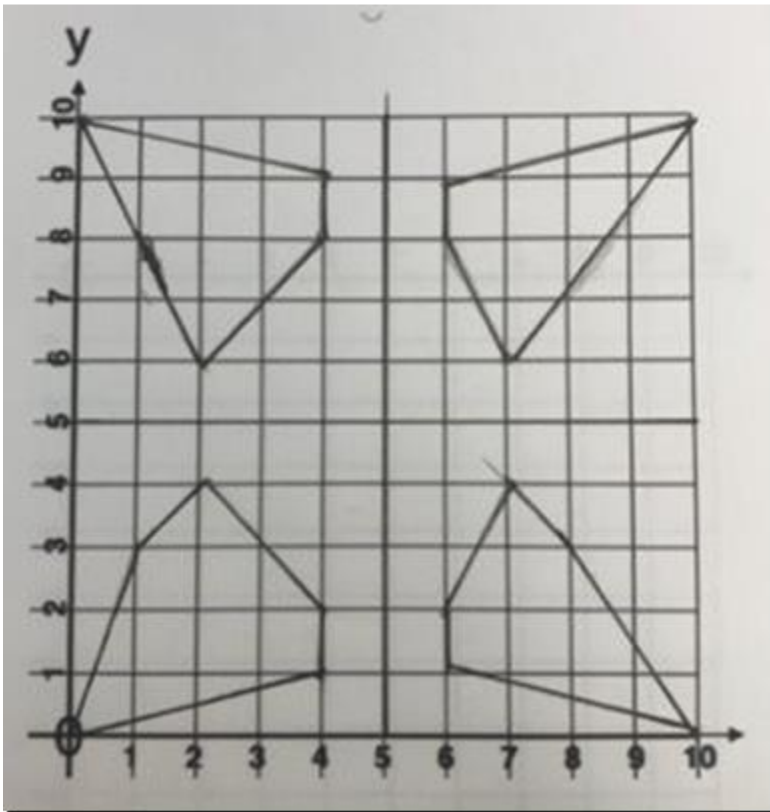
Triangle	Coordinates			Translation
Triangle A	(1,6)	(2,8)	(4,6)	$\binom{0}{0}$
Triangle B	(4,3)	(5,5)		$\binom{3}{-3}$

Example of "closed" activity with geoboards in AR Cycle 2.

Bridging activities. Bridging activities were built into the lessons in AR Cycle 2 in the form of asking students to copy down work from their geoboards onto graph paper. Bridging activities support the formalization process from the concrete experience of using a manipulative to more formal written mathematics which can often fail (Hart,1993). Some teachers felt that not enough time was spent engaging in bridging activities and this may reflect some of the negative feedback on appropriateness for exam preparation.

Transferring work from the geoboard to graph paper proved more difficult than expected, with Teacher A reporting that many students' translations work became incorrect when copied down onto paper (see picture below). A learning support assistant commented:

"Having to draw the shape out on a smaller grid than the geoboard kind of changed everything."



Students work from AR Cycle 2. A correctly translated pattern becomes distorted when drawn onto paper.

Conclusions and Recommendations

Conclusions

Manipulatives can be suitable for post-16 GCSE maths learners, supporting their engagement and enjoyment in lessons and helping them access mathematics further. However, successful facilitation by teachers and use by students requires several key considerations.

Teachers must be confident to get into often already challenging situations with new resources and new teaching styles. Confidence can be gained through training and CPD but having enough time for teachers to plan and familiarise themselves with resources is essential, as well as getting into the classroom. Initial confidence gains were sometimes eroded when lessons did not go to plan but consistently gathering data from students allowed a more balanced view to be built up.

Student engagement and enjoyment in lessons appeared to be boosted by the use of manipulatives, especially the geoboards which they had time to familiarise themselves with; were more applicable to a wide range of topics; are less prescriptive in their use than algebra tiles; and were used alongside more "open" activities.

Many students reported that the manipulatives helped their learning and understanding of mathematics and some could see relevance to their exam preparation. More work could be done to support this by focusing on bridging activities and the iterative movement between concrete-pictoral-abstract ways of working. Students who already had working methods were less willing to take up manipulatives however there may be ways to overcome this by planning activities which can be extended to a suitable level for them.

There is still more work to do on looking into how students with additional learning needs can access work with manipulatives and benefit from it. We have seen some good evidence that this can happen however we have also seen some students struggle to get over a threshold to engage with activities – especially with algebra tiles.

Recommendations

1. Hands on training where teachers can try out resources with colleagues (Hurdle, 2020; Vizzi, 2016). The action research group ran a day long training session for 16 other teachers in June 2022 and have received excellent feedback with teachers feeding back that they will be using manipulatives next year.
2. Time must be made for teachers to plan for use with manipulatives, especially when teachers are new to using manipulatives.
3. A non-judgemental and supportive attitude to teachers trying new things out.
4. Gather data from students and teachers on how the manipulatives are going in lessons in order to build a balanced view.
5. Build in multiple lessons across different topics with one manipulative to help student familiarisation, build in time in lessons for students to familiarise themselves with the resource before ploughing on with the scheme of work.
6. Plan for bridging activities to support the learning process through multi directional movement between concrete-pictoral-abstract stages and to communicate the applicability of manipulatives to exam preparation for both students and teachers.
7. Plan a variety of open and closed activities but consider starting with an open activity to keep the threshold for starting low with options for extending and developing mathematical ideas.

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Appendix/Appendices

APPENDIX 1: A table of available manipulatives and suggested topic areas

Topic Area	**Manipulative you can use**
Algebra	Algebra Tiles Geoboards XY Coordinates Pegboards
Counting & Sorting	Abacus Base Ten Blocks Colour Cubes Colour Tiles Cuisenaire Rods Rekenrek Snap Cubes Two-Colour Counters Play Money
Geometry	Anglegs Attribute Blocks Colour Cubes Colour Tiles Geoboards Pattern Blocks Relational Geosolids Snap Cubes Tangrams XY Coordinates Pegboards Zometool
Fractions, Decimals & Percentages	Cuisenaires Deluxe Rainbow Fraction Circles Deluxe Rainbow Fraction Squares Fraction Tiles Fraction Tower Equivalency Cubes Two-Colour Counters
Measurement	Bucket Balance Colour Tiles Snap Cubes Measuring Spoons
Operations	Base Ten Blocks Cuisenaire Rods Rekenrek Snap Cubes Two-Colour Counters
Pattterns & Attributes	Attribute Blocks Colour Cubes Colour Tiles Cuisenaire Rods Pattern Blocks Snap Cubes Two-Colour Counters
Place Value	Abacus Base Ten Blocks Colour Tiles Cuisenaire Rods

	Snap Cubes
Probability	Two-Colour Counters
Time	Geared Clocks Clock Faces

APPENDIX 2

Folder containing resources for lessons on algebra tiles: [Algebra tiles]

APPENDIX 3 Initial teacher questionnaire

Initial Teacher Quiz - ARG Manipulatives & CPA

This quiz is to share where we all are in terms of using manipulatives and what we would like to get out of this years action research on manipulatives and CPA.

1. What is your name?

2. How long have you been teaching for, and how long have you taught in post 16 education?

3. Please briefly describe your teaching style, attidutes and values for teaching :)

4. Have you used manipulatives before in a post 16 GCSE maths setting?
   - ○ Yes, regularly
   - ○ Yes, but as a one off or not as a routine
   - ○ Not at all

5. Throughout your career which manipulatives have you used and with which classes?

6. Taking into account all of your experience in using manipulatives what do you think works well (this could include specific manipulatives but also contexts and teaching styles or pedagogies)?

7. Overall how did you find these experiences?
   - ○ Positive
   - ○ Negative
   - ○ Mixed

8. What would you say have been the main challenges with using manipulatives?

9. Have you had any recent (last 5 years) training in how to use manupulatives in a maths classroom?
   - ○ Yes
   - ○ No

10. Describe the training you have received and when?

11. Would you like more training? If so, what would be useful to you?

12. How confident are you to use manipulatives with your current classes?

☆ ☆ ☆ ☆ ☆

13. What are your main concerns with using manipulatives with your current classes?

14. What do you think are the main benefits which could be gained from using manipulatives with your current classes?

15. What is your current understanding of the CPA model?

16. How often do you use the CPA model when planning your lessons? 0-not at all; 5-all the time

☆ ☆ ☆ ☆ ☆

17. If you do use the CPA model, how do you intergrate it into your teaching?

18. What would you like to get out of this action research? What are you interested in researching?

19. What would you like your students (current and future) to get out of your participation in this action research?

20. Are there any manipulatives in particular which you would like to researcch?

21. Are there any pedagogic styles (e.g. investigation, mastery, teaching problem solving) that you are particularly interested in researching whilst using manipulatives?

22. Any other comments?

APPENDIX 4 Teacher reflective questions – AR Cycle 1

Teacher Reflective Exercise

Complete this after each session. It should take about 15-20mins.

Step 1: Take 5 minutes to sit in silence and think over the lesson. Identify significant moments or events – something which felt important to you (you are the judge of this!) We are mainly concerned with students' engagement but there may be other significant things which come up. Identify one which you want to record.

(Engagement: how well they concentrated, how well the activities held their attention and how well they completed or worked towards completing their work)

Step 2: Take 5 minutes to describe as vividly as possible a significant moment. This can be as short as one or two sentences or as long as a whole paragraph.

An example of a short account might be: ***After 5 minutes of asking students to attempt to solve equations with algebra tiles, I look around the room, I notice student A has made a symmetrical pattern with their algebra tiles!***

Step 3: Take 5 minutes to answer the following four questions

1. How did the students engage with the lesson (be as descriptive as possible)?
2. What else went well?
3. What else was challenging or could have gone better?
4. What would you do differently next time?

APPENDIX 5 Student interview questions AR Cycle 1

Student Interview Script:

- *Hello, I want to ask you two questions based on your work on Algebra Tiles in the past few sessions.*
- *We want to know about your experience in order to write our report and improve our teaching.*
- *The interview will last for 5 minutes.*
- *Your answers will be anonymized and used in our report.*
- *No one will judge you on your answers, they are not used for assessing you personally in any way.*
- *Are you ok to continue? (Student must say yes)*
- *Are you ok if I record your answers on a Dictaphone?*

Ask each student a question in turn, students may want to build on each other's answers and that may be useful for getting a more in-depth response.

Question 1*: Think of when you have used ALGEBRA TILES in lessons in the last few weeks. How well did the ALGEBRA TILES help you engage in the lessons?*

Possible Follow up questions*:*

- *Could you describe what you meant by...?*
- *Would you like to build on what ... said?*
- *Could you tell me more about that?*
- *How did that make you feel?*

Students may need a definition of "engage": *hold your attention, help you concentrate, help you complete your work*

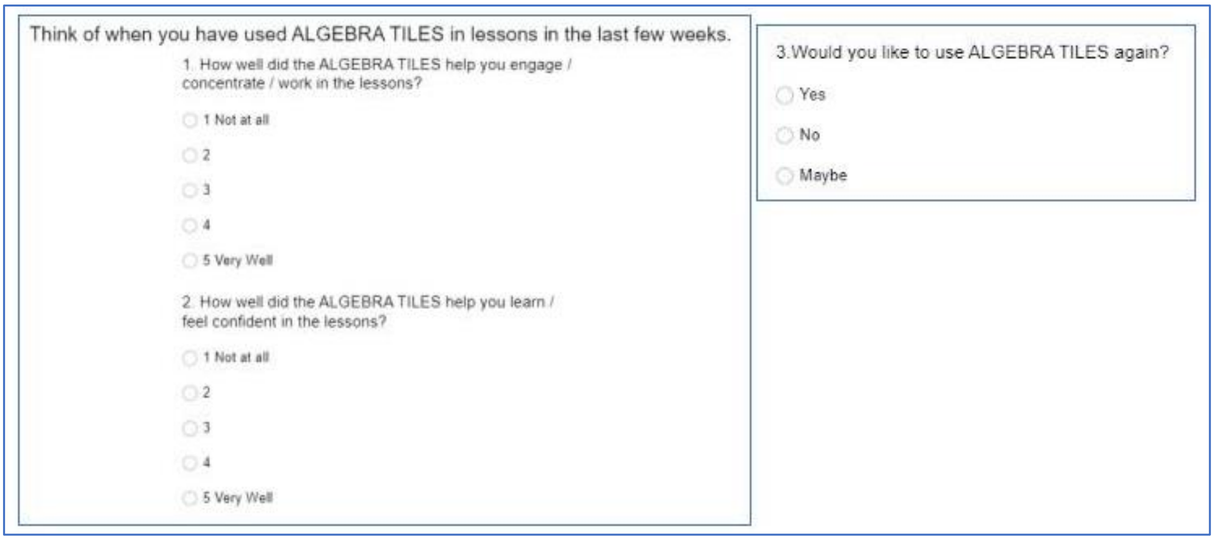
If time, try **Question 2:** *How well did the ALGEBRA TILES help you learn / feel confident in the lessons?*

Possible Follow up questions*:*

- *Could you describe what you meant by...?*
- *Would you like to build on what ... said?*
- *Could you tell me more about that?*
- *How did that make you feel?*

Question 3: *Would you like to use ALGEBRA TILES again?* (Yes or no answer)

APPENDIX 6 Student survey AR Cycle 1



Think of when you have used ALGEBRA TILES in lessons in the last few weeks.

1. How well did the ALGEBRA TILES help you engage / concentrate / work in the lessons?

- 1 Not at all
- 2
- 3
- 4
- 5 Very Well

2. How well did the ALGEBRA TILES help you learn / feel confident in the lessons?

- 1 Not at all
- 2
- 3
- 4
- 5 Very Well

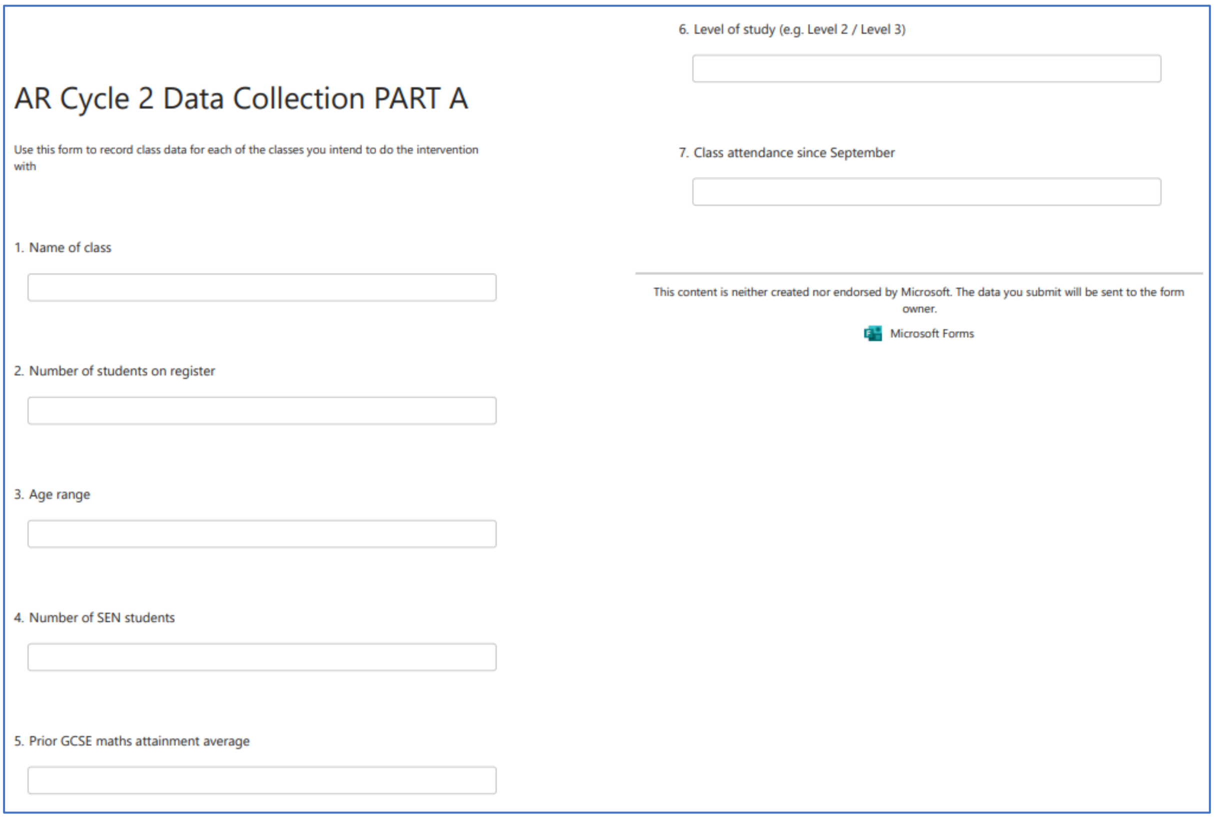
3.Would you like to use ALGEBRA TILES again?

- Yes
- No
- Maybe

APPENDIX 7

Folder containing resources for geoboards: [Geoboards]

APPENDIX 8 Ancillary class data, AR Cycle 2



AR Cycle 2 Data Collection PART A

Use this form to record class data for each of the classes you intend to do the intervention with

1. Name of class
2. Number of students on register
3. Age range
4. Number of SEN students
5. Prior GCSE maths attainment average
6. Level of study (e.g. Level 2 / Level 3)
7. Class attendance since September

This content is neither created nor endorsed by Microsoft. The data you submit will be sent to the form owner.

Microsoft Forms

APPENDIX 9 Post lesson qualitative survey including question posed to students for class discussion, AR Cycle 2

12. Question posed to students as a group at the end of the lesson. Confident student (to avoid confirmation bias that may come from teacher facilitation) to facilitate a class discussion and record student feedback on the board, grouping feedback into themes if possible:

How did you find using Geoboards in the lesson?

Please comment on your:

•Engagement

•Ability to concentrate

•Ability to complete tasks and problems

Please record student feedback either by typing it up or uploading a photo:

13. Do you have any comments to make on student feedback or their performance in the mini quiz?

14. What was your starting point/question for the students? What suplimentary questions or directions did this take you in?

15. Were there any moments in the lesson which stood out for you in terms of student engagement?

16. Do you have any photos of student work?

17. Any other comments, or anything you would do differently next time?

AR Cycle 2 Data Collection Part B

Please complete this for each topic/lesson you teach. If you teach the same topic/lesson to 2 or 3 classes only complete this form once.

This means you should complete this form 3 times.

1. Teacher name

2. Which topic did you teach?

3. What were the dates you taught this on?

4. How many different classes did you teach this lesson with?

5. What were the numbers of students in each class?

6. Class names (so we can compare to class data)

7. How confident do you feel about teaching the upcoming lesson using Geoboards?

☆ ☆ ☆ ☆ ☆

8. Do you have any comments?

9. How confident do you feel about teaching using Geoboards now you have completed the lesson(s)?

☆ ☆ ☆ ☆ ☆

10. Do you have any comments?

11. What was the average student confidence score on the topic from the start of the lesson (you may want to record each class seperately)?

APPENDIX 10 Student survey AR Cycle 3

GCSE Maths Physical Resources - Student survey 2022

Data from this survey will be used in a report on improving maths teaching co-written by your teacher at the end of the year.

Your feedback will be anonymised and will not effect your end of year grade.

All feedback welcome – please be honest about how you feel.

* Required

1

Which college do you attend? *

- CTK Emmanuel
- Shooters Hill
- East Surrey
- South Thames

2

What is your current highest grade based on an offical assessment (awarded by the exam board)?

If you have never sat an official exam, please give your end of year grade from last year *

- U - GCSE
- 1 - GCSE
- 2 - GCSE
- 3 - GCSE
- Functional Skills Level 1
- Functional Skills Level 2
- None of the above

3

This year we have been trialing physical resources such as algebra tiles and geoboards in your lessons. Would you like to use physical resources like these in lessons again? *

- Yes
- No
- Maybe
- I didn't use any physical resources

4

How many lessons did you use physical resources for this year? *

- 1 - 2 lessons
- 2 - 5 lessons
- 5 - 10 lessons
- Over 10 lessons

5

Please choose the comment you agree with most: *

- These resources are only for fun play
- These resources are useful for learning mathematics
- There resources are useful for learning GCSE mathematics for my exam
- None of the above

6

Did you use Geoboards in any lessons? *

- Yes
- No

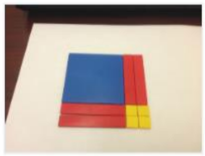
7

Please rate how much you agree with the following statements: *

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
The geoboards helped me concentrate on learning maths	○	○	○	○	○
I was confused as to what to do with the geoboards	○	○	○	○	○
I enjoyed using geoboards	○	○	○	○	○
I would rather use a different method to learn than geoboards	○	○	○	○	○
The more I used geoboards the easier it got to use them	○	○	○	○	○
The geoboards were better for learning some topics than others	○	○	○	○	○
The geoboards are mainly for younger students than me	○	○	○	○	○

8

Did you use algebra tiles in any lessons? *



- Yes
- No
- I can't remember

9

Please rate how much you agree with the following statements: *

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
The algebra tiles helped me concentrate on learning maths	○	○	○	○	○
I was confused as to what to do with the algebra tiles	○	○	○	○	○
I enjoyed using algebra tiles in lessons	○	○	○	○	○
I would rather use a different method to learn than algebra tiles	○	○	○	○	○
The algebra tiles are mainly for younger students than me	○	○	○	○	○

10

Many thanks for completing the survey, do you have any other feedback?

APPENDIX 11 Teachers interviews AR Cycle 3

Teacher Interview Script AR Cycle 3

This sheet includes questions for the teacher interviews for AR Cycle 3 which focus on teacher experience of using manipulatives. The interview should last between 5 and 10 minutes.

The interview starts with two introductory questions followed by two main questions including follow ups. The main questions are separated into two themes: the first is centered around considerations of teacher professional development; and the second is around teacher perceptions of student engagement and relevance to the curriculum.

Guidance for interviewer:

- **Read the script and questions exactly as they are on the sheet.** This will enable us to keep the questions the same for everybody, therefore standardizing the process and increasing the reliability of our results. Follow-up questions are provided as prompts for further discussion.
- Feel free to prompt further with neutral follow ups such as *"would you like to expand on that"* or *"could you tell me more about that"*
- Record the interview on teams and make notes as you go along if possible. Cross-reference your notes to the recording if needed. If you notice the interviewee says something interesting and you do not manage to write it down, record the time on the recording for ease of reference later.
- Boxes are provided below to record notes in.

Guidance for interviewee

- Take time to read the questions in advance to prepare for the interview
- Answer as honestly as possible, we are not here to judge each other.
- Any comments can be anonymized, or omitted from the report later if you do not wish them to be included.
- You will have access to the final notes made from your interview for your approval before data analysis begins.

Beginning of script

Hello and thank you for taking part in this interview. There are three introductory and two main questions for this interview, as well as some follow-up questions to stimulate further discussion.

The main questions are separated into two themes: the first is centered around considerations of teacher professional development; and the second is around teacher perceptions of student engagement and relevance to the curriculum.

As the interviewer I will stick to the scripted questions. This will help to standardize the interviews across all participants although I may prompt you to speak further on a point if appropriate.

Please answer as honestly as possible, we are not here to judge each other. Any comments can be anonymized, or omitted from the report later if you do not wish them to be included.

You will have access to the final notes made from your interview for your approval before data analysis begins.

Introductory questions

What is your name?

How many lessons or lesson segments did you conduct with each manipulative?

Would you recommend using manipulatives in lessons to your colleagues? *Yes/no/maybe*

Main Questions and follow up questions

Were there any challenges for you as a teacher in using manipulatives and how did you overcome them?

- *Were you concerned about your skill base and being able to explain the tasks clearly to the students?*
- *What was important for gaining confidence in using manipulatives with your classes?*
- *How did you find planning for using manipulatives?*

What do you think were the main outcomes of using manipulatives for our learners?

- *How do you think the manipulatives affected student engagement?*
- *Did using manipulatives effect the flexibility within the lessons for example differentiation between students and options for extension*
- *Do you feel the manipulatives helped prepare students to answer GCSE exam questions?*