



MASTERY TEACHING IN FURTHER EDUCATION:

a handbook for practitioners

2023

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Introduction to CfEM

Centres for Excellence in Maths (CfEM) was a five-year national improvement programme, funded by the Department for Education (DfE), which ended in March 2023. The programme was tasked with delivering sustained improvements in maths outcomes for 16–19-year-olds, up to Level 2, in post-16 settings.

The programme researched what works for teachers and learners, and in the latter stages focused increasingly on an adapted mastery approach to maths suitable for post-16 learners, based on five key principles. Three overarching issues emerged from this:

- **Engagement and Resilience:** How can we engage learners, helping them to overcome negative attitudes, anxiety and fixed mindsets?
- **Teaching for Mastery:** How can we develop learners' deeper understanding, reasoning and problem-solving?
- **Responsive Teaching:** How can we deliver a maths curriculum that is responsive to gaps in learners' skills and understanding?

Centres for Excellence and delivery partners

The Centres for Excellence were 21 Further Education (FE) and Sixth Form Colleges selected to drive innovation and improvement in both their institution and beyond. Each worked with a network of providers to share practice across the FE sector. You can see the 21 centres on the **Education and Training Foundation (ETF) website**.

The programme was led by the ETF working with a range of expert delivery partners, including Association of Colleges (AoC), Pearson, touchconsulting and University of Nottingham.

Processes and outputs of the programme

Key processes and outputs have been grouped under four headings:

Teaching and learning resources: practical resources for FE teachers, including schemes of learning, mastery lessons, and guidance on using mastery approaches.

Evidence: from research, including a large-scale randomised control trial, action research, and research based on scaled-up versions of promising action research interventions.

Professional development: through action research and lesson study, CfEM promoted a collaborative approach to professional development and championed practitioner networks.

Whole college approach: how to plan and implement organisational change that leads to improved mathematical outcomes.

All these outputs can be accessed through the **CfEM resources and evidence website**.

Introduction to mastery

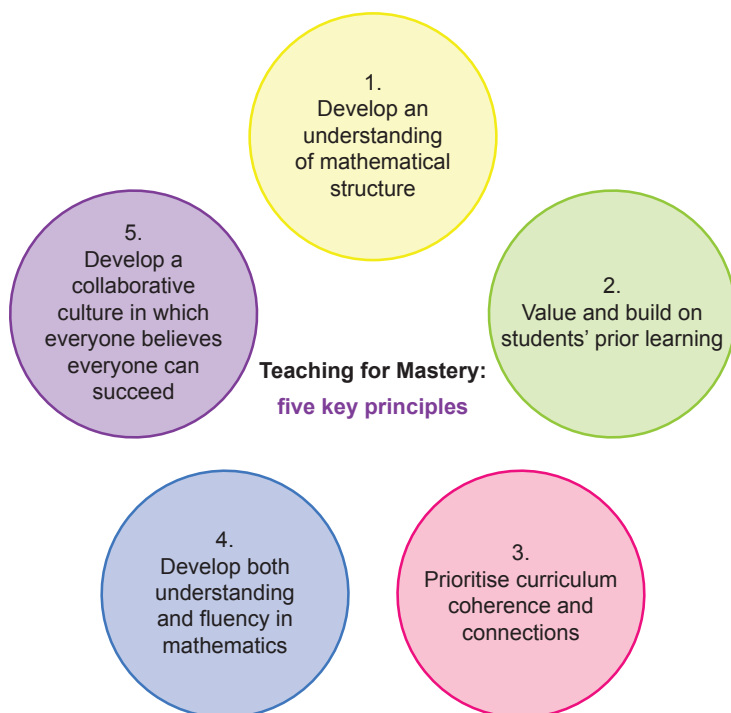
What is mastery?

When has a skill been mastered? For example, when top athletes reach the pinnacle of their sport, can they no longer improve? Do they stop making errors? Likewise, when has a mathematical skill been mastered? Is it when a student has scored full marks in a topic test? Will they necessarily still be able to show an error-free performance in a week's time, a month's time, or later?

In the context of maths, mastery needs to be distinguished from proficiency. Although the National Association of Mathematics Advisers (NAMA) note that there is no single clear definition of 'mastery', they also observe that there is much common ground. For example, when Benjamin Bloom wrote about mastery learning in the 1960s and 1970s his work was underpinned by the notion that all students, given enough time and effort, could succeed and improve. This remains a key aspect of the notion of mastery today.

The research evidence for the CfEM model of Teaching for Mastery

As part of the CfEM programme, the Centre for Research in Mathematics Education of the School of Education at the University of Nottingham carried out a large-scale randomised controlled trial of a programme that explored whether the Teaching for Mastery approach could improve GCSE resit student scores. The trial looked at two models of intervention, each of which was guided by the TfM in FE key principles and included elements of professional development as well as five lessons that exemplified how the principles can inform teaching in classrooms. In one of these models the teachers were additionally involved in a lesson study process in which they came together with colleagues to discuss their TfM in the lessons. This particular model was found to have an impact on students' GCSE scores, particularly for students from the most deprived backgrounds. The impact for these students was equivalent to an additional two months of learning. For all students taught by teachers who took part in the lesson study programme, the impact was equivalent to one additional month of learning.





Mastering maths means pupils acquiring a deep, long-term, secure and adaptable understanding of the subject¹

National Centre for Excellence in the Teaching of Mathematics (NCETM) ””

What does mastery mean for teaching FE maths?

There are inevitably going to be challenges in adopting the mastery approach at post-16. Students arrive with gaps in their knowledge; they are not often fluent in calculations and lack understanding, having relied on rote learning. Moreover, they often come with deep-seated negative attitudes towards maths based on previous experiences. The five big ideas from NCETM, however, are transferable to post-16 settings. This can be demonstrated by considering each of the four ways the term mastery is used, as described by Askew *et al.* (2015).⁴

A mastery approach

This is the belief that anyone can succeed in maths. The apparent sense of failure felt by many students who have not achieved the expected standard at GCSE can have a major impact on their motivation (Higton *et al.*, 2017)⁵. Investing in the development of a ‘can do’ attitude is key.

A mastery curriculum

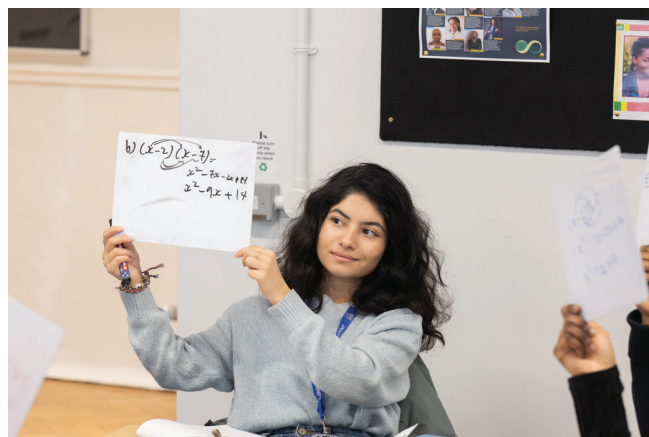
A mastery curriculum features the development of a connected pathway. In Further Education (FE), this needs to take into account both the limited time available to FE providers and the fact that much, if not all, of the curriculum has been met before.

Teaching for mastery

Pedagogic practices that teach for mastery allow students to gain proficiency and understanding. As shown by the NCETM’s five big ideas, some of these strategies are already embedded in FE practice and can be developed further.

Achieving mastery

This means that students know ‘why’, ‘what’ and ‘how’ and are able to use their knowledge flexibly rather than just memorise procedures. In particular, this aspect empowers students to better access the problem-solving-type questions that are often omitted by students who attain lower grades.



Key principles

Taking the research and recommended pedagogic ideas into account, there are five key principles that this handbook will consider, demonstrating the implementation of mastery in an FE setting.

Key principles

Key principle 1: Mathematical structure

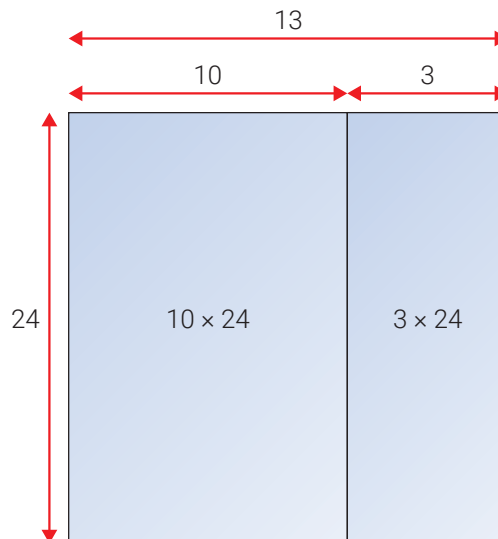
Teaching that allows students to develop an understanding of mathematical structure

The key point of this principle is to understand how representation can be used to unlock understanding so that students know the 'why' and not just the 'how'. Representations can both clarify the meaning of a concept and provide access to the structure of mathematical problems.



Bar model

Similarly, an area model can be used to illustrate that $13 \times 24 = 10 \times 24 + 3 \times 24$



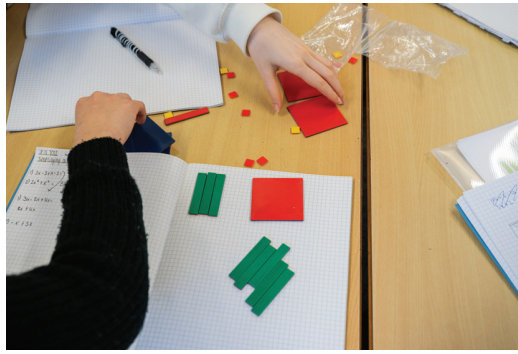
Area model

This can lead on to illustrate the expansion of brackets: $a(b + c) = ab + ac$

What does the research show us?

Evidence from both Bruner (1966)⁶ and the EEF clearly shows the use of manipulatives and visual representations help support problem solving for those struggling with maths. Bruner described the most effective way to explore mathematical concepts using *concrete* manipulatives, *pictorial* methods (use of pictures) and solving problems using abstract *details* (*numbers, letters and symbols*). Whilst the EEF report 'Improving mathematics and Key Stages 2 and 3 significantly notes that 'manipulatives and representations can be used to support students of all ages'.⁷

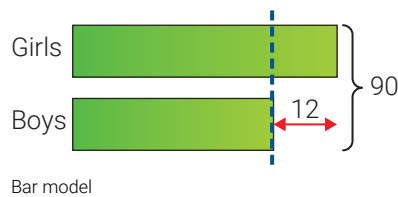
How to put this into practice



The use of representations must be specifically taught. For example, consider this problem.

There are 90 students at a party.
There are 12 more girls than boys.
How many boys are at the party?

At first sight many teachers will see this as an algebraic problem that could possibly be solved through simultaneous equations, but this can be easily represented as a bar model.



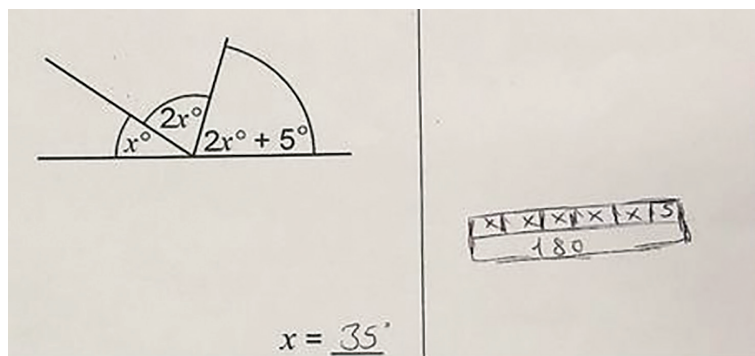
The bar model does not solve the problem, but it shows the way forward: $90 - 12 = 78$ for the two equal parts, so there are $78 \div 2 = 39$ boys. Students are unlikely to come up with this representation independently and teachers will, at first, need to supply the model and use careful questioning to support them with its use.



Questions you can ask your students

What do we know about these two bars?
Which bar represents the boys?
Where does the 90 go?
Where does the 12 go?
What can we work out now?

As students become more experienced with the modelling they will become adept, particularly if they are used to employing similar models in other areas of the curriculum.



This is an example of a student's work during a trial lesson, through the University of Nottingham's Teaching for Mastery trials.

Key principle 2: Prior learning

Valuing and building on students' prior learning

Teachers can celebrate and build on what students already know and make maximum use of the teaching time available to fill in key gaps in knowledge and understanding. There may be a number of misconceptions that need to be unpicked.

What does the research show us?

Higton et al. (2017) stress the need to examine prior learning to identify gaps in knowledge and develop appropriate learning plans.⁸ They also note that 'peer-to-peer activities allow students with deficit in a given concept to learn from another student and allow the more advanced student to embed their own knowledge'. Smith et al. (1994)⁹ explore how misconceptions arise and the need to build on current knowledge to develop deeper understandings, unpicking where errors occur and building from there. Likewise, Ryan and Williams (2007)¹⁰ state that correcting errors alone is unlikely to be an effective response, and that teachers need to tackle misconceptions at a deeper level.

How to put this into practice

The extent of prior learning can be checked in several ways without the need to use large amounts of curriculum time carrying out long formal assessments.

- Assessment for learning during lessons is key. A lesson can start with exploring what is known about a topic and careful questioning throughout can establish the class and individual needs. Multiple methods may emerge which will give you deeper knowledge of prior understanding and common misconceptions. This could encourage comparison of methods that develop students' mathematical thinking.
- Question-level analysis of students' performance in previous GCSE sittings can reveal key areas for improvement.
- Diagnostic questions are a useful tool to check understanding and can be used flexibly throughout a class. A well-structured multiple-choice question will have the correct answer and a series of wrong answers that each reveal a specific misconception. For example:

Work out $-3 + (-2)$:

- A** -5 (correct answer)
- B** -1 (student has done $-3 + 2$)
- C** 5 (thinking the two negatives 'cancel each other')
- D** 1 (student has done $3 + -2$)

Doubling two lengths makes the volume four times bigger



Charlie



Dee

Doubling all three lengths makes the volume six times bigger

Diagnostic teaching

Particularly when teaching post-16 learners, it is important that schemes of learning are used flexibly to take account of learners' prior knowledge and experience of maths. Using Question Level Analysis (QLA), at both learner and teacher level, can be an effective way of identifying any gaps in knowledge, skills and understanding. Teachers are then able to make the connections between any gaps identified and adapt their teaching and scheme of work to reflect these.



Learners received back a checklist of gaps and teachers did additional small-group sessions to fill those gaps identified in knowledge and skills, as required.

(Action research)

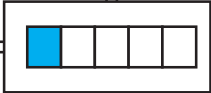
How do FE maths learners respond to diagnostic teaching? East Kent College [video](#)

Name: _____

ENTRY TICKET

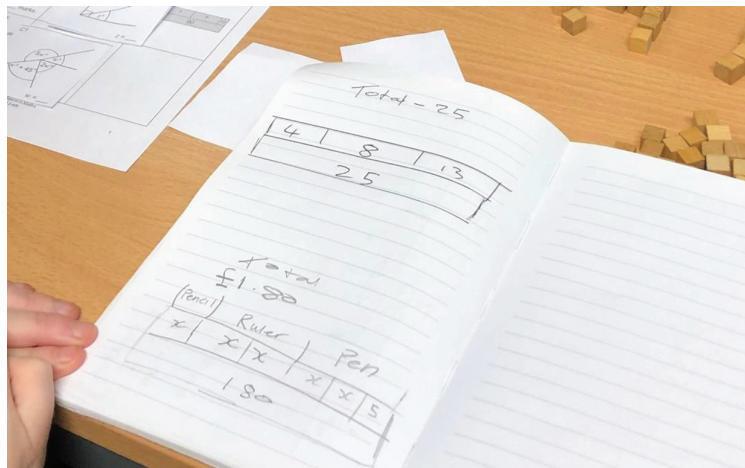
What proportion of the shape is shaded?

Describe as a Fraction	Write down 3 equivalent fractions
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Describe as a Percentage	Describe as a Decimal
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Use of entry and exit tickets

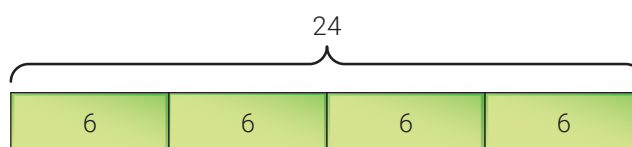


Key principle 3: Curriculum coherence

Prioritising curriculum coherence and connections

Students need to be encouraged to see the links between mathematical concepts (for example, ratio, scale, and similarity) rather than seeing them as separate content that needs to be individually learned. As well as these curricular links, using familiar representations across different topics develops flexibility in their use and supports problem-solving.

For example, consider this bar model:



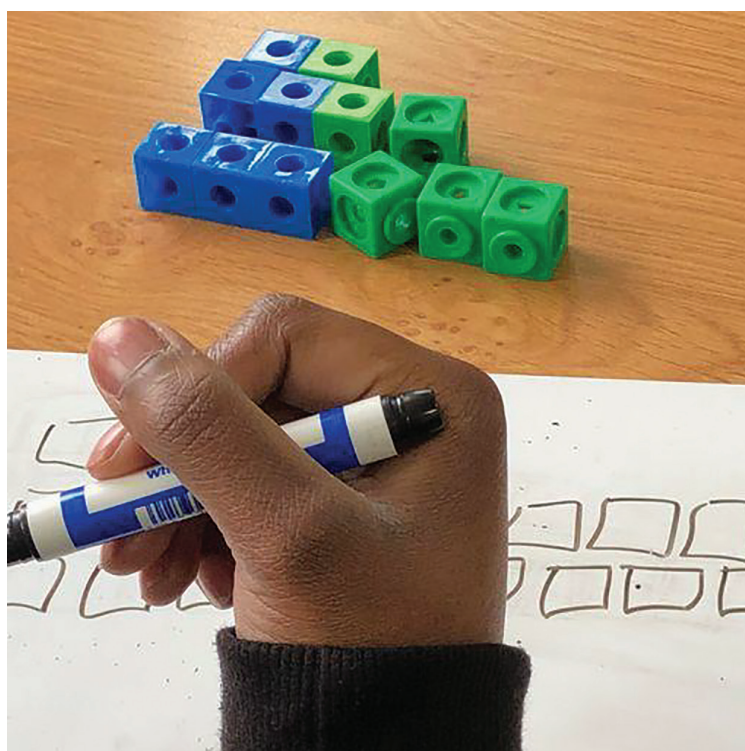
As well as the links between addition and subtraction, repeated addition, and multiplication and

division, the model can also be used to consider fractions $\left(\frac{1}{4} \text{ of } 24, \frac{2}{4} \text{ of } 24, \frac{3}{4} \text{ of } 24, \frac{4}{4} \text{ of } 24\right)$.

This can easily be extended to link other key areas of the curriculum such as ratios and percentages. Using a consistent model helps to establish links and make the journey through the curriculum more coherent for students.

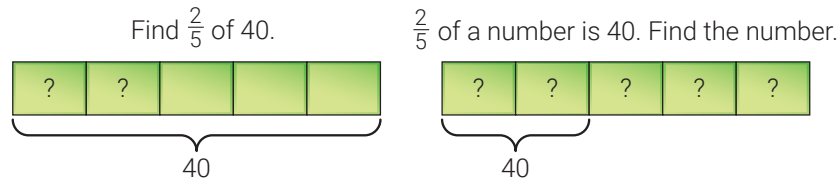
What does the research show us?

Oates (2011)¹¹ argues curriculum coherence is more than just the order of content, but also needs to link to assessment, pedagogy and teaching materials. It is important that teaching emphasises the connections between mathematical facts, procedures and concepts.¹² The NCETM suggests that to have an effective mastery curriculum, small, connected steps must be carefully sequenced, allowing students to move on to the next stage.



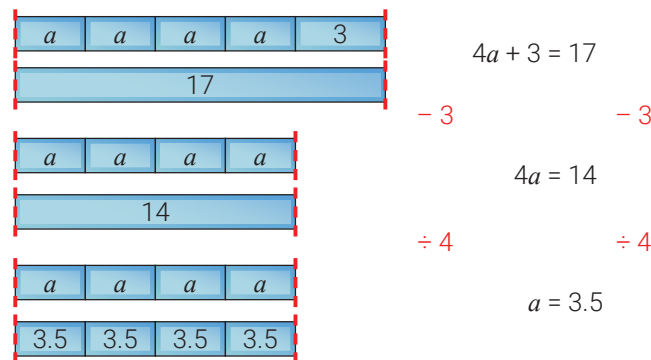
How to put this into practice

Maths is a network of connected ideas and skills. When planning lessons, consider what skills underpin the lesson. Identifying the correct starting points will be vital.

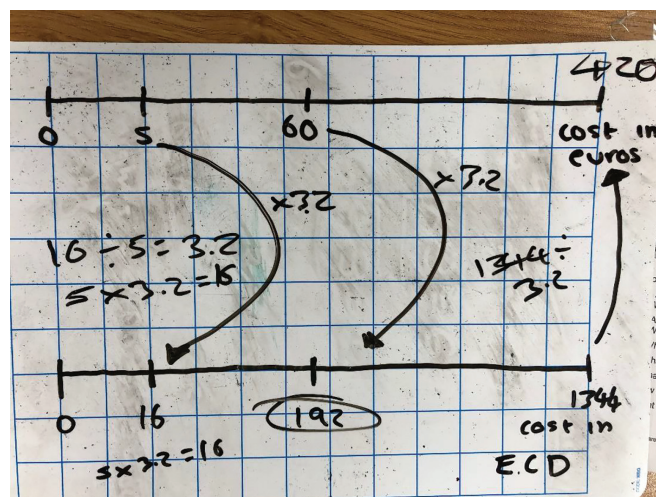
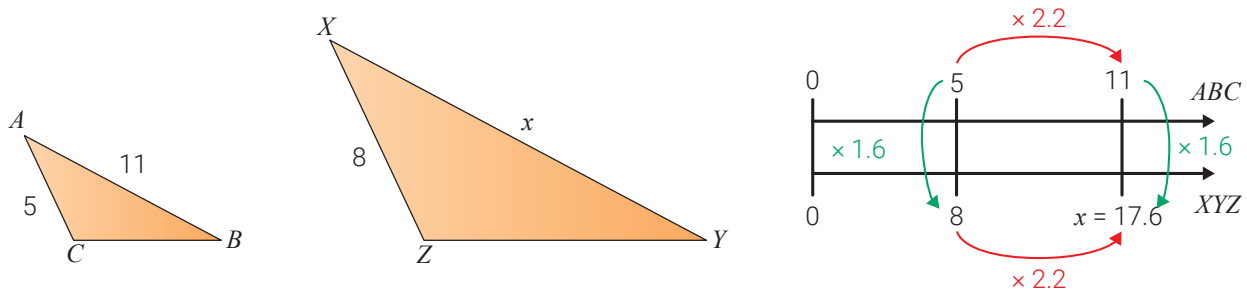


The same model could be used to consider sharing in ratio, increasing or decreasing by a percentage, or finding the original amount.

More than one bar can be used to compare quantities, for example, as an alternative method for sharing in a ratio or for modelling solving equations alongside the abstract method.



Another key representation is the double number line. This is particularly useful for multiplicative reasoning, a key curriculum area with many links. The double number lines below illustrate their efficacy in exchange rates and similarity.



An example of a students' working using a double number line during the University of Nottingham's Teaching for Mastery trials.

Key principle 4: Fluency and key ideas

Developing both fluency and understanding of key ideas

Covering key content in depth to attain fluency and understanding that can be applied in different contexts is preferable to superficial coverage of a larger amount of material.

It is important to remember that fluency is not just about knowing facts and procedures, but also how and when to use them.

One of the three key aims of the national curriculum for maths is that students 'become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately'.¹³ Consider this problem:

The table below shows the probabilities that a die lands on the numbers 1, 2, 3 and 4.

Score on die	1	2	3	4	5	6
Probability	0.1	0.25	0.2	0.05	x	$2x$

Work out the probability that the die lands on 6.

To solve the problem, you need to:

- recall that the sum of the probabilities of all outcomes of an event is 1
- know how to simplify an expression containing numbers and algebraic terms
- be able to add decimals accurately
- form and solve an equation in x
- multiply x by 2 to obtain the required answer.

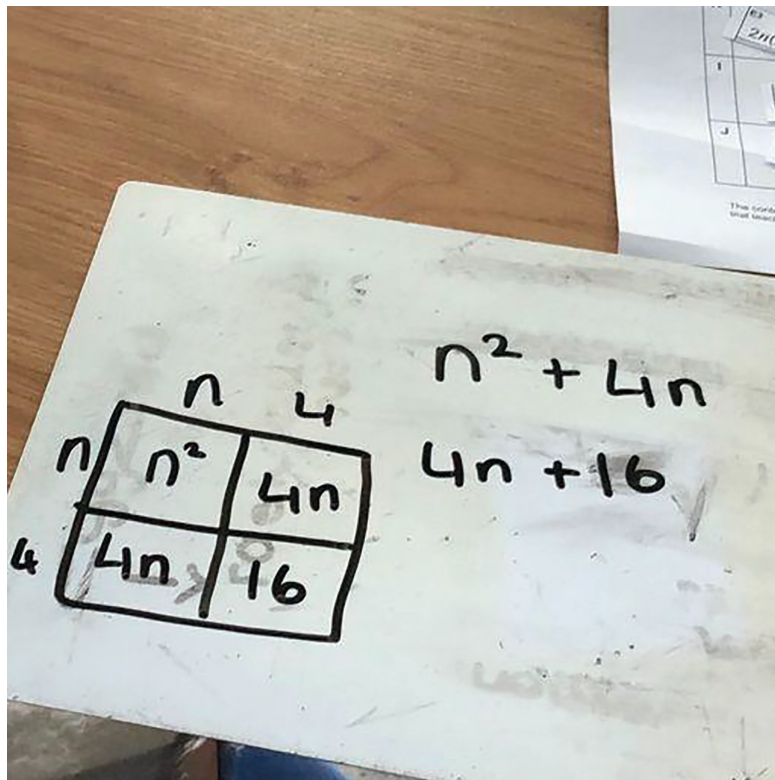
There is a lot of maths involved in this comparatively short and simple problem, and the greater the fluency with each step, the more likely a student is to find an accurate solution promptly.

What does the research show us?

Russell (2000)¹⁴ describes three aspects of fluency:

1. efficiency (easily carried out so that the logic of the strategy is not lost)
2. accuracy (including knowledge of number facts and relationships)
3. flexibility (choosing an appropriate strategy for the current problem).

Lynne McClure, writing for NRIC in 2014¹⁵, cites Hiebert (1999)¹⁶ stating that it is difficult for students to see the meaning in maths when they are just memorising mathematical rules. She suggests the use of manipulatives (concrete objects), talking about their work and consolidating in meaningful contexts as ways of supporting fluency.



How to put this into practice

- GCSE maths is a very broad curriculum, which cannot easily be covered in the 30 weeks available to most resit learners. Instead of trying to cover everything superficially, try focusing your scheme of learning on a limited number of key concepts which can make the biggest difference to learners' chances of achieving a grade 4.
- Mathematical talk is a strong vehicle to develop fluency and encourage flexibility in the classroom. Students could be challenged to work in pairs to consider how many ways they can find to mentally work out a calculation such as 35% of 80. Typical responses might include:

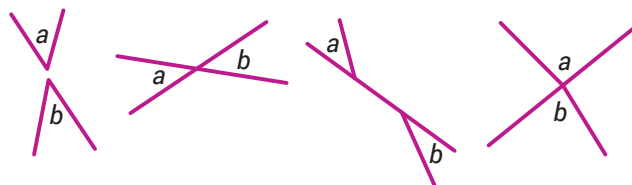
10% of 80 = $80 \div 10 = 8$
 30% of 80 = $3 \times 8 = 24$
 5% of 80 = $8 \div 2 = 4$
 So 35% of 80 = $24 + 4 = 28$

10% of 80 = $80 \div 10 = 8$
 25% of 80 = $80 \div 4 = 20$
 So 35% of 80 = $8 + 20 = 28$

10% of 80 = $80 \div 10 = 8$
 20% of 80 = $2 \times 8 = 16$
 5% of 80 = $8 \div 2 = 4$
 So 35% of 80 = $8 + 16 + 4 = 28$

Students could then be presented with similar questions (work out 45% of..., 75% of..., 90% of..., 11% of...) where the focus of learning is not just the fluency in calculation but the choice of method.

Students' understanding is established as they consider not only examples of what a concept is, but also examples of what it isn't. This initially challenging task can be scaffolded by providing a series of examples and non-examples for the students to categorise, for example, for the concept 'vertically opposite angles'.



Key principle 5: Belief in success

Developing a culture in which everyone believes everyone can succeed

There is a widely held view that maths is a subject that you are either 'good at' or not, and that nothing can be done to change that situation. Post-16 resit students often believe that they have already in some sense 'failed' at maths. Indeed, many may have been in low-attaining lesson sets for a number of years, reinforcing their negative connotations about the subject.

To challenge this, it is necessary to create a belief that everyone can succeed and that, through effort, improvement in maths is possible, just as in any other challenge. The culture needs to recognise all students' starting points and emphasise that the first step is improvement, and that this takes time. In their first year of post-16 study an improvement from, say, grade 1 to grade 2 is an achievement that should be celebrated as a step on the journey.

Given students' prior experiences and attitudes, creating this culture is not easy. Shared language, understanding and a collaborative approach to learning can help to establish a low threat/high challenge culture that will increase students' self-belief.

Find out more

The CfEM Motivation and Engagement Handbook links to this key principle and has more on the importance of increasing students' self-belief.

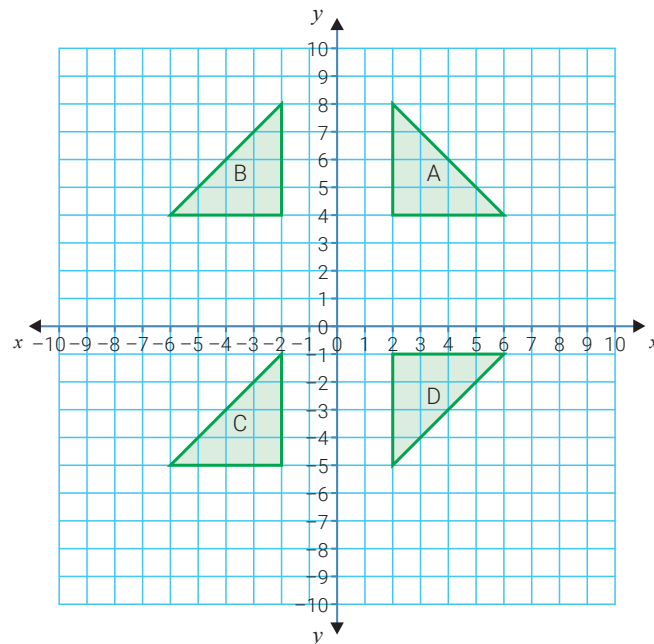
What does the research show us?

Hough et al. (2017) note that GCSE resit students experience challenges because their prior experience of learning maths has had a negative effect on their confidence and motivation.¹⁷ Likewise, Highton et al. (2017) state that cultivating more positive student attitudes is key, and advocate peer learning as a strategy to refine ideas and build confidence.¹⁸ There has been much work on developing growth mindsets, popularised by the psychologist Carol Dweck, arguing that ability is not fixed and that changing students' and teachers' beliefs has a positive effect.¹⁹ Boaler also argues that valuing incorrect answers is useful and helps form stronger connections in the brain.²⁰



How to put this into practice

Overcoming students' self-perceptions as 'failures' can be challenging, and many may be reluctant to engage in mathematical activity at all for fear of 'getting it wrong'. Try using goal-free problems. For example, students could be presented with this diagram:



Rather than a series of closed questions such as 'Describe the transformation to get from shape B to shape D', ask students 'What can you find?'. The threat of getting a wrong answer is removed as students can name coordinates, discuss the properties of the shapes, spot different transformations, and so on. The threat is reduced further by setting the task to be done in pairs or groups, encouraging peer-to-peer collaborative learning. In addition, this type of activity links to Key principle 2, as the responses give the teacher a good understanding of what students do and don't know.



We feel part of the learning process, and even when I make a mistake, I do not feel I am being judged but encouraged to try in different ways.

Student from Harlow College ”

Further reading

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- ² Education Endowment Foundation (2017), *Improving Mathematics in Key Stages 2 and 3*. Available at: <https://educationendowmentfoundation.org.uk/tools/guidance-reports/maths-ks-2-3>
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- ⁴ Askew, M., Bishop, S., Christie, C., Eaton, S., Griffin, P. and Morgan, D. (2015), *Teaching for Mastery: Questions, tasks and activities to support assessment* (Oxford University Press: Oxford)
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- ⁸ Higton, J., Archer, R., Dalby, D., Robinson, S., Birkin, G., Stutz, A., Smith, R. and Duckworth, V. (2017), *Effective Practice in the Delivery and Teaching of English and Mathematics to 16–18 year olds* (Department for Education: London, UK). Available at: <https://www.gov.uk/government/publications/english-and-maths-for-16-to-18-year-olds-effective-teaching>
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- ¹¹ Oates, T. (2011), *Could Do Better: Using International Comparisons to Refine the National Curriculum in England* (Cambridge Assessment: Cambridge, UK). Available at: <https://doi.org/10.1080/09585176.2011.578908>
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- ¹³ Department for Education (2014), *Mathematics programme of study: key stage 4 National curriculum in England* (Department for Education: London, UK). Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/331882/KS4_maths_PoS_FINAL_170714.pdf
- ¹⁴ Russell, S.J. (2000), 'Developing Computational Fluency with Whole Numbers', *Teaching Children Mathematics* 7(3), 154–158. Available at: <https://www.jstor.org/stable/41197542>
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Credits

Photos

The Education and Training Foundation; Shutterstock.com: Vectorfair.com

City College Plymouth: Alternative Learning Environments – Exploring maths teaching outside the traditional classroom environment.

University of Nottingham: The images are taken from the 'teaching for mastery randomised control trial' carried out by the University of Nottingham during 2021–2022.

Text Acknowledgements

The Education and Training Foundation
