Lesson Plan

Factors and multiples

Contents

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1. Rationale

For many students, factors and multiples are abstract concepts that are difficult to relate to. As a result, they often confuse factors and multiples, and highest common factors (HCF) and lowest common multiples (LCM). They rely on remembering a method or technique for finding these, rather than understanding the principles underpinning the techniques. This lesson uses the context of chocolate bars and packing trays to support the development of thinking about factors as the dimensions of a rectangular array and multiplication as the area of the array. Developing an understanding of **mathematical structure** (Key Principle 1) through mathematical representations such as arrays is a key part of the teaching for mastery approach. The use of arrays aims to highlight the **links between mathematical concepts** (Key Principle 3).

2. GCSE curriculum

Number

Structure and calculation

N4 use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem.

3. Lesson objectives

- Find factors and multiples of numbers less than 100
- Find common factors and common multiples of two numbers
- Solve problems involving HCF and LCM
- Understand how to use representations to provide insight into solving problems

4. Starting points

The lesson assumes that students have some knowledge of multiples and factors.

5. Research questions

Pedagogic focus

How does the teacher use the student tasks to help develop understanding of mathematical structure?

Maths focus

In what ways do students use representations to consider mathematical structure to develop their mathematical understanding of highest common factor and lowest common multiple?

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6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	10	Begin the lesson by checking students' understanding of the terms 'factors' and 'multiples'. Introduce the context of chocolate bars and show students a '6 by 4' 24 piece chocolate bar (referred to as a 24-bar). Ask students to work in pairs to find other possible configurations of 24-bars.	Mini whiteboards Squared paper (optional) Slides 2–4
Discuss	10	Ask students with different rectangular arrays to share their thinking. Discuss how finding all possible 24-bars gives the factors – and factor pairs – of 24, emphasising how the relationship between factors and multiples is evident in the dimensions and area of a rectangular array.	Slides 5–8
Explore	40	Introduce the context of packing chocolate bars in trays. Ask students to work in pairs on the main task. Students find the dimensions of a 96-tray that can be used to pack different 24-bars. They then explore trays that can be used to pack both 24-bars and 30-bars.	'Chocolate factory' handout Squared paper (optional) '24-bars' and '96-trays' handouts (optional) Slides 9–19
Review	20	Once students have completed the tasks, discuss their different approaches. Reinforce key mathematical concepts that students have been working with.	Slides 20–27
Practice question	10	Ask students to answer an exam question and after a few minutes discuss their methods.	'Practice question' handout Slide 28

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7. Teacher guidance

Introduction

Aim	Students identify factor pairs of 24 using an array
Materials	Mini whiteboards, squared paper (optional)
Slides	Slides 2–4
Time	10 minutes

A key element of teaching for mastery is the '**concrete, pictorial, abstract**' (CPA) approach. Students develop their understanding of abstract mathematical concepts and structures, starting with concrete objects, moving on to pictorial representations and then linking to abstract mathematical symbols. In this section of the lesson, students use the context of a 24 piece chocolate bar (24-bar) to explore the factors of 24.

Another fundamental aspect of the mastery approach is **building on existing knowledge** (Key Principle 2). Observing the students as they work on the task will provide you with some understanding of what they already know.

What the students might do and what you might do

Slide 2 Before introducing the context for the lesson, spend a few minutes checking students' understanding of the terms 'factors' and 'multiples'.

Slide 3 Tell students that a factory makes chocolate bars of various sizes. Ask them how many pieces are in the bar shown on the slide. Ask them to explain how they worked out the number of pieces. Explain that a chocolate bar containing 24 pieces of chocolate is described as a 24-bar.

Slide 4 Ask students to work in pairs to find other possible 24-bars and draw them on their mini whiteboards. Some students may find it helpful to use squared paper.

Students may struggle to get started, but once they know what to do, it is likely that they will find all possible 24-bars. However, they may not be systematic. Some students may draw a rectangular array, whereas others may simply draw a rectangle with labelled dimensions.

Discuss

Aim	To highlight connections between factors and multiples and the dimensions and area of an array
Slides	Slides 5–8
Time	10 minutes

In this section, students share the 24-bars that they found and the methods they used to find them. Establish that finding all possible 24-bars gives the factor pairs of

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24, emphasising how the relationship between factors and multiples is evident in the dimensions and area of a rectangular array. Exploring this connection supports students in **developing an understanding of mathematical structure** (Key Principle 1).

What the students might do and what you might do

Slide 5 Ask students to share the other 24-bars they have found and the methods they used to find them. Make sure that all the different configurations of a 24-bar are discussed.

Asking students to explain how they worked out the dimensions of the 24-bars encourages them to think about mathematical structure. Emphasise the connection between factors of 24 and the dimensions of the array (the number of pieces of chocolate in each row and column) and, in this way; **representations are used to unlock understanding** (Key Principle 1).

Tell students that the factory decides that it is not practical to make chocolate bars that are only one piece wide. Establish that the bars that the factory produces will be at least 2 pieces wide.

Slide 6 If students have drawn their arrays in different orientations, you may want to discuss multiplicative commutativity. Ensure that students understand that a ‘6 by 4’ bar is the same as a ‘4 by 6’ bar and that we can multiply two numbers in any order.

Slide 7 Ensure that students understand the mathematical vocabulary of ‘factor’, ‘multiple’ and ‘common multiple’.

Slide 8 Reinforce that all numbers have a factor of 1 and the number itself.

Explore

Aim	Students solve problems involving factors and multiples
Materials	‘Chocolate factory’ handout, mini whiteboards, squared paper, ’24-bars’ and ’96-trays’ handouts (optional)
Slides	Slides 9–19
Time	40 minutes

In this section of the lesson, students work in pairs to solve problems involving factors and multiples in the context of packing chocolate bars into trays. The trays are represented as labelled rectangles rather than rectangular arrays. This exposes the direct connection with the two-dimensional area model and supports students in **making connections between different mathematical concepts** (Key Principle 3). Working through this task requires students to think in terms of common factors and multiples; you might want to point out where they are using highest common factors and lowest common multiples.

Directing students to take turns when completing the ‘Chocolate factory’ handout encourages a **collaborative culture** where students work together and share their understanding while providing opportunities for students to explain their thinking to each another (Key Principle 5).

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What the students might do and what you might do

Slides 9–18 introduce the context that the students will need for the tasks on the 'Chocolate factory' handout.

Slides 9–11 Introduce the cardboard trays used to pack the chocolate bars. Explain that the trays are described by the number of pieces of chocolate that fit the base layer. Establish that a 96-tray is so-called because 96 pieces of chocolate fit into the base layer. Explain that the chocolate factory wants to use 96-trays and makes two different trays. Show students the two trays and ask them to explain why they are 96-trays.

Slide 12 Tell students that the factory wants the base layer of each tray to be full when packing the chocolate bars. Ask students to identify which 24-bars are being packed in this '24 by 4' 96-tray and discuss how the '12 by 2' bars have been packed. Check that students are clear about what the 12 and 2 refer to on the '12 by 2' chocolate bars.

Tell students that the factory wants to pack each tray with chocolate bars of the same size. Ask students whether any other 24-bars can be packed into the '24 by 4' tray. You may want to distribute the '24-bars' and '96-trays' handouts for students to use.

Show students the diagram of four '6 by 4' bars packed in the tray. Check that students can explain why the '8 by 3' bars cannot be packed into this '24 by 4' tray. Remind students that the base layer needs to be full.

Slide 13 Give students a few minutes to determine which 24-bars can be packed into the '16 by 6' 96-tray. Reveal the answers as appropriate, depending on the order in which students suggest arrangements.

- Left: Packing 8-by-3 bars
- Right: Packing 4-by-6 bars

Slide 14 After showing how '8 by 3' bars and '4 by 6' bars can be packed into a '16 by 6' tray, check that students can explain why '12 by 2' bars cannot be packed into a '16 by 6' tray.

Remind students that the factory does not want to mix bars with different dimensions in the same tray (so they cannot fill the space that would remain after packing three '12 by 2' bars with a '4 by 6' bar, for example).

Slides 15 and 16 After establishing that '12 by 2' and '6 by 4' bars can be packed into a '24 by 4' 96-tray, use the animations on slide 15 to explain to students that bars with dimensions of 6 and 12 will fit into a tray with a side of 24 because 24 is a common multiple of 6 and 12. Similarly, 4 is a common multiple of 2 and 4.

Slide 16 uses the same approach as slide 15 to explain why '8 by 3' and '4 by 6' bars can be packed into a '16 by 6' 96-tray; 16 as a common multiple of 8 and 4, and 6 is a common multiple of 3 and 6.

Slides 17 and 18 Ask students if there are any other possible 96-trays that could be made, in addition to the two trays that have been explored. Remind students that the factory only produces chocolate bars that are at least 2 pieces wide so trays need to have a width of at least 2. Slide 18 shows the other possible 96-trays. Identify all possible 96-trays, highlighting to students that a '96 by 1' tray is not included as the factory only produces chocolate bars that are a least 2 pieces wide. You may want to write the factor pairs for 96 on the board.

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There are three tasks (A to C) on the ‘Chocolate factory’ handout. Task A reinforces the concepts previously discussed but is not essential. If you are short of time, you may ask students to start at Task B, which students should be able to complete quite quickly, before moving on to Task C. It is important that students have time to attempt Task C as this is reviewed in the next section of the lesson.

Slide 19 Distribute the ‘Chocolate factory’ handout and ask students to work on the tasks in pairs. It is important that they work collaboratively and that they are both able to articulate their collective thinking, as either student may be called upon to explain their work as part of the discussion that follows. Some students may find using squared paper helpful.

As students work, circulate amongst them and observe what they are doing. Make a mental note of which pairs’ work you might like to share in the class discussion later. Let students work without your help, but if they are clearly struggling to get started, ask some questions to nudge their thinking.

Deepening understanding Do students recognise the relationship between the product of the highest common factor and lowest common multiple for a pair of numbers and the product of the numbers themselves?

Review

Aim	To consolidate students’ thinking and establish key mathematical concepts
Slides	Slides 20–27
Time	20 minutes

After students have had sufficient time to work on the tasks, discuss different student approaches. Use this discussion to reinforce key mathematical concepts.

What the students might do and what you might do

Ask a pair of students (identified as you circulated during the exploration activity) to describe their approach to the ‘Chocolate factory’ tasks and explain their thinking. Discuss other students’ methods and identify any common understanding. Students may want to find out whether their chosen tray is correct, but the focus of this discussion should be on the connections made and any struggles they faced. Use the discussion to draw out the key ideas and support students in making sense of the relationship between the factors and multiples of the numbers considered.

Slide 20 Students may have eliminated some 96-trays without any further exploration by considering the properties of the 24-bars that need to be packed.

Check that students can explain why each of the 24-bars can/cannot be packed into each of the 96-trays. Confirm that all three types of 24-bars can be packed into a ‘12 by 8’ tray.

Slide 21 When discussing Task B, highlight the value of being systematic and the links with factor pairs of 30.

Remind students that the chocolate factory only produces bars that are at least 2 pieces wide so a ‘30 by 1’ bar does not need to be considered.

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Slide 22 Check that students have correctly identified all of the different types of 24-bars and 30-bars that need to be packed.

Slides 23–26 Discuss these two questions as a class, making sure that students understand the mathematical vocabulary of ‘highest common factor’ and ‘lowest common multiple’. Draw out that the highest common factor of 24 and 30 is 6, and that this means that a tray that is 6 wide could be used to pack 24-bars and 30-bars. The lowest common multiple of 24 and 30 is 120; this gives the shortest length of a tray that could fit all possible 24-bars and 30-bars. Check that students can explain how each of the 24-bars and 30-bars can be packed into a ‘120 by 6’ 720-tray.

Slide 27 Bring the discussion to a close by exploring other possible trays that could be used. Students may have found a different tray when completing Task C. Check that the 24-bars and 30-bars can all be packed in the tray and establish that a 720-tray is the smallest size of tray that can be used to pack all the 24-bars and 30-bars.

Practice question

Aim	Students apply their knowledge to an unfamiliar task
Materials	‘Practice question’ handout. It is not necessary to print this out: the question can be displayed on the board or read out.
Slides	Slide 28
Time	10 minutes

Distribute a copy of the ‘Practice question’ handout to each student. Give students a couple of minutes to work on the question individually and then discuss their thinking.

What the students might do and what you might do

Slide 28 Students may draw diagrams to help them to answer the exam question. The solution has been provided, but rather than using the discussion merely as a check of accuracy, it is important to focus on students’ thinking.

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