CENTRES FOR EXCELLENCE IN MATHS

To explore the effectiveness of using representations in an FE setting. Specifically in the context of answering the AO3 Questions.

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OUR PARTNERS

FUNDED BY

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Acknowledgements

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About CfEM

Centres for Excellence in Maths (CfEM) is a five-year national improvement programme aimed at delivering sustained improvements in maths outcomes for 16–19-year-olds, up to Level 2, in post-16 settings.

Funded by the Department for Education and delivered by the Education and Training Foundation, the programme is exploring what works for teachers and students, embedding related CPD and good practice, and building networks of maths professionals in colleges.

Summary

This Action Research project sought to address the low marks attained by GCSE resit learners in A03 problem solving questions. We continued work that had been started last year, exploring the effectiveness of representations in FE settings and focussed on the use of these as a tool for approaching multistep problems, specifically using ratio tables and bar models and working on ratio, proportion, fraction and percentage topics.

The research worked through three cycles: - using ratio tables, introducing bar models and then focussed use of bar models in ratio. Ratio tables were introduced over a couple of weeks developing proportional reasoning, learners then developed their understanding of bar models (supported by centicubes) for four weeks, before the final focus on ratio for a further four weeks.

Assessment data were collected through learners completing an Edexcel A03 question (the Mints Question) before and after the interventions; this data was triangulated with two further A03 questions (the Bungalow and Cakes questions) and learner and teacher appreciative enquiry interviews.

The data from students who had completed the before and after assessment question showed that 70% of learners improved the marks they achieved after the interventions, and 55% rated their confidence in attempting these questions as improved. Common themes from the learners' feedback were that the interventions helped with their understanding and helped them attempt more questions. Teachers reported the same and additionally that student engagement and resilience was improved along with personal teacher enjoyment also.

The research concluded that physical and visual representations have a positive impact on student confidence and achievement in AO3 questions. The use of these needs to be introduced early, starting with familiar topics, and be sustained through the scheme of learning and revisited regularly. It also found that using open ended questions, such as 'If we know …., then what else do we know?" were critical for developing learner's own thinking and efficacy.

Contents

Introduction and Background

The Colleges:

The local populations of the five colleges involved in the Action Research project are mostly white, with a low proportion of ethnic minorities, which reflect populations. There are mostly high employment levels within the South-West, but all colleges also have some areas of high deprivation. Weston and South Devon are dependent on seasonal tourism. The maths progression for the colleges is generally improving and from 2017 to 2019, progression data ranged between -0.21 to +0.28. Two of the colleges taking part in this study showed significantly higher progression than the national average of +0.09. The other colleges were below the national average ranging from -0.6 to -0.21. (If the score is positive students are on average making progress, if negative they have on average achieved lower grades from their entry point). www.compare-school-performance.service.gov.uk/compare-schools?for=16to18

The development of this Action Research Project.

This work followed on from the work we did last year. Cockerton (2021) highlighted the problems involving proportional relationships, specifically those involving direct proportion. This year we wanted to focus on exploring the effectiveness of using representations in an FE setting whilst answering the multistep AO3 Questions involving Ratio, proportion and rates of change. These questions are generally answered poorly or ignored by learners in FE. The table below shows national achievement data for the three questions we used as an assessment measure.

Edexcel AO3 Questions	Average marks achieved for this Question	Marks achieved from Grade 4 learners.
Mint Question 5 marks	2.25	3.7
Bungalow Question 3 marks	0.88	1.75
Cake Question 5 marks	2.24	3.8

In this action research, we continued to explore different ways of thinking about and visualising proportional relationships. Ratio tables were introduced as a tool for illustrating different methods for operating on proportional relationships and for investigating the multiplicative nature of these relationships. The main findings from last year were that there was a significant improvement in student outcomes on number and the use of the Ratio Table was acknowledged by the teachers as having influenced this.

The results of this research indicated the effectiveness of our mastery approach and we wanted to extend the project by combining models in one exam question to further help learners to be able to answer, or at least gain more marks for multi-step GCSE maths questions for all exam boards.

Literature Review

AR1: Effectiveness of Representatives in an FE Setting

Conclusions and Recommendations

- Representatives and manipulatives have a positive impact on mathematics learning and learner confidence and success in word problem-solving.
- The introduction of bar models should be made using mathematical topics learners are already confident with before introducing their use for new mathematical concepts.
- The use of bar modelling is particularly effective when learners develop their own representations for problems.
- The understanding of the use of bar models is supported by the use of concrete materials.
- The timescales, teaching activities and verbal questioning techniques used with the introduction of representatives should allow sufficient time to learn how to use the representatives.
- Teachers' professional development is a key aspect in the successful use of representatives, teachers need to develop their own experience and confidence.
- Teachers needed to develop questioning techniques so not to give answers but encourage learners to explore their own deeper thinking. Teachers can ask questions such as "What else do you know?" rather than "How much for 30 people?"

Introduction

Our Action Research aimed to explore the effectiveness of using representatives in a Further Education setting, particularly in our learner's attempts to answer the problem solving (AO3) questions in the GCSE exam papers.

This literature review examines articles which support the choice of this area as a worthwhile Action Research project and then seeks to learn from what the current literature can tell us about how best to plan, design and carry out our Action Research interventions. The key themes have been grouped for ease of reading.

In mathematical terms, we use the word representations (or representatives) as one way of labelling the many methods and strategies we may encounter. The National Council of Maths Teachers' definition of representations may be useful in understanding this further:

"The act of capturing a mathematical concept or relationship in some form and to the form itself."

Our primary focus is on the use of bar modelling; however we also consider other representatives such as the double number line and ratio tables, and concrete manipulatives such as centicubes and algebra tiles.

The Effectiveness of Representatives in teaching and learning mathematics

Use of the concrete (hands-on, practical manipulatives) and the pictorial (visual representations) in the teaching and learning of mathematics has long been common in primary schools and has been shown to have a statistically significant positive effect on learning when compared to teaching with only abstract mathematical symbols (Back, 2019)

In his book for teachers, Visible Maths, Mattock (2019) describes and promotes a range of representatives which support the teaching and learning of the vast majority of the UK maths curriculum from primary school to GCSE, sharing his experience gained as Secondary maths teacher. He refers to the evidence for and promotion of representatives by organisations such as Ofsted and the National Centre for Excellence in Teaching Mathematics (NCETM.)

> Ofsted also recognises the importance of the use of manipulatives and representations to support flexibility in pupil thinking. In their Mathematics: Made to Measure report from 2012, it is noted that schools should choose 'teaching approaches and activities that foster pupils' deeper understanding, including through the use of practical resources and visual images.

> The NCETM lists representation and structure as one of the 'Five Big Ideas' in teaching for mastery. The NCETM make it clear that using appropriate representations in lessons can help to expose the mathematical structure being taught, allowing pupils to make connections between and across different areas of maths.

Saraswati et al (2016) researched the use of algebra tiles in solving linear equations finding this to have a successful impact on learner achievement in this topic, in particular they found that using these manipulatives minimised common mistakes and that student's understanding developed from an informal to a formal level.

However, both Back (2019) and Hough et al. (2019) identified that manipulatives and representatives can still become a taught procedure that learners blindly follow to arrive at an answer. During Back's research she often saw one specific manipulative appear to teach a given concept and then disappear never to be seen again in the classroom, leaving learners mystified about what approach to use. Likewise, Hough et al.'s reflection on the Singapore Bar approach, which requires learners to be able to identify the correct bar model for the problem-type (a learned, rote process), found that this tends to result in lowerattaining learners misusing them, as they try to replicate another method they haven't quite understood or remembered correctly.

Mattock (2019) also identifies this issue, that the use of the representative is not the end goal in itself and if learners don't make the leap to a deeper understanding of the concept, they can just become another misunderstood and therefore often misused procedure. He advocates that to avoid this, multiple representations need to be used to ensure learners have a range of tools available to them.

> Representations are tools that provide a window into the underlying structure of a concept. They are a window that pupils can keep coming back to look into, but they are not a window they should continually have to stare through. There is a danger that representations become another procedure that pupils have to remember and apply without understanding; this must be avoided at all costs if pupils are going to work towards mastery of mathematical concepts.

Gifford (2020) also advocates the need for a range of representatives to develop deeper mathematical thinking.

> We also need to challenge the notion that there's only one right way to approach a problem. It's a reminder that mathematics is wholly creative, and we should embrace the differences in the representations we see.

We wanted to investigate how representatives can deepen mathematical understanding and specifically enable GCSE resit learners to feel confident to successfully tackle problem solving questions. We wanted to see if, and how, representatives can be more than 'just another method' and be an approach which will improve our leaners' long term mathematical thinking.

Learner Led Representatives

Hough et al. (2019) concluded that the introduction and encouragement to use bar models and other representatives needs to be a bottom-up approach, building from students' own models for problems, not just another method presented to students which they may or may not remember and apply correctly. Similarly, Back (2019) saw learners using manipulatives as a rote learned procedure with learners not gaining a sense of the ways in which the concrete and pictorial reflected mathematical structures.

Dickinson et al. (2014) describe the term progressive formalisation as a

> "learning sequence that begins with informal strategies and knowledge, and then develops into pre-formal methods that remain linked to concrete experiences, models and strategies. These models and strategies then develop progressively into more formal and abstract mathematical procedures."

They explain that it is crucial within this process for learners to experience a shift in their view and use of their models – from a 'model of' a situation (where the model is closely connected to the context) to a 'model for' (where the model is more abstract and can be used as the basis for mathematical reasoning and problem solving.)

An example of this, where the sharing of sandwiches evolves into a fraction bar (bar model) and then a double number line, is shown below.

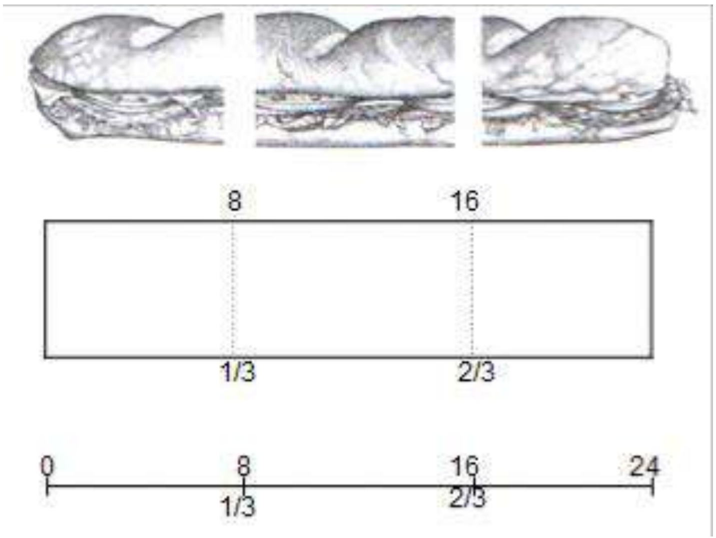


Figure 1. Progressive formalisation of models (Dickinson and Eade, 2005.)

Dickinson et al. believe that progressive formalisation is crucial in the use of representatives in the classroom. Teachers rightly desire, and expect, their learners to understand formal methods and procedures but Dickenson et al recognised that learners need opportunities and time to work towards this aim through the use of representatives and visualisations.

This suggests that, in developing learning activities and resources, it is best to start with contexts and models that naturally lend themselves to being represented with a bar, number line, ratio table etc. And to then introduce a wider range of contexts and problems, allowing learners the opportunity to try developing models which work for them.

Both Hough et al. and Back promote the need for learners' to develop and use representatives which they are familiar with and have meaning for them, enabling them to make sense of a problem.

> Manipulatives do have a place as computational tools to support various calculation strategies and demonstrational tools to expound a procedure but it is only when learners themselves use the artefacts to support their own sense making processes that they will begin to see their power as tools for calculation and not just rely on them as crutches to support them in blindly following taught procedures (Back, 2019)

Dickinson et al. (2014) similarly found that,

> It is as important as ever that students are allowed to work informally and 'make sense' of situations before developing more formal mathematics.

The challenge for us, as Action Research teachers, is to introduce representatives in such a way as not to suggest they are 'the one correct way' of answering a question, but in such a way to encourage learners to develop their own use of representatives that make sense to them.

Problem Solving

Widyasari and Rosiyanti (2018) reviewed the suitability of materials developed to promote problem-solving through bar-modelling. They defined problem-solving, and some of the issues with current teaching methods, as being:

> " not merely a form of being able to apply rules that have been mastered through past learning activities, but more than that is the process of obtaining rules at a higher level … low ability of problem-solving due to the lessons used by teachers is still dominated by ordinary learning models that give less emphasis on the application of mathematics in everyday life, and students are not accustomed to [being] involved in solving problems. Students tend to solve a problem by mimicking the problem solving that the teacher exhibits when discussing the problem."

They developed and trialled teaching material around bar modelling for problem-solving. The resources are aimed at grade 4 Indian children (not written in English), so not resources we can trial or adapt in our own Action Research project. However, they were found to:

- be well designed,
- correspond well to the theme/problem/context,
- include easy to understand language,
- increase interest in problem solving,

And these are all elements we need to consider in the teaching resources and materials we use or produce to trial in our Action Research.

Osman et al.'s (2018) research also looked specifically at the use of bar modelling as a tool to support the solving of word-problems. Along with finding that using bar models inspired creativity in problem solving, their research revealed that students could overcome their dislike of work problem-solving questions by using the bar model method.

> There were positive impacts not only on the results but also on the participants' attitude towards the questions. This method helped students to build and elevate their confidence as they were excited and courageous enough to try answering all the questions given.…It enhances students' understanding regarding problem-solving as it helps to visualise and solve the problems. Other than that, it can be applied to many mathematical

> operations and topics. It can help students to apply these concepts to a variety of challenging word problems questions.

As we designed the data collection and analysis for our Action Research interventions, we collected not only data which captured the extent to which the use of representatives and manipulatives developed learners' mathematical skills but also the extent to which their use increased learners' view of their self-confidence in tackling problem-solving questions and, further, whether there is a correlation between the two.

Pedagogical considerations

Dickinson et al. also identified that discussion was a key feature in teachers' appraisal of what had changed and improved with the use of representatives.

> One teacher described how discussion had enabled him to 'stop driving some elements of the lesson ... you can allow people to have time to find things out for themselves, to discuss what they're doing.' He goes on to acknowledge his changing teacher identity: 'It's a lot less about me driving them through something, explaining something for them to regurgitate .... I just kind of perch and find interesting things that are going on, then periodically stop people and say, "right, let's have a look at what you're doing". This teacher would appear to be beginning to develop the complex teacher role of framing student contributions.

This highlights the need for a very open approach to teaching when using representatives, using open questions, allowing learners time to explain their models and methods, being willing to allow the learners to shape the direction of the lesson (which can be a challenge in an intense one-year resit scheme of learning when teachers can feel under pressure to cover as much of the curriculum as possible.)

Ahmed (online) advises teachers on some of the pitfalls and common mistakes in teaching and using bar models. As a consultant for the Singapore Bar Model approach, his approach is prescriptive, and therefore somewhat contrary to the research of Dickinson et al, and other findings, but does contain some practical points worth considering in our Action Research. He advocates that core foundations have to be carefully taught through high-end questioning and visual modelling to get the most out of the bar model approach. Our Action Research interventions must be well planned, starting with basic concepts, and our deep and effective questioning techniques.

Another relevant point Ahmed makes is the need for questioning which edg the learner's thinking forward so that by the end of a question they feel they are the ones that solved the problem. He believes that the common mistake can be to give too much away, so that learners are not making progress themselves. This is a common potential teaching pitfall and one we sought to remain mindful of as we balance the desire for quick progress with the acquisition of deep, long-term understanding.

Sufficient time must be allowed for learners to learn how to use manipulatives and representatives and become confident in using them to solve algebra problems. Amidu et al.'s (2020) research investigated the effect of using algebra tiles as manipulatives in the teaching and learning of basic algebra (with pre-service teachers.) Their literature review found that in a previous similar study (Magruder, 2012) the control group actually showed a statistically significant better outcome, which was felt to be due to the time it takes to learn how to operate the manipulatives. In another study, (Akpalu, Adaboh and Boateng (2018)), there were statistically significant post-test scores in favour of the experimental group who were taught using the algebraic tiles for four weeks. This gives an indication of the timescales needed to plan for learners to become competent and confident in using these approaches. Amidu et al.'s own research concluded that learners needed time and support in becoming familiar with the tools.

> At the beginning, some of them were confused which colour-tile represented negative and which colour-tile represented positive, and also which piece represented x and which piece represented 1. After they learned these representations, they did not have any difficulty in using them.

We need to introduce representatives and manipulatives at the right pace and over a sufficient period of time, to allow learners to develop a good working understanding of how they can use them to solve algebra problems.

Ahmed also believes that not fully using the Concrete-Pictorial-Abstract approach is the first major mistake when introducing bar models and that it is important for teachers and children to use concrete materials for bars when attempting to use bar modelling as a strategy to solve problems. This suggested, for our Action Research, the need to provide and encourage the use of physical blocks by our learners.

Mattock (2019) also has some practical advice for teachers using representatives in the classroom, which were helpful to our Action Research interventions. He recognises that students can find it difficult to develop understanding of the use of a representative if it is introduced at the same time as a new mathematical concept. He recommends introducing the representative with a concept that learners already feel comfortable with, before using that representative to explore a new concept, using the representative as a 'bridge' to the new concept. In designing our Action research intervention, this suggested we needed to introduce bar-models to basic mathematical concepts first, before moving onto more complex concepts or problems.

Dickinson et al. (2014) concluded in their research that 'Professional Development [was] essential' if the use of representatives was to be effective. They commented that this is particularly true for teachers feeling the pressure of examinations, and that successful CPD in this area is a need for further study.

Ahmed also identifies that it is not just learners that need to gain experience with the approach and questions how we allow teachers to build their own confidence. This is particularly apposite for Further Education teachers who may have little exposure to this approach. To be effective in our teaching of the bar model approach, or any other manipulative or representative, we, the Action Research teachers, developed our own understanding of and confidence in these techniques. In reality, these only fully developed once we immersed ourselves in the Action Research and learned from our, and the learners' experience.

Conclusion

Reviewing the literature provided a strong rationale for this Action Research project, confirming the value of manipulatives and representatives in maths teaching and learning for all ages, and specifically identifying in previous research that these support learners in approaching and solving AO3 questions. It drew our attention to the need for providing sufficient time to allow learners (and teachers) to gain experience and confidence, so we introduced and repeated our interventions over a number of weeks and months. It also drew our attention to the suggestion that concrete and pictorial representations are best introduced to familiar topics before widening their use to aid understanding of new concepts, so we introduced bars for simple concepts we believe our learners were already confident in. Learners were encouraged to use manipulatives and draw representations which have meaning for them, rather than rote learning an approach, as when introduced well manipulatives and representatives are an effective tool to develop deeper understanding.

Methods

The Students and Teachers:

190 students experienced the Action Research Interventions however only **109** completed both the "before and after" AO3 Questions in the Action Research project. Here is a table showing their levels at outset and their colleges.

Colleges	GCSE grade U-2	GCSE grade 3
Weston College	**1**	**12**
New College, Swindon	**10**	**3**
Heart of Worcestershire	**28**	**17**
Wiltshire College	**1**	**24**
South Devon College	**4**	**9**
Totals	**44**	**65**

The Aims and Objectives:

We aimed to explore the effectiveness of using representations in an FE setting. Specifically in the context of answering the AO3 Questions.

We aimed to develop conceptual understanding of the following topics:
Number, especially four operations (N2), primes, factors & multiples (N4), fraction operations (N8)
Ratio, proportion & rates of change, especially. percentages (R9), compound measures & units (R1), ratio (R5), ratio with fractions (R8), direct & inverse proportion (R10)
Simplifying & manipulating algebraic expressions (A4)
Area & Volume (G16)

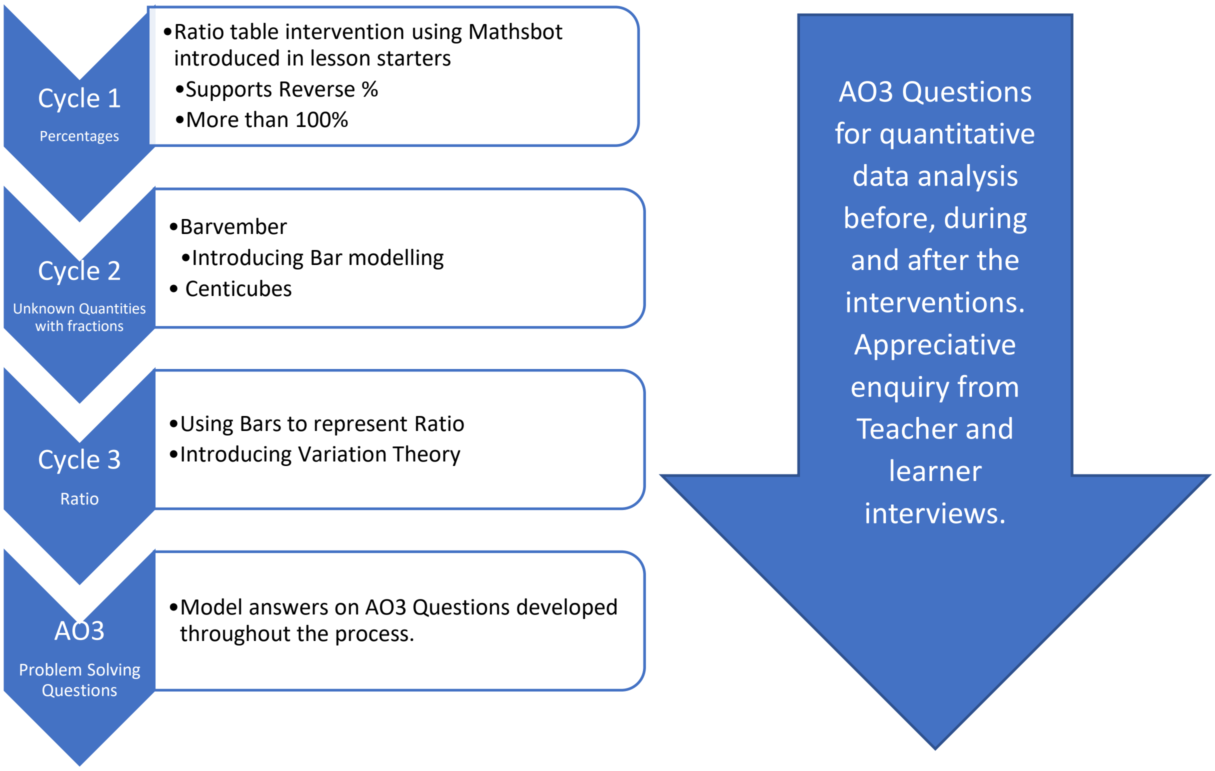
In practice, we focussed on Ratio, proportion & rates of change, and next year we intend to look at **Simplifying & manipulating algebraic expressions** (A4)

The Cycles:

We conducted 3 Cycles of collaborative planning, teaching, gathering evidence and reflection, based on our research. We practised and made sure we had mastered the techniques before teaching the lessons to our classes. It was vital that we were all confident so we could address any misconceptions or problems which may have arisen. We took photos of students' work and the Action Research Lead observed some lessons during teaching. After delivering each lesson we reflected on the lessons informally in the shared workroom and formally in writing and discussion. As part of this process we discussed how the lesson could be improved for next teaching.

Appendix A shows the lessons developed over Cycles 1,2 and 3

The table below shows each Cycle and how it developed and the processes we used for gathering evidence for evaluation.



During Cycle 1 teachers identified Mastery Approaches. We used the Ratio Tables. At this point teachers introduced the use of the centicubes. Teachers also developed questioning techniques so not to give answers but encourage learners to explore their own deeper thinking: with the support of the ratio table structure, we used questions such as What else do you know? What else could you find out? In Cycle 2 teachers then wanted to explore how bar modelling could be used to support learners in answering AO3 Questions. We used a readymade resource from White Rose Maths: Barvember. In Cycle 3 teachers were realising the usefulness of using bar models to answer both algebraic questions and ratio questions. We wanted to focus more on the ratio questions, so we developed a resource to scaffold learners using the same structure as the Barvember resource, however we used very small changes in the question. Throughout this process we used AO3 questions at the start and end of the Cycles. The interventions can be found in Appendix A

Collecting Evidence:

As well as collecting examples of student work in the lessons, we conducted interviews with the students to establish what they thought of the approaches using appreciative enquiry. We also used the same technique to gather evidence from teachers (See Appendix B)Throughout the process we used The Thinking Environment Kline,N (2009) as a collaborative meeting process. This enabled teachers to speak freely and share their experiences. We recorded the interviews with the teachers and learners and the data have been coded in Appendix C

Informed Consent Form sent to all students can be found in Appendix D

Data Collection templates can be found in Appendix B

Adverse Impacts on the research:

Covid: Teachers from some colleges were stretched and were unable to meet up on various occasions. Learners often missed their lessons due to isolation or symptoms of Covid. The Cohort was extremely weak with Grade 3 learners struggling as if they were grade 2 learners and the November resit showed this. This has been discussed in the results.

Results and Discussion

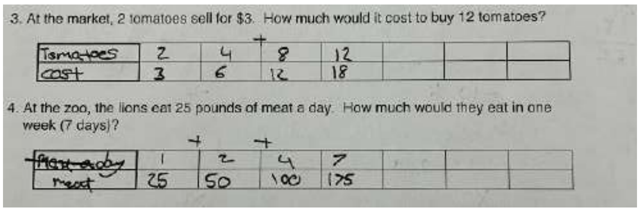
Description of interventions

The table below shows the timeline of the Action Research and at what point each intervention took place.

Month	Intervention	Measure	Detail
September	Ratio Table LTM Ratio Table booklet Activity 9 and 10. Completed and developed Appendix A	Learner Scripts	Teachers were given the Ratio Tables to try out for themselves and then to try out on learners. Reflections on these were captured for the first data analysis workshop. These were from Frykholm (2013). The reason teachers chose this intervention first was not all teachers were familiar with it.
October	Ratio Table Mathsbot examples in Appendix A	Learner Scripts and ongoing teacher reflections	Teachers continued to use ratio tables in the lessons as starters. Exposing the structure to the learners on a regular basis throughout the intervention period.
November	Barvember Examples in Appendix A	AO3 Question Mints	Teachers were not sure how to use bar models effectively. This intervention was used weekly for at least 2 months. Learners had to be constantly reminded to use the bar models so the teachers made scaffolded and un-scaffolded questions. Appendix A
December	Barvember	Ongoing teacher reflections	Teachers adapted materials to make sure learners were able to use the models on topics they understood before progressing to more difficult questions.
December:- Data Analysis Workshop			
January	Adaptations of Barvember and introducing Centicubes Appendix A	AO3 Question Bungalows	Teachers acknowledging that learners were still unable to gain significant marks in the AO3 Questions so further adaption required.
February	Ratio worksheets using conceptual variation. Appendix A	AO3 Question Mints AO3 Question Cakes	Teachers and learners were growing more used to using bar models. After the AO3 Bungalow Question which was answered poorly AR teachers switched from using Barvember to using a ratio sharing worksheet which had conceptual variation. Learners then started to use bar models independently on assessment questions
March	Teacher and learner interview scripts Appendix B	Teacher and Learner Interviews	Teachers interviewed each other with the resources used placed at the bottom of the scripts as reminders. Teachers interviewed a selection of voluntary students for appreciative enquiry.
April:- Data Analysis Workshop			

Examples of some of the Interventions.

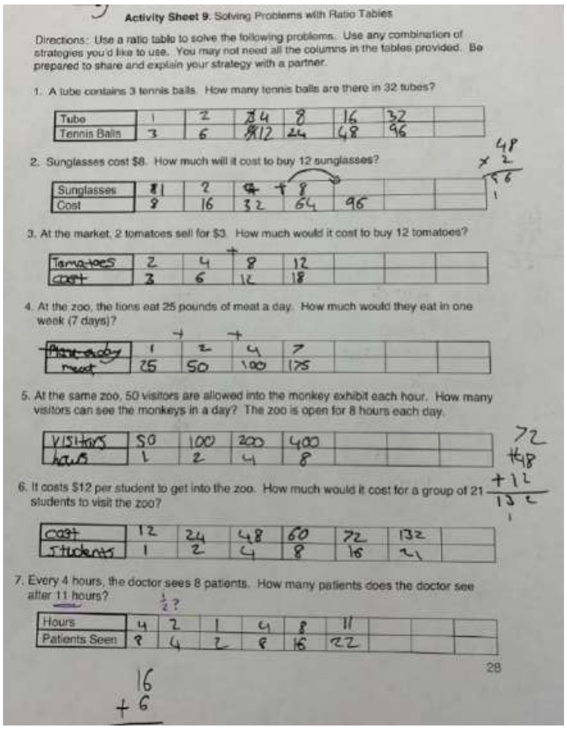
Ratio Tables


3. At the market, 2 tomatoes sell for $3. How much would it cost to buy 12 tomatoes?

Tomatoes	2	4	8	12			
cost	3	6	12	18			

4. At the zoo, the lions eat 25 pounds of meat a day. How much would they eat in one week (7 days)?

~~days~~	1	2	4	7			
meat	25	50	100	175			


Activity Sheet 9: Solving Problems with Ratio Tables

Directions: Use a ratio table to solve the following problems. Use any combination of strategies you'd like to use. You may not need all the columns in the tables provided. Be prepared to share and explain your strategy with a partner.

1. A tube contains 3 tennis balls. How many tennis balls are there in 32 tubes?

Tube	1	2	4	8	16	32	
Tennis Balls	3	6	12	24	48	96	

2. Sunglasses cost $8. How much will it cost to buy 12 sunglasses?

Sunglasses	1	2	4	8			
Cost	8	16	32	64	96		

3. At the market, 2 tomatoes sell for $3. How much would it cost to buy 12 tomatoes?

Tomatoes	2	4	8	12			
cost	3	6	12	18			

4. At the zoo, the lions eat 25 pounds of meat a day. How much would they eat in one week (7 days)?

~~days~~	1	2	4	7			
meat	25	50	100	175			

5. At the same zoo, 50 visitors are allowed into the monkey exhibit each hour. How many visitors can see the monkeys in a day? The zoo is open for 8 hours each day.

visitors	50	100	200	400			
hours	1	2	4	8			

6. It costs $12 per student to get into the zoo. How much would it cost for a group of 21 students to visit the zoo?

cost	12	24	48	60	72	132	
students	1	2	4	8	16	21	

7. Every 4 hours, the doctor sees 8 patients. How many patients does the doctor see after 11 hours?

Hours	4	2	1	4	8	11		
Patients Seen	8	4	2	8	16	22		

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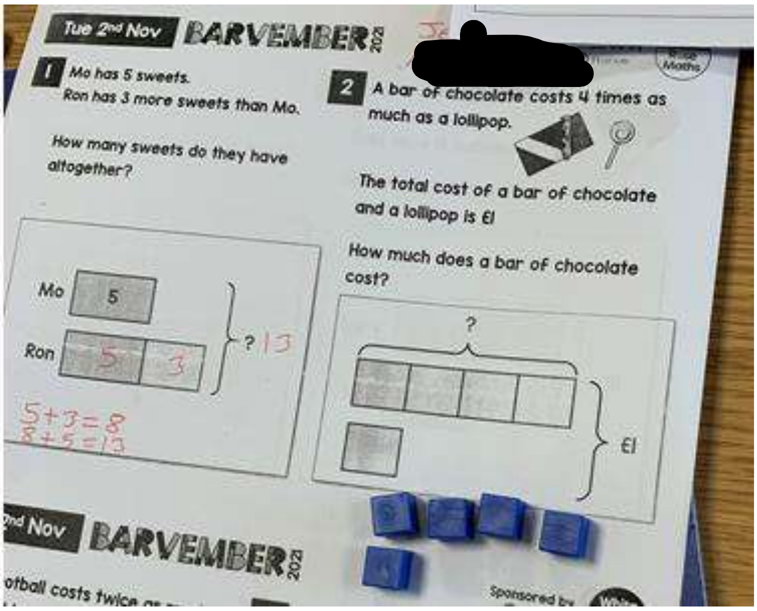
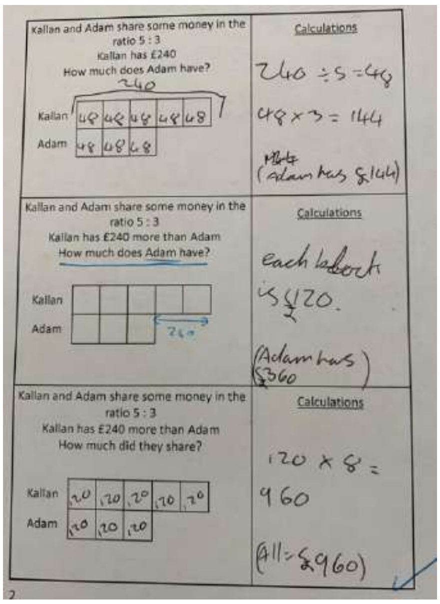
Ongoing use of ratio table using Mathsbot Missing proportion grid

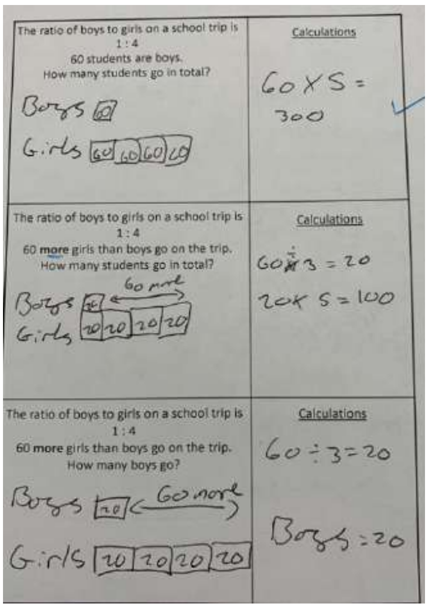
100%	50%	25%	75%	10%	20%	5%
£480						
	£680					
		30 ml				

4 weeks of Barvember with Centicubes



3-4 weeks of work sheets on ratio using Conceptual Variation





Method of Assessing Progress.

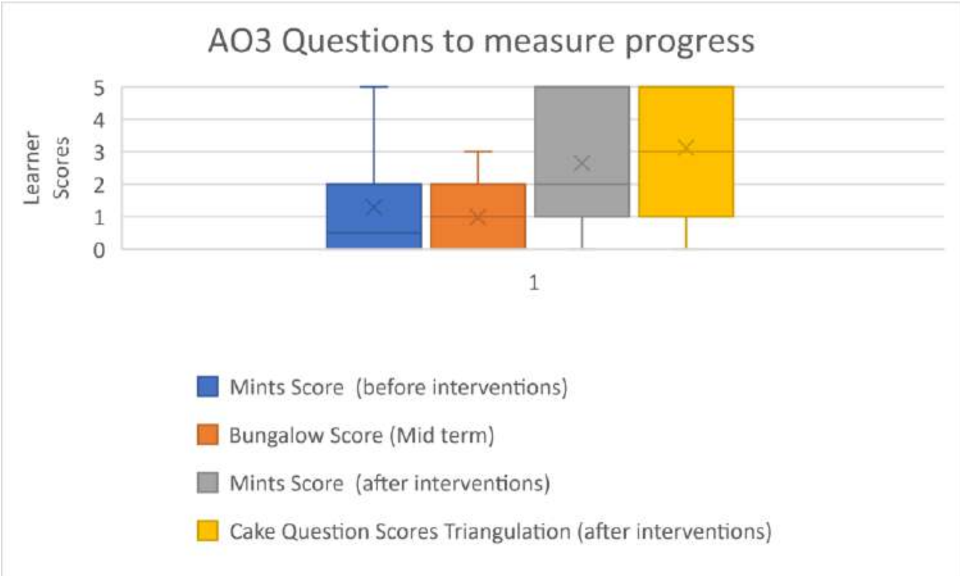
We used three AO3 Questions from Edexcel to assess progress and model solutions. The full modelled power points can be found in Appendix B.

Costcorp sells packets of mints to shop owners. On Monday three shop owners buy mints from Costcorp. Each shop owner buys small packets, medium packets and large packets of mints. Alan buys 400 packets of mints. 32% are small packets. 40% are large packets. Beryl buys 500 packets of mints. $\frac{3}{10}$ are small packets. $\frac{1}{10}$ are large packets. Charlie buys 150 small packets of mints so that number of small packets : number of medium packets = 3 : 4 Work out the total number of medium packets of mints these shop owners buy. You must show all your working. Mints 3F June 19 5 marks	In a village the number of houses and the number of flats are in the ratio 7 : 4 the number of flats and the number of bungalows are in the ratio 8 : 5 There are 50 bungalows in the village How many houses are there in the village? Bungalow 3F June 19 3 marks	Daniel bakes 420 cakes. He bakes only vanilla cakes, banana cakes, lemon cakes and chocolate cakes. $\frac{2}{7}$ of the cakes are vanilla cakes. 35% of the cakes are banana cakes. The ratio of the number of lemon cakes to the number of chocolate cakes is 4:5 Work out the number of lemon cakes Daniel bakes. Cake 3F June 19 5 marks

Learner Achievement whilst answering AO3 Questions

The graphs below show the achievements of learners over time.

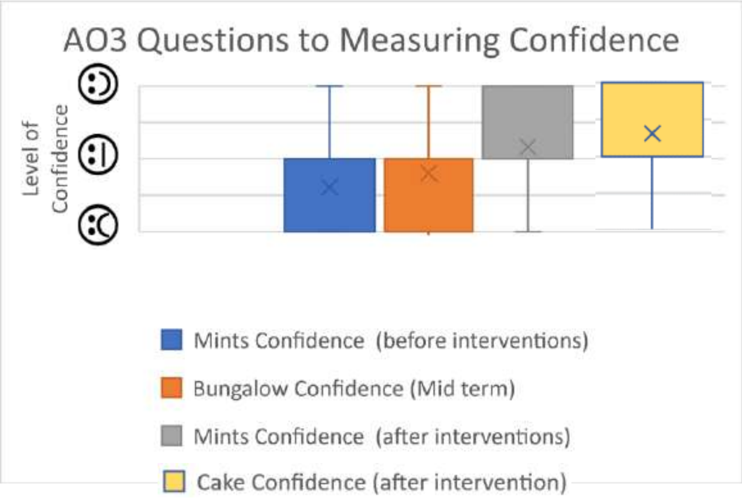
(The same 109 learners answered the" before interventions" and "after interventions" Mints Question. 73 of the 109 learners answered the bungalow Question, and 66 learners of the 109 answered the cake question.)



- **The Mints Question** before the intervention saw a mean score of 1.4 marks out of 5. The first to third quartiles learners scored 0-2 marks with a median of 1. After the intervention the mean score was 2.8 out of 5. The first to third quartiles learners scored 1-5 marks with a median of 3.
- **The Bungalow Question** completed in January (midway through the interventions) saw a mean score of 1.0 marks out of 3. The first to third quartiles learners scored 0-2 marks with a median of 1.
- **The Cake Question** was completed at the end of the intervention saw a mean score of 3.1 marks out of 5. The first to third quartiles learners scored 1-5 marks with a median of 3.

Learner Confidence whilst answering AO3 Questions
The graphs below show the confidence of the learners over time.

(Not all 109 learners input a confidence rating for the" before interventions" and "after interventions" Mints Question. 68 of the 109 learners input a confidence rating for the bungalow Question, and 59 learners of the 109 input a confidence rating for the cake question.)

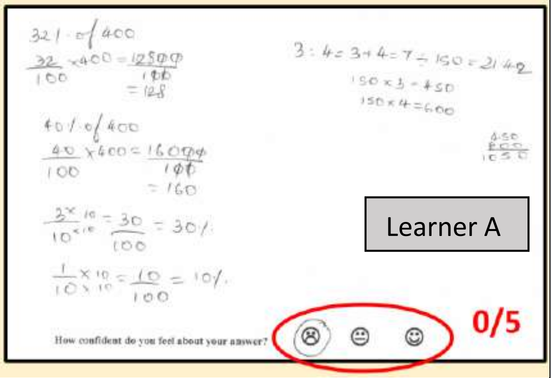
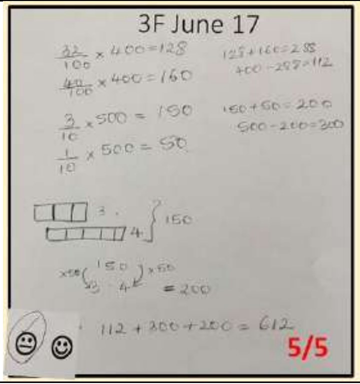
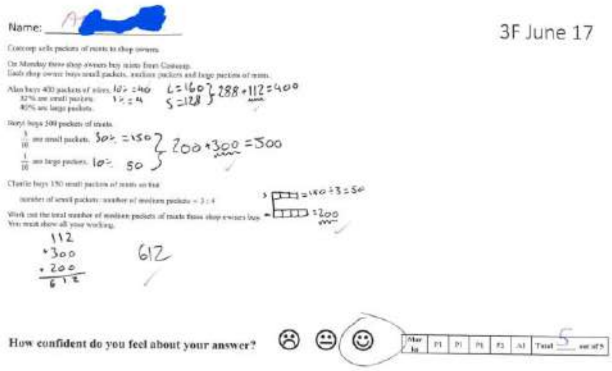


The Mints Question before the intervention saw a mean confidence of 1.6 marks and 1.8 for the Bungalow Question between sad and flat expression. The first to third quartiles learners' confidence was between sad and flat expression for both questions.

After the intervention the mean confidence was 2.2 for the Mints Question and 2.3 for the Cake Question (which is between and flat and happy face.) The first to third quartiles of learners' confidence were between flat and happy expression.

Samples of Learners Answers whilst completing the AO3 Questions

See below pictures of two learners work answering the Mints Question before and after the interventions.

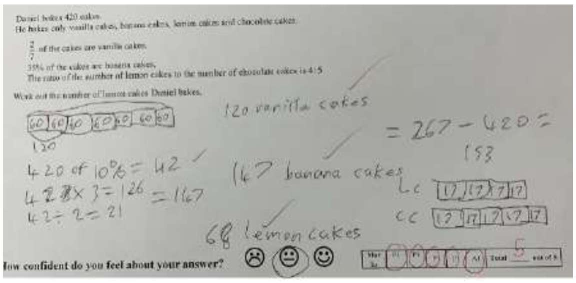
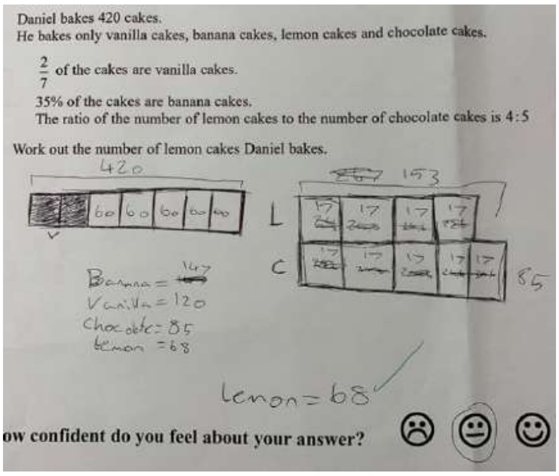
Before intervention	After intervention
 Learner A	 Learner A
 Learner B	 Learner B

Before the intervention "Learner A" was unable to show the answer which was required either 28% or 112 sweets. He converted the fractions to the percentage without further workings then incorrectly answered the ratio part of the question. After the intervention he was able to complete all the calculations correctly, again finding each percentage then remembering to take it away from the original number of sweets. He converted the fraction to a percentage and again followed through the working. He then drew bars to show the calculations clearly. Appendix B shows teachers modelled solutions, the mark-scheme and examiners comments for this question.

Learner B did not engage in this question before the interventions, aside from putting his own name on the paper. However, he was engaged in the answering each part of the question correctly after the intervention. It is clear to see him using the bar models as we had modelled them in the interventions.

More Learners Work

See below a triangulation of two learners work after the intervention, an alternative AO3 Cake Question.

Learner D	
Learner E	

Learner D used bars to calculate the fraction of the quantity, labelling their own diagram correctly. They divided by 10 and halving to find 35%. Then they used the bars with labels to find the ratio.

Learner E displayed some incorrect working but was able to double back and correct their calculations.

Summary of learner scores

The table below shows individual learners scores when answering the Mints Question in November and then again in February. (Appendix B Processes, Appendix C Raw Data.)

The Mints Question.	Number of learners 109.	Percentage of learners overall.
Marks improved	76	70%
Marks stayed the same	24	22%
Marks reduced	9	8%

Most of the learners improved their marks, a quarter of the learners' marks stayed the same and only a small proportion scored less when answering the AO3 question.

Summary of learners' own confidence in answering the Questions

The table below shows individual learner's confidence when answering the Mints Question in November and then again in February. (Appendix B Processes, Appendix C Raw Data.)

Mint Question.	Number of learners 109.	Percentage of learners overall.
Confidence improved	60	55%
Confidence stayed the same	45	41%
Confidence reduced	4	4%

Over half of the learners had an improved confidence rating. A very small proportion had a lower confidence rating. Teachers as a group did not explore the later learners experiences, but might be something we could do in the future.

Comparing the national picture

The results below show the national picture and then our own findings.

Edexcel AO3 Questions	National Average marks achieved for this Question	Average Marks achieved by our learners		
		Before the interventions	Mid-point of interventions	After the interventions
Mints 3F June 19 5 marks	2.25	1.4		2.8
Bungalow 1F June 18 3 marks	0.88		1.0	
Cake 3F June 17 5 marks	2.24			3.1

Before the interventions learners achievement on the Mints Question was below the national average. The marks achieved were doubled for the Mints Question after the interventions and both Mints and Cake Questions were significantly above the national average.

The Bungalow Question was a mid-term assessment that guided us to their understanding of ratio in a wider context. This gave us the information that teachers needed to continue working with the learners as they were still showing little improvement.

The table below shows one teacher's November resit grades. All but one of these learners had teacher assessed grades (TAGs) of a Grade 3. One had a TAG of a grade 2.

	Not taken	Grade 1	Grade 2	Grade 3	Grade 4	Total
Nov Resit 2021	2	3	13	8	1	27

The November resit results graded learners well below their TAG. These learners were mainly graded at a grade 2 when sitting an examination. Teachers found that the TAGs levelled their learners a grade higher than they would have normally achieved in an exam. This may have been to do with exam anxiety, or learners reaching the lower end of the grade so they were given the benefit of the doubt. However, many vocational teachers were reporting that their learners maths aptitude was much lower than in previous years and the learners were unable to access the Maths required for the level 3 courses.

What the teachers said during the appreciative enquiry.

The table below shows the results from the appreciative enquiry interviews with teachers. The comments made by teachers were themed. See Appendix C Raw Data.

Out of all 6 teachers interviewed	Theme	Example of comments
6	Helped with understanding	“My students had eureka moments that they suddenly understood ratio”
5	Student Engagement	“Great to see learners engaging with A03 chunky questions by drawing blocks and bars”
5	Teacher Enjoyment	“I’ve really enjoyed the interventions, I feel more confident. It’s easy to do the same old ways, this has given a different way of thinking”
3	Interlinking	“The cubes were useful for other topics; plans & elevations and shapes”
5	Planning & introducing early	“Would want to start earlier in the year with physical resources, ratio tables, algebra tiles and include percentage and fractions”

The results of the teacher’s interviews show the impact the interventions had in their practice. All the teachers inferred that using the interventions helped with learner understanding and most of the teachers said independently that the learner engagement increased, and that planning and early intervention would be necessary to be able to include more representatives. Half of the teachers mentioned seeing connections between the topics.

What Learners said during the appreciative enquiry

Out of 14 learners interviewed	Theme	Example of comments
9	Helped with understanding	“Ratio tables have helped me to understand percentage better. I will use it more in the future because of carpentry”
8	Helped with attempt more questions	“I used to just ignore these questions on an exam – now I would be happy trying to have a go at them”
3	Helped with Confidence	“It surprised me how to do it and I felt smarter than I ever had in maths, and it just helped me”
3	Fun	“It made maths fun and interesting”
4	Visualisation	“Instead of writing it down it’s not just seeing them, you can move them around, It really helps”
4	Repetition	“Doing the sheet of random questions at the start of each lesson helps to keep other topics you might not be working on in class anymore fresh in your head and gets your brain going”

From the table of learner comments

- Most learners independently said that they found the interventions helped in their understanding of the topics.
- Most learners independently said that the interventions helped with their resilience in answering more of the question.
- Some of the learners said the lessons were fun.
- Some of the learners said that the interventions helped them visualise the questions.
- Some of the learners said that the repetition helped the information stick.

Discussion

How does our research match up with the literature review?

By using the multiple representations such as the centicubes, bar-models and ratio table in different forms we were able support the learners to answer the proportional reasoning problem solving questions. The structure that teachers modelled in the classroom proved to be a valuable process when answering the questions. The learners said that they were able to recognise how the different models had helped them to understand the maths. All teachers agreed that the structure also helped them understand the maths. This matched up with the literature review which said that the use of multiple representations was needed to avoid rote learning of the process, it supported the understanding and the exposing of the mathematical structures. The literature review also indicated that making connections enabled a deeper understanding with practical approaches to embed structures. Mattock (2019), Back (2019), Hough (2019) NCETM.

Teachers and learners needed time to assimilate each representative when applying it and then remember to use it. Teachers found that the learners needed the models to be sustained and this agreed with the literature review. Magruder (2012) and Amidu et al (2020) both said that teacher and learner confidence grew over time and one-off lessons were ineffective.

Bar models needed to be introduced with concepts the learners understood. This was effective through the bar modelling with the Barvember resource (Appendix A.)

In our earliest findings we found that the ratio-table questions needed to be open ended otherwise the learners would not explore the numbers. Dickenson et al (2019) said that it was important to stop driving the lesson on and instead ask questions to develop learners thinking rather than just getting the right answers. We adapted our questions to be open rather than closed. “What else can you find?” rather than “How much for 30 people?”

Osman et al (2018) and Amidu et al (2020) also found that the learners grew in confidence and their elevated confidence allowed them to apply the models to many other topics. We also found that our learner scripts were suddenly filled with workings and reworkings as they worked things through using the bar models. When taking the secondary Cake Question, the learners wanted to have a go and make progress with it. Their own confidence and resilience grew throughout the interventions.

Teachers found that using multiple representatives was something that they weren’t familiar with and the Barvember activities which were ready made which gave them the knowledge of the structure and they wanted to continue with this structure. Teachers were also able to see the link between the different representations such as using bar models to solve algebraic problems not just ratio problems.

Conclusions, Limitations and Recommendations

Conclusions

- Physical manipulatives and visual representations have a positive impact on student confidence in answering AO3 questions (for our research we focused on ratio, fractions and percentages.)
- Physical manipulatives and visual representations have a positive impact on achievement.
- Students are more confident and are more engaged when answering challenging questions
- The bar models and ratio tables must be sustained with activities needing to be revisited regularly
- The use of the conceptual variation questions was a game changer for most teachers and learners.
- Students need to be trained to use the structure of the bar model with simpler problems.
- Teachers need to develop questioning techniques so not to give answers but encourage learners to explore their own deeper thinking. Teachers can ask questions such as "What else do you know?" rather than "How much for 30 people?"

Limitations

- The **student attendance** had an impact on the journey made
- Concerned that the **time** spent on this was to the detriment of other teaching subjects
- We couldn't produce the final interventions until January/February due to the project cycle time frame and fitting in with the exam dates

Recommendations

- Teachers need to develop questioning techniques so not to give answers but get learners to explore their own deeper thinking by using open ended e.g. "If we know…. what else do we know...?" Rather than "how much for 30 people?"
- Teacher training in the use of these and other representatives is essential, e.g., Algebra tiles
- To plan the regular use of representatives from the beginning of the academic year and in a wider range of topics (by developing the SOW in preparation over summer)
- To share the interventions and resources with other colleagues (produce a Padlet from the project with recommended resources)

Reflections

- We feel more enthusiastic, and this rubs off on our learners
- We have enjoyed being more creative in our teaching
- We feel more confident as teachers
- Networking with other colleges and sharing ideas has been a useful experience

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Appendices

Appendix A Interventions

- Ratio Tables
- Barvember
- Ratio Worksheets (using conceptual variation)

Appendix B Processes

- AO3 Questions
- Teacher Interview sheet
- Learner Interview sheet

Appendix C Raw data

- Learner interview data
- Teacher interview data
- Data from AO3 Scripts

Appendix D Informed Consent

Appendix A Interventions

Power Points can be found in our padlet [2021_2022 Action Research Mastery at CfEM Weston (padlet.com)]

Ratio Table interventions

Introduction to Ratio Tables

Cakes	3	6	12	24	48	
Cost	£2	£4	£8	£16	£32	

Boxes	80	40	20	10	5	
Screws	4000	2000	1000	500	250	

A combination of techniques

Books	3	30	300	150	9	159
Cost	£12	£120	£1200	£600	£36	£636

Hours	20	10	5	40	35	
Cost	£440	£220	£110	£880	£770	

Introducing Ratio Tables

Ratio Table Techniques

Doubling
Three cakes cost £2, how much would 48 cakes cost?

Cakes	3	6	12	24	48			
Cost	£2	£4						

Halving
Eighty boxes hold 4000 screws, how many screws are there in five boxes

Boxes	80	40	20	10	5			
Screws	4000	2000						

Multiplying by 10
Five pens cost £3. how much would 5000 pens cost?

Pens	5	50	500	5000				
Cost	£3							

Dividing by 10
Gary exchanges £1000 into 8770 Danish Krone. What is £1 worth in Danish Krone?

£	1000	100	10	1				
Krone	8770	877						

Multiplying by 3
Three cakes cost £4, how much would 27 cakes cost?

Cakes	3	9	27					
Cost	£4	£12						

Dividing by 3
A doctor saw 30 patients in 9 hours, how many did they see in 3 hours?

Hours	9	3						
Patients	30							

Combining a variety of these techniques (including adding and subtracting)

Example: Three books cost £12, how much would 159 books cost?

Books	3	30	300	150	9	159		
Cost	£12	£120	£1200	£600	£36	£636		

Example: Twenty hours of driving lessons cost £440, how much would 35 hours cost?

Hours	20	10	5	40	35			
Cost	£440	£220	£110	£880	£770			

Learning to Think Mathematically with the Ratio Table

Name: ____________________

Activity Sheet 10: Ratio Table Strategy Review

You have solved many problems in this book with ratio tables. You probably used familiar strategies like these:

Multiply by 10

1	10
15	150

Multiplying

2	6
50	150

Doubling

4	8
15	30

Halving

20	10
30	15

Adding

1	2	3
25	50	75

Subtracting

1	10	9
12	120	108

Find the missing numbers in the shaded boxes in the ratio tables below. Then write which of the above strategies you used.

1.

1	10	9
18	180	

Strategy: ____________________

2.

1	10	5
18	180	

Strategy: ____________________

3.

1	10	20
12	120	

Strategy: ____________________

4.

2	12
8	

Strategy: ____________________

5.

4	40
18	

Strategy: ____________________

6.

2	10	12
8	40	

Strategy: ____________________

7.

4	8
18	

Strategy: ____________________

8.

12	
8	4

Strategy: ____________________

Learning to Think Mathematically with the Ratio Table

Name: ______________________

Activity Sheet 9: Solving Problems with Ratio Tables

Directions: Use a ratio table to solve the following problems. Use any combination of strategies you'd like to use. You may not need all the columns in the tables provided. Be prepared to share and explain your strategy with a partner.

1. A tube contains 3 tennis balls. How many tennis balls are there in 32 tubes?

Tube							
Tennis Balls							

2. Sunglasses cost $8. How much will it cost to buy 12 sunglasses?

Sunglasses							
Cost							

3. At the market, 2 tomatoes sell for $3. How much would it cost to buy 12 tomatoes?

4. At the zoo, the lions eat 25 pounds of meat a day. How much would they eat in one week (7 days)?

5. At the same zoo, 50 visitors are allowed into the monkey exhibit each hour. How many visitors can see the monkeys in a day? The zoo is open for 8 hours each day.

6. It costs $12 per student to get into the zoo. How much would it cost for a group of 21 students to visit the zoo?

7. Every 4 hours, the doctor sees 8 patients. How many patients does the doctor see after 11 hours?

Hours									
Patients Seen									

Combining a variety of these techniques (including adding and subtracting)

Three T-shirts cost £45, how much would 35 T-shirts cost?

T-shirts	3							
Cost	£45							

Fifty boxes hold 20000 nails, how many nails are there in 22 boxes?

Boxes	50							
Nails	20000							

Five pens cost 70p, how much would 38 pens cost?

Pens	5							
Cost	70							

Anni exchanges £100 into 1500 Japanese Yen. How many Yen would she get if she exchanged £320?

£	100							
Yen	1500							

Five cakes cost £15, how much would 27 cakes cost?

Cakes	5							
Cost	£15							

A doctor saw 24 patients in 6 hours, how many did they see in 2.5 hours?

Hours	6							
Patients	24							

Stretch (calculator)

3.5 kg of sweets cost £5.25 What would 2.5 kg of sweets cost?

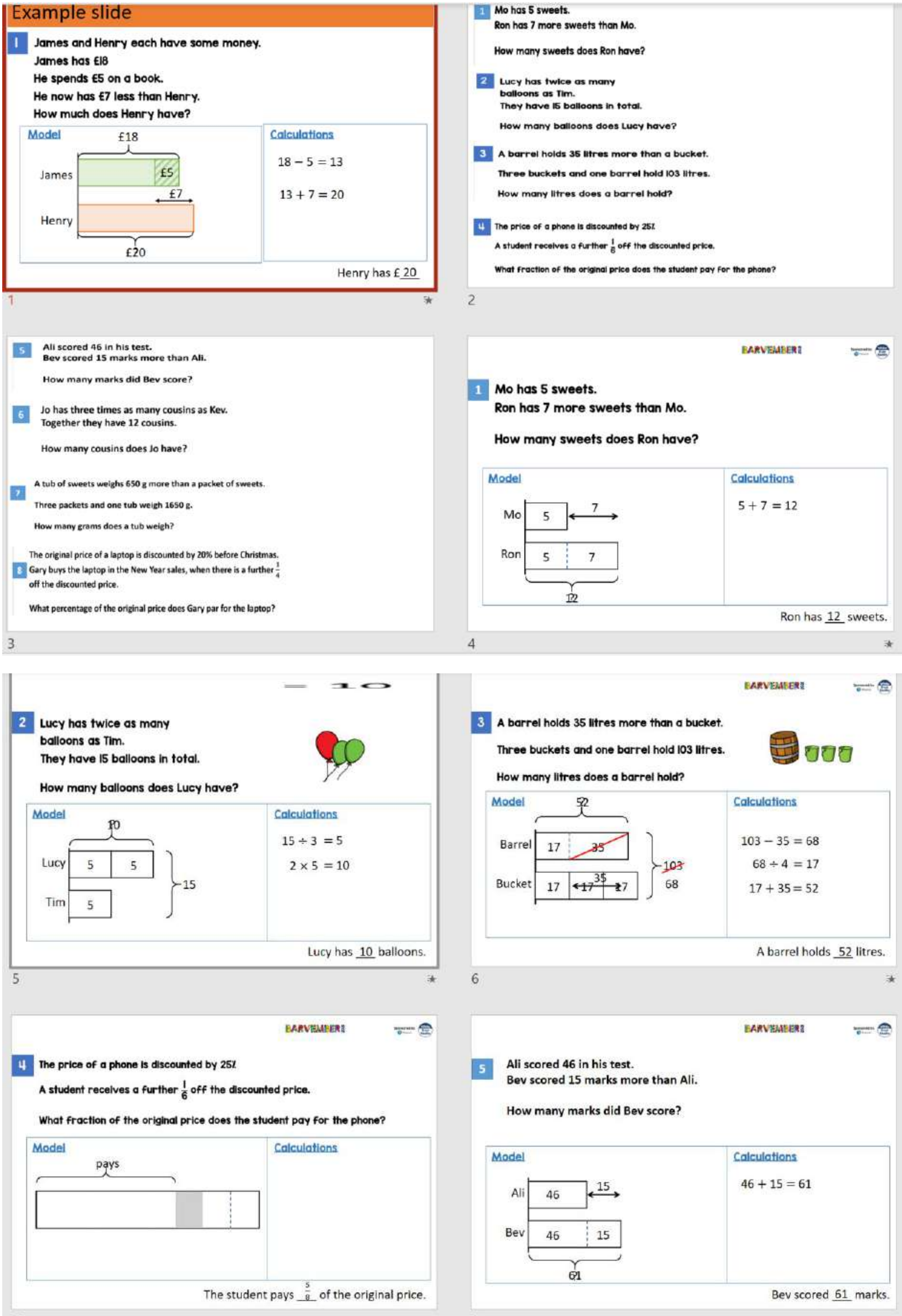
Kg	3.5							
Cost	£5.25							

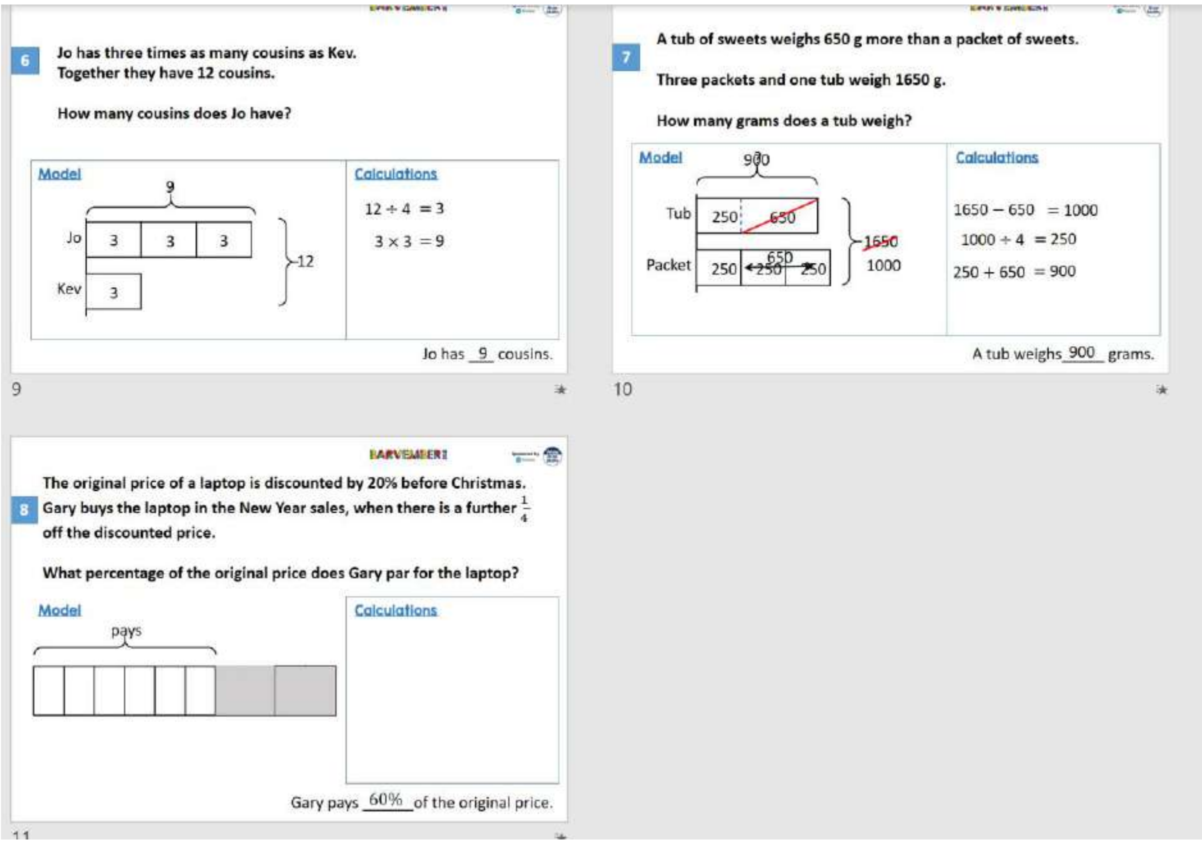
4.5 litres of petrol cost £5.40 What would 12.5 litres of petrol cost?

Litres	4.5							
Cost	£5.40							

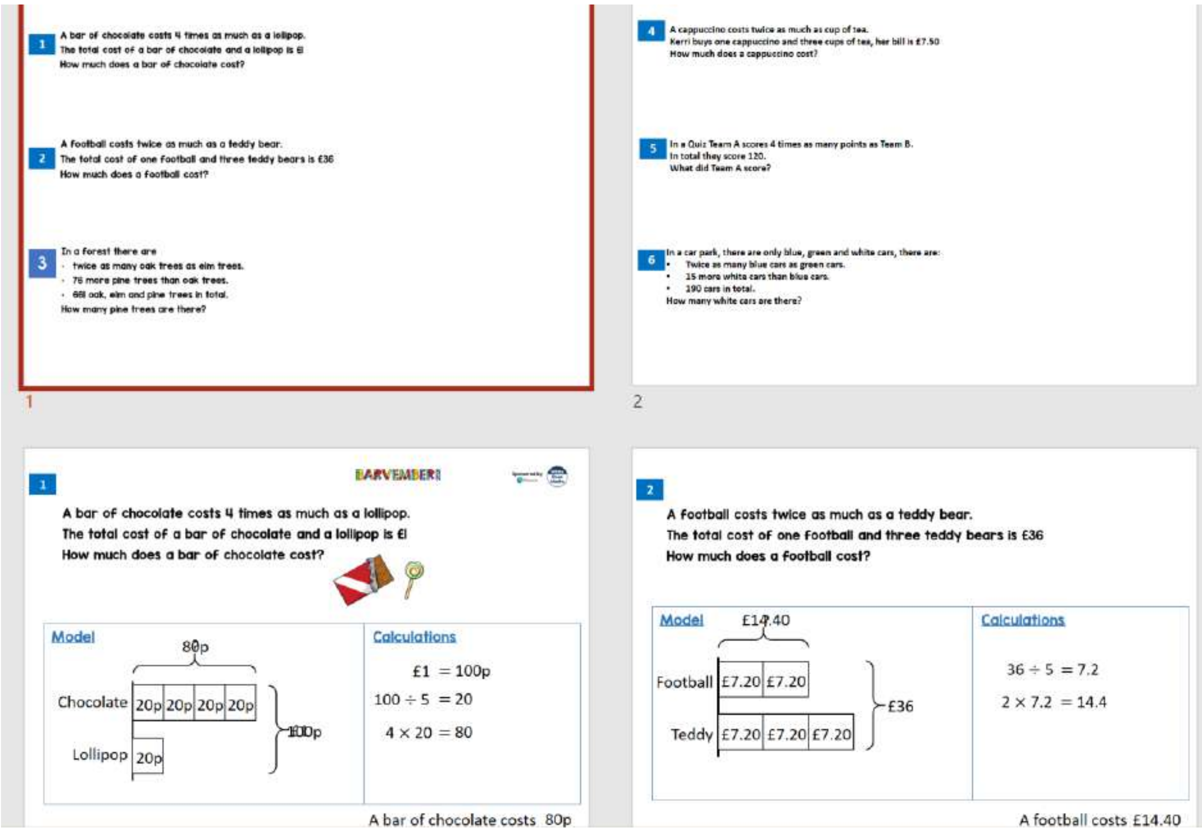
Bar Modelling Resources

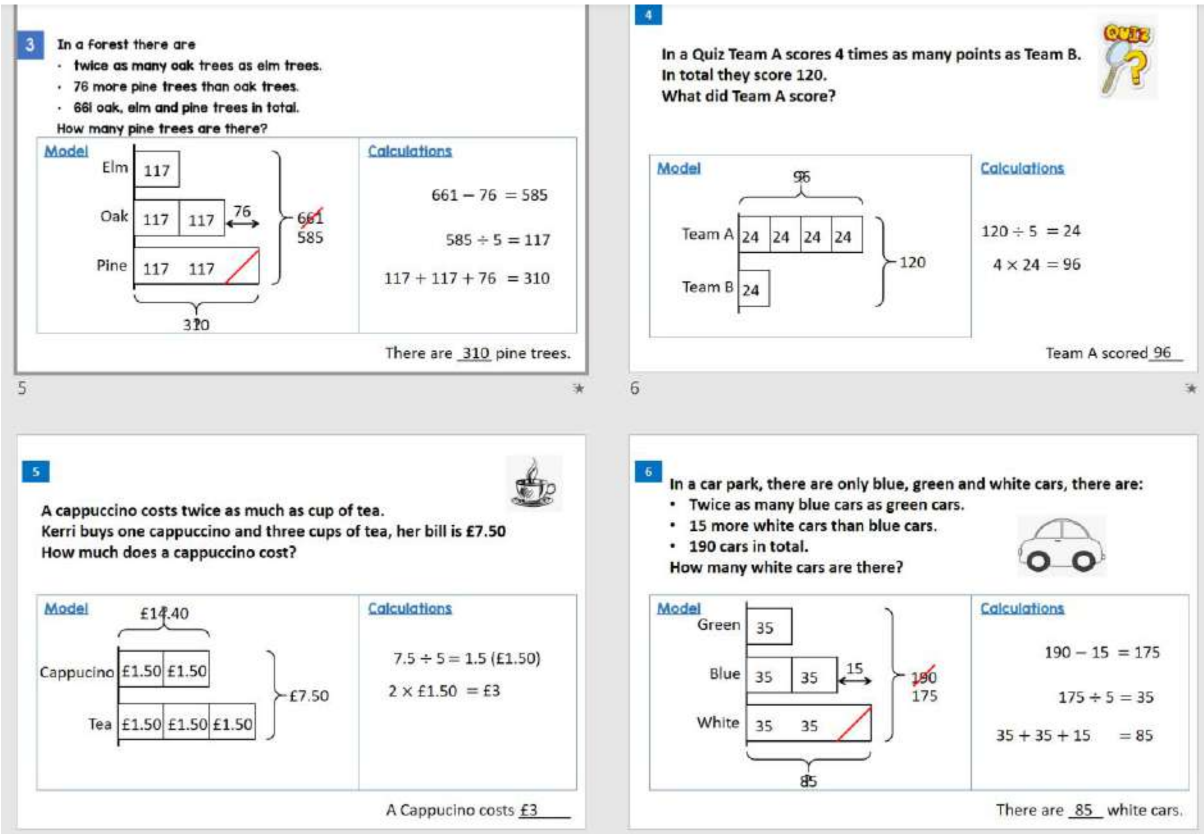
PP 1


Example slide
1 James and Henry each have some money.
James has £18
He spends £5 on a book.
He now has £7 less than Henry.
How much does Henry have?
Model
£18
James
£5
£7
Henry
£20
Calculations
18 − 5 = 13
13 + 7 = 20
Henry has £ 20
1
1 Mo has 5 sweets.
Ron has 7 more sweets than Mo.
How many sweets does Ron have?
2 Lucy has twice as many balloons as Tim.
They have 15 balloons in total.
How many balloons does Lucy have?
3 A barrel holds 35 litres more than a bucket.
Three buckets and one barrel hold 103 litres.
How many litres does a barrel hold?
4 The price of a phone is discounted by 25%
A student receives a further 1/6 off the discounted price.
What fraction of the original price does the student pay for the phone?
2
5 Ali scored 46 in his test.
Bev scored 15 marks more than Ali.
How many marks did Bev score?
6 Jo has three times as many cousins as Kev.
Together they have 12 cousins.
How many cousins does Jo have?
7 A tub of sweets weighs 650 g more than a packet of sweets.
Three packets and one tub weigh 1650 g.
How many grams does a tub weigh?
8 The original price of a laptop is discounted by 20% before Christmas.
Gary buys the laptop in the New Year sales, when there is a further 1/4 off the discounted price.
What percentage of the original price does Gary par for the laptop?
3
BARVEMBER!
1 Mo has 5 sweets.
Ron has 7 more sweets than Mo.
How many sweets does Ron have?
Model
Mo
5
7
Ron
5
7
12
Calculations
5 + 7 = 12
Ron has 12 sweets.
4
2 Lucy has twice as many balloons as Tim.
They have 15 balloons in total.
How many balloons does Lucy have?
Model
10
Lucy
5
5
Tim
5
15
Calculations
15 ÷ 3 = 5
2 × 5 = 10
Lucy has 10 balloons.
5
BARVEMBER!
3 A barrel holds 35 litres more than a bucket.
Three buckets and one barrel hold 103 litres.
How many litres does a barrel hold?
Model
52
Barrel
17
35
Bucket
17
17
17
103
68
Calculations
103 − 35 = 68
68 ÷ 4 = 17
17 + 35 = 52
A barrel holds 52 litres.
6
BARVEMBER!
4 The price of a phone is discounted by 25%
A student receives a further 1/6 off the discounted price.
What fraction of the original price does the student pay for the phone?
Model
pays
Calculations
The student pays 5/8 of the original price.
BARVEMBER!
5 Ali scored 46 in his test.
Bev scored 15 marks more than Ali.
How many marks did Bev score?
Model
Ali
46
15
Bev
46
15
61
Calculations
46 + 15 = 61
Bev scored 61 marks.

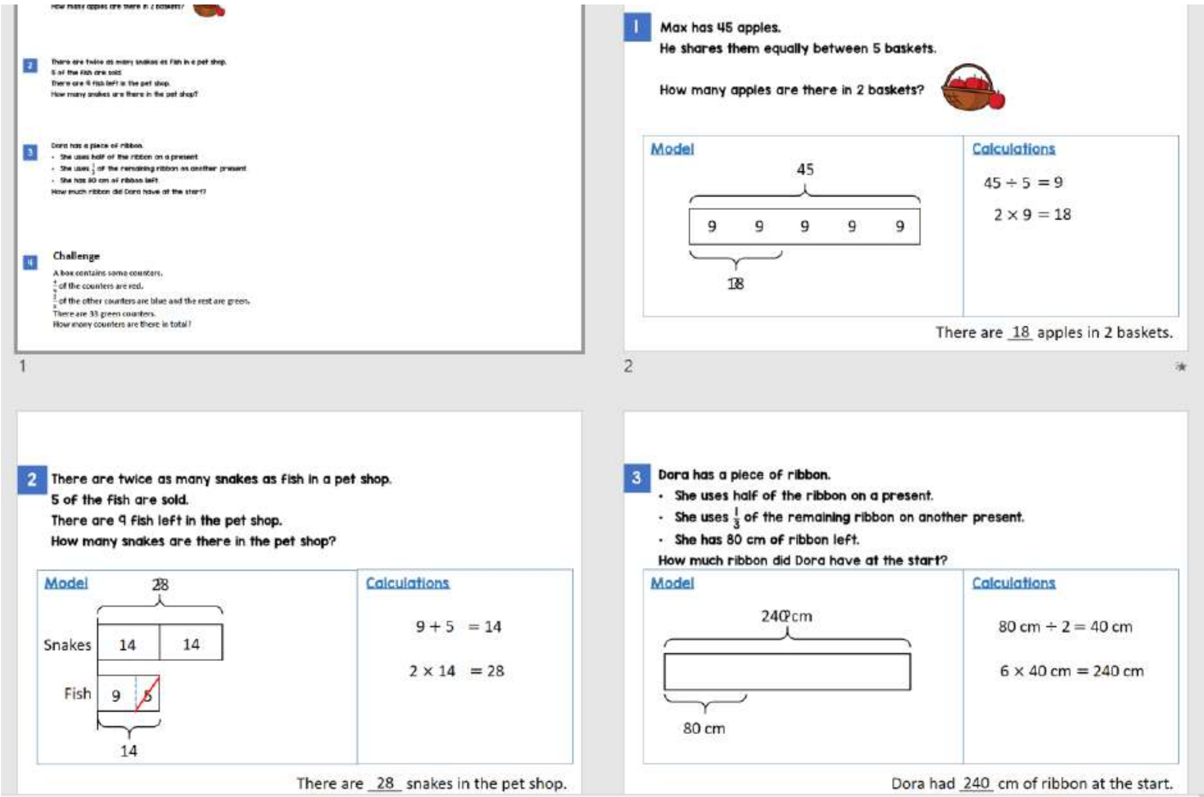

6
Jo has three times as many cousins as Kev.
Together they have 12 cousins.
How many cousins does Jo have?
Model
9
Jo 3 3 3
Kev 3
12
Calculations
12 ÷ 4 = 3
3 × 3 = 9
Jo has 9 cousins.
9
7
A tub of sweets weighs 650 g more than a packet of sweets.
Three packets and one tub weigh 1650 g.
How many grams does a tub weigh?
Model
900
Tub 250 650
Packet 250 250 250
1650
1000
Calculations
1650 − 650 = 1000
1000 ÷ 4 = 250
250 + 650 = 900
A tub weighs 900 grams.
10
8
The original price of a laptop is discounted by 20% before Christmas.
Gary buys the laptop in the New Year sales, when there is a further 1/4 off the discounted price.
What percentage of the original price does Gary par for the laptop?
Model
pays
Calculations
Gary pays 60% of the original price.

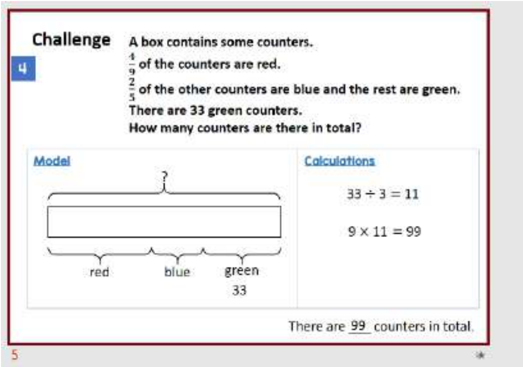
PP2


1
A bar of chocolate costs 4 times as much as a lollipop.
The total cost of a bar of chocolate and a lollipop is £1
How much does a bar of chocolate cost?
2
A football costs twice as much as a teddy bear.
The total cost of one football and three teddy bears is £36
How much does a football cost?
3
In a forest there are
twice as many oak trees as elm trees.
75 more pine trees than oak trees.
680 oak, elm and pine trees in total.
How many pine trees are there?
4
A cappuccino costs twice as much as cup of tea.
Kerri buys one cappuccino and three cups of tea, her bill is £7.50
How much does a cappuccino cost?
5
In a Quiz Team A scores 4 times as many points as Team B.
In total they score 120.
What did Team A score?
6
In a car park, there are only blue, green and white cars, there are:
Twice as many blue cars as green cars.
15 more white cars than blue cars.
190 cars in total.
How many white cars are there?
1
2
1
A bar of chocolate costs 4 times as much as a lollipop.
The total cost of a bar of chocolate and a lollipop is £1
How much does a bar of chocolate cost?
Model
80p
Chocolate 20p 20p 20p 20p
Lollipop 20p
100p
Calculations
£1 = 100p
100 ÷ 5 = 20
4 × 20 = 80
A bar of chocolate costs 80p
2
A football costs twice as much as a teddy bear.
The total cost of one football and three teddy bears is £36
How much does a football cost?
Model
£14.40
Football £7.20 £7.20
Teddy £7.20 £7.20 £7.20
£36
Calculations
36 ÷ 5 = 7.2
2 × 7.2 = 14.4
A football costs £14.40

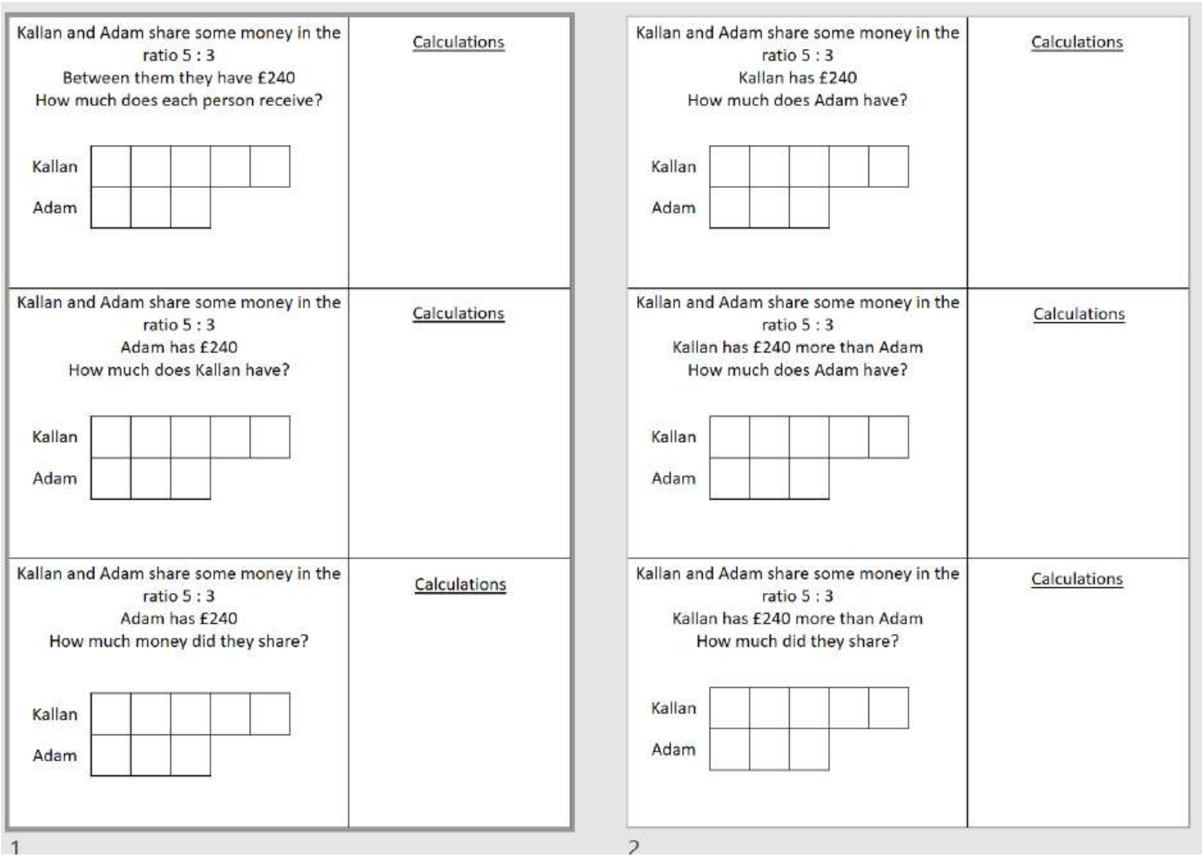

3
In a forest there are
twice as many oak trees as elm trees.
76 more pine trees than oak trees.
661 oak, elm and pine trees in total.
How many pine trees are there?
Model
Elm 117
Oak 117 117 76
Pine 117 117
661
585
310
Calculations
661 − 76 = 585
585 ÷ 5 = 117
117 + 117 + 76 = 310
There are 310 pine trees.
5
4
In a Quiz Team A scores 4 times as many points as Team B.
In total they score 120.
What did Team A score?
Model
96
Team A 24 24 24 24
Team B 24
120
Calculations
120 ÷ 5 = 24
4 × 24 = 96
Team A scored 96
6
5
A cappuccino costs twice as much as cup of tea.
Kerri buys one cappuccino and three cups of tea, her bill is £7.50
How much does a cappuccino cost?
Model
£14.40
Cappucino £1.50 £1.50
Tea £1.50 £1.50 £1.50
£7.50
Calculations
7.5 ÷ 5 = 1.5 (£1.50)
2 × £1.50 = £3
A Cappucino costs £3
6
In a car park, there are only blue, green and white cars, there are:
Twice as many blue cars as green cars.
15 more white cars than blue cars.
190 cars in total.
How many white cars are there?
Model
Green 35
Blue 35 35 15
White 35 35
190
175
85
Calculations
190 − 15 = 175
175 ÷ 5 = 35
35 + 35 + 15 = 85
There are 85 white cars.

PP3


Challenge
1
1
Max has 45 apples.
He shares them equally between 5 baskets.
How many apples are there in 2 baskets?
Model
45
9 9 9 9 9
18
Calculations
45 ÷ 5 = 9
2 × 9 = 18
There are 18 apples in 2 baskets.
2
2
There are twice as many snakes as fish in a pet shop.
5 of the fish are sold.
There are 9 fish left in the pet shop.
How many snakes are there in the pet shop?
Model
28
Snakes 14 14
Fish 9 5
14
Calculations
9 + 5 = 14
2 × 14 = 28
There are 28 snakes in the pet shop.
3
Dora has a piece of ribbon.
She uses half of the ribbon on a present.
She uses 1/3 of the remaining ribbon on another present.
She has 80 cm of ribbon left.
How much ribbon did Dora have at the start?
Model
240 cm
80 cm
Calculations
80 cm ÷ 2 = 40 cm
6 × 40 cm = 240 cm
Dora had 240 cm of ribbon at the start.



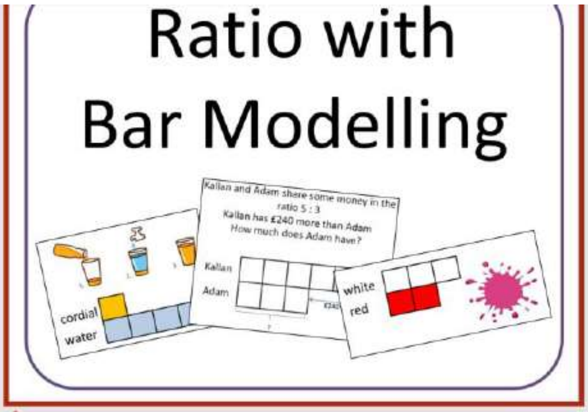
Worksheets with conceptual variation. Scaffolded and un-scaffolded

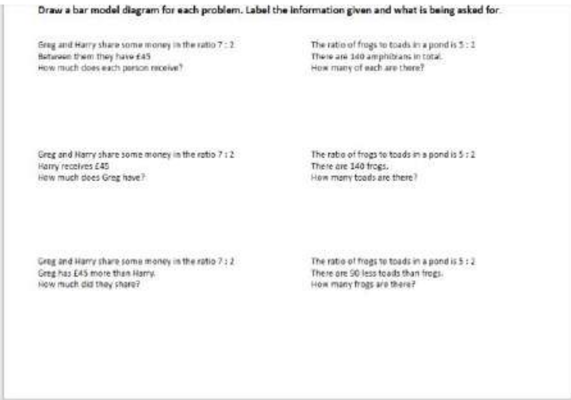


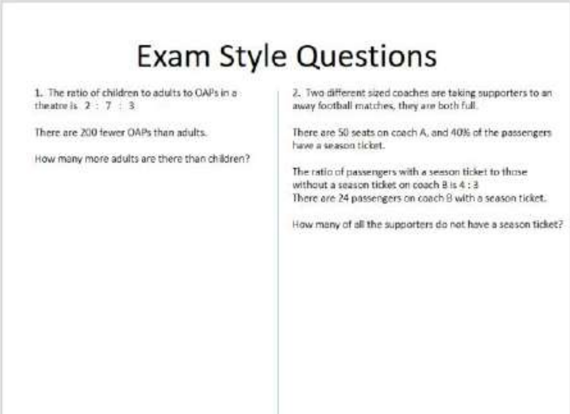
The ratio of boys to girls on a school trip is 1 : 4 60 students go on the trip. How many girls go?	Calculations
The ratio of boys to girls on a school trip is 1 : 4 60 students are boys. How many girls go on the trip?	Calculations
The ratio of boys to girls on a school trip is 1 : 4 60 students are girls. How many boys go on the trip?	Calculations

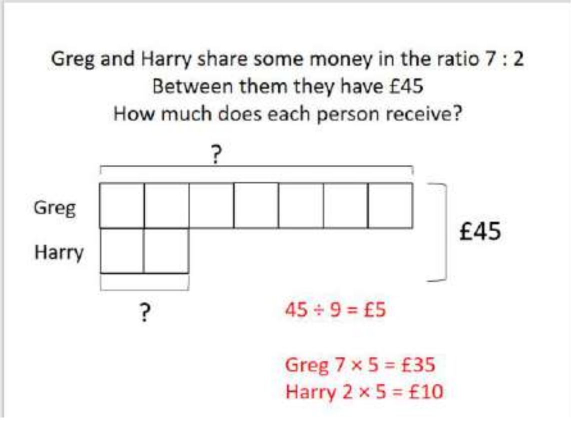
The ratio of boys to girls on a school trip is 1 : 4 60 students are boys. How many students go in total?	Calculations
The ratio of boys to girls on a school trip is 1 : 4 60 **more** girls than boys go on the trip. How many students go in total?	Calculations
The ratio of boys to girls on a school trip is 1 : 4 60 **more** girls than boys go on the trip. How many boys go?	Calculations

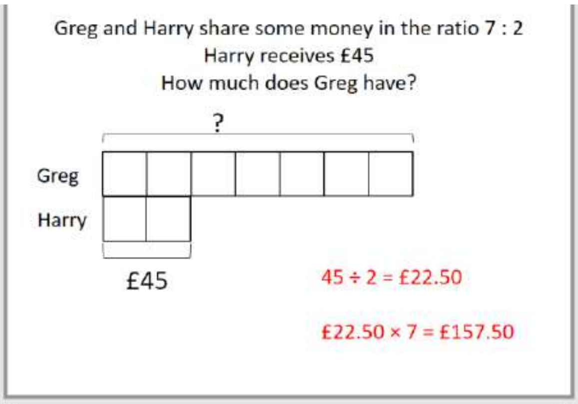
Modelled Solutions

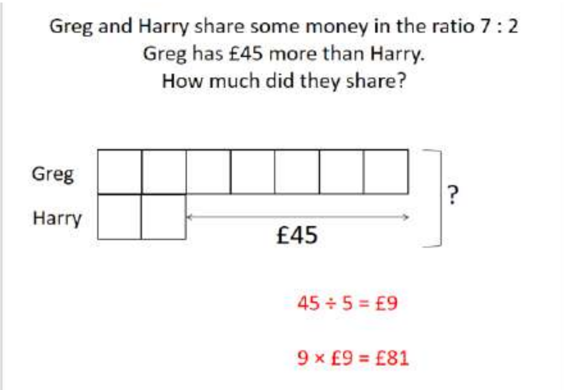

Ratio with
Bar Modelling
Kallan and Adam share some money in the ratio 5 : 3
Kallan has £240 more than Adam
How much does Adam have?
cordial
water
Kallan
Adam
£240
white
red

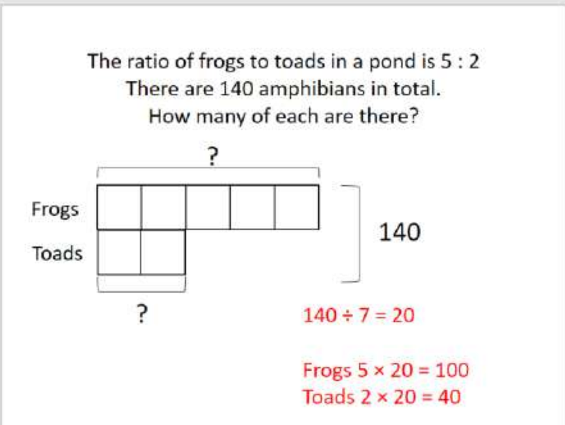

Draw a bar model diagram for each problem. Label the information given and what is being asked for.
Greg and Harry share some money in the ratio 7 : 2
Between them they have £45
How much does each person receive?
The ratio of frogs to toads in a pond is 5 : 2
There are 140 amphibians in total.
How many of each are there?
Greg and Harry share some money in the ratio 7 : 2
Harry receives £45
How much does Greg have?
The ratio of frogs to toads in a pond is 5 : 2
There are 140 frogs.
How many toads are there?
Greg and Harry share some money in the ratio 7 : 2
Greg has £45 more than Harry.
How much did they share?
The ratio of frogs to toads in a pond is 5 : 2
There are 90 less toads than frogs.
How many frogs are there?

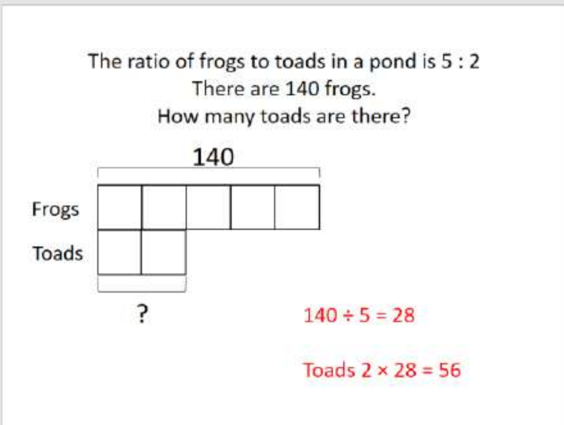

Exam Style Questions
1. The ratio of children to adults to OAPs in a theatre is 2 : 7 : 3
There are 200 fewer OAPs than adults.
How many more adults are there than children?
2. Two different sized coaches are taking supporters to an away football matches, they are both full.
There are 50 seats on coach A, and 40% of the passengers have a season ticket.
The ratio of passengers with a season ticket to those without a season ticket on coach B is 4 : 3
There are 24 passengers on coach B with a season ticket.
How many of all the supporters do not have a season ticket?

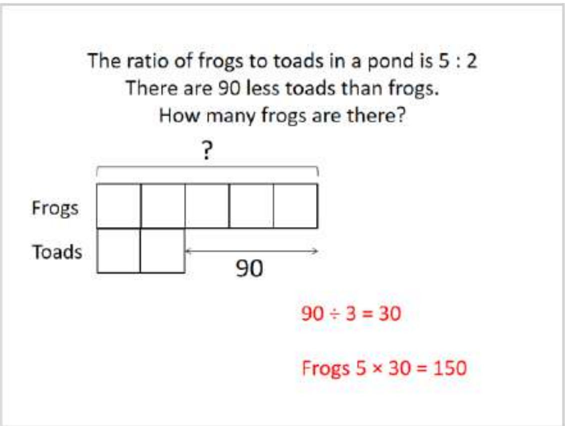

Greg and Harry share some money in the ratio 7 : 2
Between them they have £45
How much does each person receive?
?
Greg
Harry
£45
?
45 ÷ 9 = £5
Greg 7 × 5 = £35
Harry 2 × 5 = £10


Greg and Harry share some money in the ratio 7 : 2
Harry receives £45
How much does Greg have?
?
Greg
Harry
£45
45 ÷ 2 = £22.50
£22.50 × 7 = £157.50

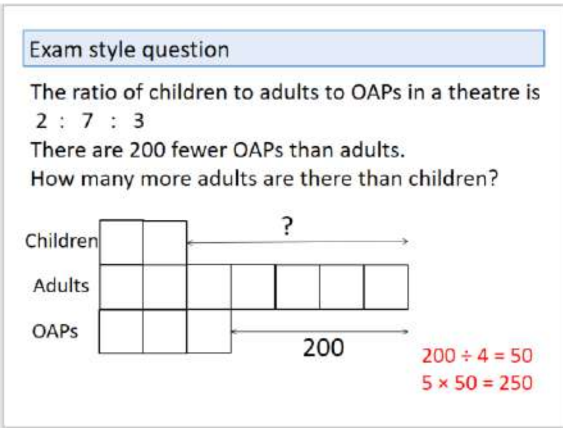

Greg and Harry share some money in the ratio 7 : 2
Greg has £45 more than Harry.
How much did they share?
Greg
Harry
£45
?
45 ÷ 5 = £9
9 × £9 = £81


The ratio of frogs to toads in a pond is 5 : 2
There are 140 amphibians in total.
How many of each are there?
?
Frogs
Toads
140
?
140 ÷ 7 = 20
Frogs 5 × 20 = 100
Toads 2 × 20 = 40

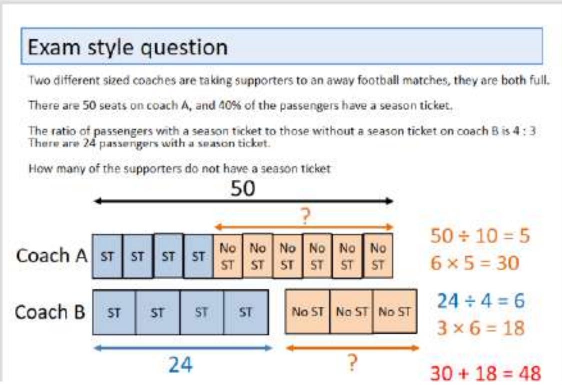

The ratio of frogs to toads in a pond is 5 : 2
There are 140 frogs.
How many toads are there?
140
Frogs
Toads
?
140 ÷ 5 = 28
Toads 2 × 28 = 56


The ratio of frogs to toads in a pond is 5 : 2
There are 90 less toads than frogs.
How many frogs are there?
?
Frogs
Toads
90
90 ÷ 3 = 30
Frogs 5 × 30 = 150

9

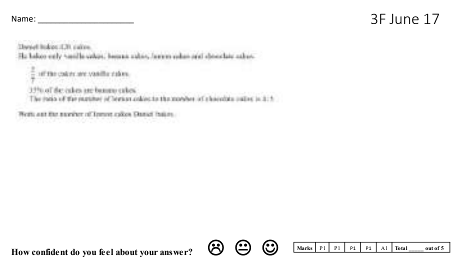

Exam style question
The ratio of children to adults to OAPs in a theatre is
2 : 7 : 3
There are 200 fewer OAPs than adults.
How many more adults are there than children?
?
Children
Adults
OAPs
200
200 ÷ 4 = 50
5 × 50 = 250

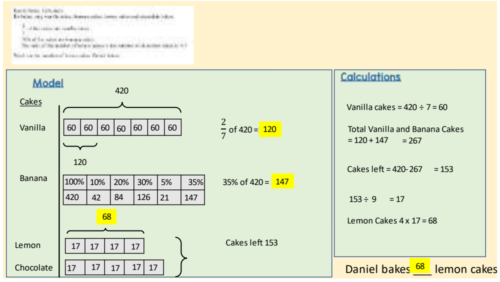
10

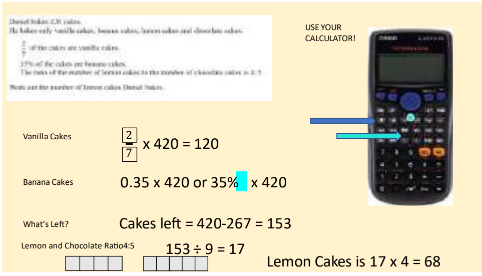

Exam style question
Two different sized coaches are taking supporters to an away football matches, they are both full.
There are 50 seats on coach A, and 40% of the passengers have a season ticket.
The ratio of passengers with a season ticket to those without a season ticket on coach B is 4 : 3
There are 24 passengers with a season ticket.
How many of the supporters do not have a season ticket
50
?
Coach A
ST
No ST
50 ÷ 10 = 5
6 × 5 = 30
Coach B
ST
No ST
24 ÷ 4 = 6
3 × 6 = 18
24
?
30 + 18 = 48

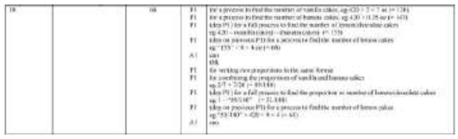
Appendix B Processes

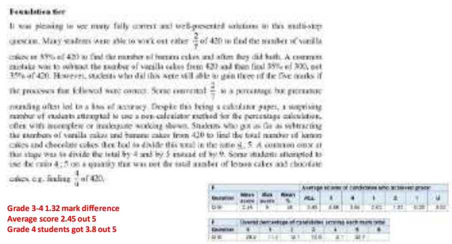
AO3 Cake Question PP



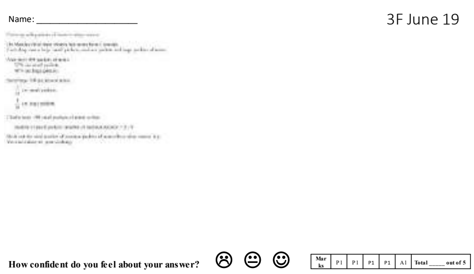


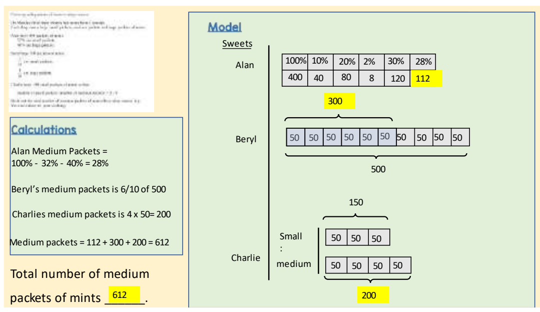


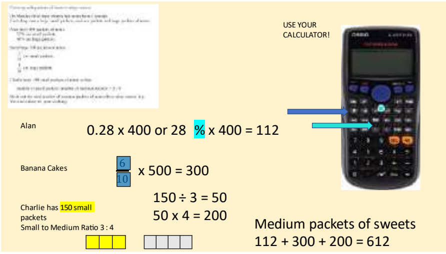


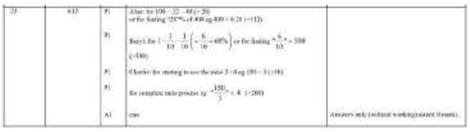


AO3 Mint Question PP









Mean score: 2.25

Examiner comment This was a problem -solving question in the context of Number and Ratio. The question was structured in order to aid students in dealing with the three shop owners. This could be done by finding remainders in the case of the first two, and undertaking a division by ratio for the third, which is the more challenging aspect of the question.

Grade 3-4 1.31 mark difference
Average score 2.25 out 5
Grade 4 students got 3.7 out 5

AO3 Bungalow Question PP

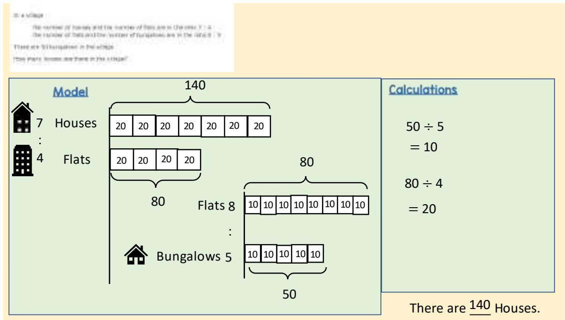
1f June 18

In a village

the number of houses and the number of flats are in the ratio 7 : 4
the number of flats and the number of bungalows are in the ratio 8 : 5

There are 50 bungalows in the village

How many houses are there in the village?



Examiners report

Just over half of students found the demand of this question too great. Some students were able to correctly work with the ratio 8 : 5 resulting in 80 flats but could not correctly link this with the ratio 7 : 4 to find the number of houses; an incorrect answer of 70 was very common indeed. Some students used diagrams to help understand the question with some pleasing success. Very few students tried to convert the two ratios to a triple ratio and a ratio such as 14 : 8 : 5 was a rarity.

Question	Answer	Mark	Mark scheme	Additional guidance
	140	P1	for beginning to solve the problem eg 50 ÷ 5 × 8 (= 80) or 14 : 8 : 5 oe or 14 : 8 and 8 : 5 oe (linked)	80 may be seen in the ratio 80 : 50
		P1	for a full process to solve the problem eg "80" ÷ 4 × 7 or $\frac{50}{5}$ × "14" or 140 : 80 : 50	
		A1	cao	If 140 clearly identified as houses in working award full marks

Teacher interview March 22

(adapted dependent upon teacher or learner being interviewed)

Teacher notes Interview a Colleague

- Please record the interview in the AR Team General which will auto transcribe the interview. This would have the most impact when presenting our findings. Jot down memorable episodes to help collection of data.
- Time taken could be 20 minutes
- Have on display the interventions to remind colleagues. Attached
    - AR1: Ratio table, Barvember, centicubes, ratio with bar modelling (AO3 Questions?)
    - AR2: Ratio Table, Petrol lesson Human Number Line, Twigs and Leaves, matching activity. (Separate questionnaire needed for the Voc Teachers with the V-M board)

Evaluation

Read to Colleagues: Please be as honest as you can and remember there are no right or wrong answers. All responses are really valuable to help me to understand your experiences. The recording will only be shared with permission from you. Anything said will be anonymised.

1. *Think about how we have taught Maths this year. Try to remember a time in classes when you were most positive about your teaching.*
   In your own words, tell me about that time.

> Follow up Questioning on the responses.
> What happened in the lead up to this moment?
> What were you feeling?
> What were others doing that helped?
> What were you doing that helped?
> What was happening at/during the moment?
> What made it a positive experience?

2. *Are there any other positive examples you would like to tell me about?*

3. *Have you shared any of these interventions with other colleagues.*

Prompt Questioning on the responses.
What happened in the lead up to this moment?
What were you feeling?
What were others doing that helped?
What were you doing that helped?
What was happening at/during the moment?
What made it a positive experience?

4. *Are there any other positive examples you would like to tell me about?*

5. *Thinking about your teaching in the future. Which ideas from the interventions this year do you think might be useful to you?*

6. *Is there anything you would wish to have been different?*

7. *Is there anything that anyone could do that might help you develop your ideas in maths that could help you this year?*

8. *Is there Is there anything else you would like me to feed back?*

Ending the interview

- *Thank you for taking part.*
- *May I share the notes and recordings with our colleagues in the research program and other teachers?*
  (May I share just the anonymised notes with our colleagues in the research program?)

- *May we have your permission use your story anonymously in a report about the programme?*

- *Is there anything you don't want me to write up or things you don't want me to share with other people anonymously?*
- *Thank you very much for letting me interview you.*

Colleague to sign
I give permission for you to share the anonymised notes of me and recordings of me

Yes / No

I give my permission for my story to be anonymously included in the report.
Yes / No

In the report I do not want you to include comments about

……………………………………………………………………………………………

Colleague signature ………………………………………………

Learner Interview

AR1 Teacher notes

- Voice Recordings of the interview would be most valuable. This would have the most impact when presenting our findings. Jot down memorable episodes to help collection of data.
- Time taken could be 20 minutes so ask learners if they would like to contribute. 4/5 per teacher would be good.
- Have on display the interventions to remind learners. Attached
    - AR1: Ratio table, Barvember, centicubes, ratio with bar modelling (AO3 Questions?)
    - AR2: Ratio Table, Petrol lesson, Human Number Line, Twigs and Leaves, matching activity. (Separate questionnaire needed for the Voc Teachers with the V-M board) Graphs.
    -

Evaluation

Read to Learners: Please be as honest as you can and remember there are no right or wrong answers. All responses are really valuable to help me to understand your experiences. The recording will only be shared with permission from you. Anything said will be anonymised.

9. *Think about how we have taught you Maths this year. Try to remember a time in classes when you were most positive about your learning.*
   In your own words, tell me about that time.

<u>Follow up Questioning on the responses.</u>
What happened in the lead up to this moment?
What were you feeling?
What were others doing that helped?
What were you doing that helped?
What was happening at/during the moment?
What made it a positive experience?

10. *Are there any other positive examples you would like to tell me about?*

11. *Can you think of any time outside of Maths when you have used maths knowledge you have gained this year?*

<u>Prompt Questioning on the responses.</u>
What happened in the lead up to this moment?
What were you feeling?
What were others doing that helped?
What were you doing that helped?
What was happening at/during the moment?
What made it a positive experience?

12. *Are there any other positive examples you would like to tell me about?*

13. *Thinking about your life in the future. Which ideas from maths lessons this year do you think might be useful to you?*

14. *Having been in maths, is there anything you would wish to have been different?*

15. *Is there anything that anyone could do that might help you develop your ideas in maths that could help you this year?*

16. *Is there Is there anything else you would like me to feed back?*

Ending the interview

- *Thank you for taking part.*
- *May I share the notes and recordings with our colleagues in the research program and other teachers?*
  (May I share just the anonymised notes with our colleagues in the research program?)
- *May we have your permission use your story anonymously in a report about the programme?*

- *Is there anything you don't want me to write up or things you don't want me to share with other people anonymously?*
- *Thank you very much for letting me interview you.*

Learner to sign
I give permission for you to share the anonymised notes of me and recordings of me
Yes / No
I give my permission for my story to be anonymously included in the report.
Yes / No

In the report I do not want you to include comments about
....

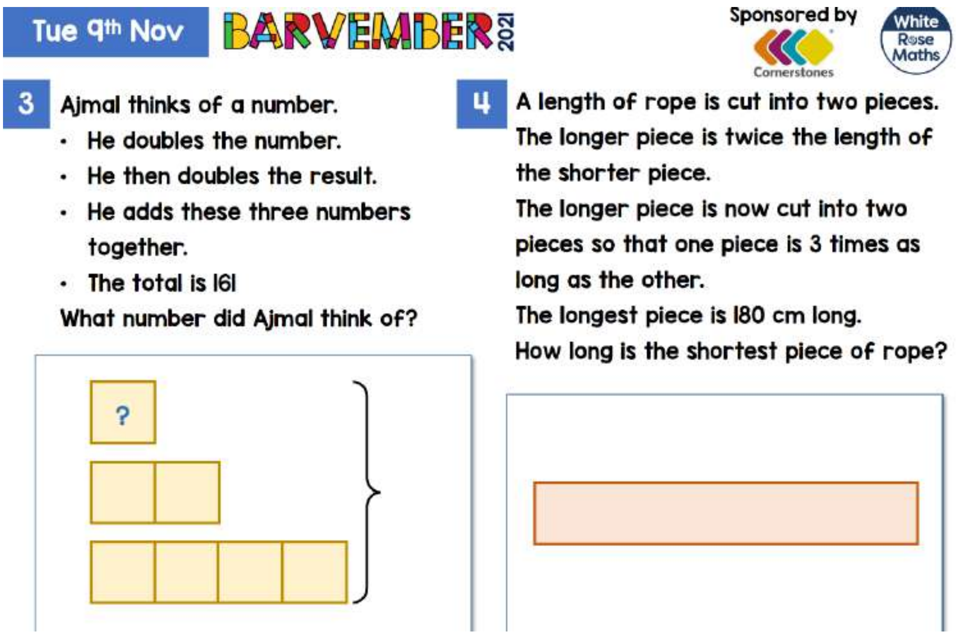
Learner signature ………………………………………………

Using the ratio tables

100%	50%	25%	75%	10%	20%	5%
£480						
	£680					
		30 ml				

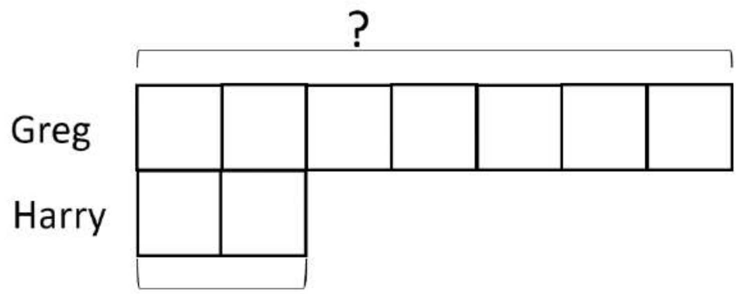
Using the Centicubes

Using Barvember Jan



Using bar modelling for ratio questions

Greg and Harry share some money in the ratio 7 : 2
Harry receives £45
How much does Greg have?



Teachers to add their own interventions...

Appendix C Raw data

AO3 Questions Raw Data

Learner	Starting grade	Mints Score 1st attempt	Confidence level (1st time)	Bungalow score	Bungalow confidence	Mints Score 2nd attempt	Confidence level (2nd time)	Cakes Score	Confidence Level	% Attendance	Difference between the 1st & 2nd attempt Mints	Confidence Difference between the 1st & 2nd attempt Mints
1	3	0	1			5	2	2	2		5	1
2		0	2			2	2	5	3		2	0
3	3	5	2			5	3	5	3		0	1
4	2	4	2			5	3	5	3		1	1
5	2	1	2			0	1	3	2		-1	-1
6		2	3			5	3	2	2		3	0
7	3	5	1			5	3	2	1		0	2
8	-	5	3			5	3	5	3		0	0
9		1	1			5	2	5	2		4	1
10		4	3			2	2	5	3		-2	-1
11		2	2			5	3	5	2		3	1
12		5	3			5	3	5	3		0	0
13	2	1	1			2	1	1	1		1	0
14	3	1	1	0	1	3	2	5	2	100	2	1
15	3	0	2	3	3	5	3	1	3	96	5	1
16	3	2	2	0		2	2	5	3	100	0	0
17	3	5	3	0	1	5	3	5	3	70	0	0
18	2	1	1			0	2	0		73	-1	1
19	3	2	2	1	2	5	2	5	3	100	3	0
20	3	5	2	3	2	5	2	5	3	82	0	0
21	3	5	2	0	1	4	3	1		54	-1	1
22	3	2	2	3	3	0	2	1	2	96	-2	0
23	3	3	1			0	1	1		85	-3	0
24	3	0	2			1	2	1	1	92	1	0
25	3	1	2			1	1	0		69	0	-1
26	3	3	2	1	1	2	2			96	-1	0
27	3	2	2			0	2	0	2	81	-2	0
28	3	4	2	1	3	5	2	5	2	81	1	0
29	3	2	2	3	3	3	2			87	1	0
30	3	0	2	3	3	5	3			86	5	1
31	3	0	2	0	3	5	3			88	5	1
32	3	0	2	0	1	5	3			89	5	1
33	3	0	3	0	1	2	1			79	2	-2
34	3	0	2	1	2	2	3			83	2	1
35	3	4	2	0	1	4	3	5	3	100	0	1
36	3	0	2	0	1	2	3			86	2	1
37	3	1	1	0	1	2	2	-	-	96	1	1
38	L1 FS	0	1	5	2	4	2	1	2	100	4	1
39	2	3	2	1	2	3	3	-	-	92	0	1
40	2	2	2	1	2	2	2	-	-	88	0	0
41	2	0	1	1	3	1	1	-	-	92	1	0
42	3	2	2	1	2	4	3	-	-	92	2	1
43	2	1	1	1	1	2	3	-	-	83	1	2
44	2	0	1	0	2	1	2	-	-	96	1	1
45	L1 FS	0	1	0	1	1	1	-	-	96	1	0

46	1	0	1	-	-	5	2	-	-	96	5	1
47	2	3	3	-	-	5	3	5	3	87	2	0
48	3	0	1	0	2	1	1	2	3	92	1	0
49	2	1	1	0	1	1	2	-	-	84	0	1
50	2	0	1		1	3	3	-	-	96	3	2
51	2	0	1	0	2	4	3	3	3	100	4	2
52	2	0	1	0	1	3	3	2	2	96	3	2
53	2	0	1	5	3	4	3	-	-	88	4	2
54	3	4	2	-	-	5	3	-	-	60	1	1
55	3	2	2	0	2	2	2	-	-	84	0	0
56	3	2	2	1	3	2	2	-	-	92	0	0
57	3	2	2	1	2	5	3	1	2	100	3	1
58	2	0	2		1	2	3	-	-	64	2	1
59	2	0	2	0	1	2	2	0	2	100	2	0
60	2	0	1	1	2	1	2	-	-	56	1	1
61	2	2	3	3	3	4	3	-	-	80	2	0
62	2	0	2	3	2	1	2	-	-	100	1	0
63	3	0	2	1	2	2	2	-	-	96	2	0
64	2	0	1	1	2	0	1	-	-	96	0	0
65	3	0	1	2	3	1	2	-	-	84	1	1
66	3	0	1	1	2	1	2	-	-	72	1	1
67	2	2	1	3	3	4	3	-	-	92	2	2
68	2	0	1	0	1	1	2	-	-	80	1	1
69	3	0	1	-	-	1	2	5	3	79	1	1
70	2	0	1	0	2	0	1	-	-	88	0	0
71	3	0	1	0	2	0	1	5	3	100	0	0
72	3	2	3	2	3	4	3	5	3	90	2	0
73	3	0	1	0	1	0	1	-	-	96	0	0
74	3	0	1	3	2	3	2	-	-	96	3	1
75	2	0	2	1	1	1	2	-	-	92	1	0
76	3	0	1	0	1	2	2	5	2	100	2	1
77	2	0	1	0	1	0	1		1	100	0	0
78	2	1	1	0	2	2	3	1	2	100	1	2
79	2	0	1	0	2	0	2	-	-	92	0	1
80	2	0	1	0	1	1	1	-	-	100	1	0
81	2	0	1	0	1	0	1	1	1	92	0	0
82	3	0	1	2	2	0	2	-	-	100	0	1
83	2	0		0		1	1	2	2	99	1	1
84	3	0				5	1	2	1	90	5	1
85	2	0		0		2	1	2	2	92	2	1
86	1	0				1	1	1	1	78	1	1
87	2	0	1	0	1	1	2	2		90	1	1
88	3	2	1	2	3	5	3	5	3	3	3	2
89	2	1	2	3	3	3	2	3	2	2	2	0
90	2	3	2	3	3	5	3	5	3	3	2	1
91	1	0	1	3	3	3	2	3		2	3	1
92	2	3	2	3	3	5	3	5	3	3	2	1
93	2	4	2			5	3	5	3	3	1	1
94	2	1	2	1		3	2	3		3	2	0
95	1	2	2			3	2	3	2	3	1	0
96	1	3	2			5	2	3	2	3	2	0
97	3	2	1	3	1	4	2	5	2		2	1
98	3	1	2	3	1	3	2	1	3		2	0
99	3	2	1	0	1	2	2	2	1		0	1
100	3	3	1			5	3	5	3		2	2
101	3	2	1	1	1	3	2	3	2		1	1
102	3	1	1	no answer		2	1				1	0
103	3	3	3			5	3	5	3		2	0
104	3	3	1			0	1	1	2		-3	0
105	3	1	1	1	1	5	1	3	1		4	0
106	3	2	1	no answer	0	5	3	5	3		3	2
107	3	3	1	no answer	1	3	3				0	2
108	3	2	1			5	2	1	2		3	1
109	2	2	2			5	3	5	3		3	1

Teacher interviews Raw Data

Teacher	Bar modeling	Ratio Table	centicubes	Double number line	Useful for sharing at the live event?	Next Year
A		When we did the last two worksheets using the small variations, my students had eureka moments that they suddenly understood ratios	I could see the students developing their skills and understanding the Maths	One learner set up the Centicubes when finding volume, he did a 4x4x4 cube and then said I don't believe there are 64 cubes in this.	When first showing the students a foreign currency question, none could answer the question. Afterwards, all but one could answer the question, using th double number line.	
		When the students used the bars on the exam questions, that really made a difference	When calculating 130% the students could see the structure more easily	Some teachers are reluctant to use these but I have found the students have been fairly responsible with these.		
		I gave the bar modelling intervention to a teacher and they said it went so well, she felt proud of them drawing their boxes. They were able to use the method successfully and independently				

B	Barvember was a good set although it was too easy or too hard.	Student doing a percentage problem, The student could easily understand and he was excited about it	The cubes for a few students liked them.		No CPD time.	
			Used cubes for plans and elevation, making your own shapes.		I have enjoyed getting together as we can share the problems, attendance, learner understanding and motivation.	
			Used them for the bars. Across the ratio type questions.		Any collaboration is positive, having the discussion makes you think of about it and working together is better than working on your own	
			They needed help with putting the scaffold in place.		been great working with everyone.	
			You could see they could understand the question		keeping in touch with the AR1 group to continue to refine resources.	
			Useful to have them with me.		I will share all the resources with other teachers.	
C	Using the TES short questions. Using bar modelling made the learners really think about what ratios meant (not just about adding ratios). Great to see learners engaging with AO3 chunky questions by drawing blocks and bars. Light bulb moments.	The early ratio tables were used by some in November resit.Was able to bring in other questions using topics such as pie charts. Made it a positive experience of maths.	Some learners engaged but on the whole did not engage, would need to prompt more and direct their use if used again.	Had not used very much this year but was looking forward to using them more next year. At her college has a strict SOW so would need to be built into it for next year, although learning mentors had been shown and thought it was useful.	General: Had shared resources with learning mentors. Had enjoyed the experience and felt it had helped learners confidence.	Next year: Would start the year off with ratio tables and physical resources. Was looking forward to exploring Algebra tiles next year and building in what she had learnt into the SOW.
D	Doing the barvember week by week, seeing the joy in the students faces, being able to build up over time. It felt more fun.	Beneficial for learners as a building up method.	Allowing learners to be in the moment, to be more creative and allowing them thinking time. With prompting, students were then drawing down representations of the blocks. I was more enthusiastic.	Groups preferred the bar modelling approach.	General: Shared some resources with Learning coaches and mentors who are using them with lowest ability learners. I've really enjoyed the interventions, I feel more confident. It's easy to do the same old ways, this has given a different way of thinking.	Next year: Would like to try using the Algebra tiles from early in the year. Would want to start off earlier in the year with physical resources, ratio table, algebra tiles etc, include % and fractions.

E	Maths 4 life has been good. Having the visual alongside the question was good for introducing bar modelling	Sharing ratio table interventions with my colleagues has been beneficial. Embedding them in all starters has been good. repetition	time constraints meant I was unable to use Centicubes. I would like to try them next year early on in my teaching to introduce topics.	It was good to try different interventions, I just wished we had narrowed down quicker so we could have done multiple tests on the same intervention. More time to dedicate to the AR		
	Bar models for wordy questions helps the learners find the difference in harder questions	Introduce ratio tables early on for induction weeks to set the tone for the learning		The SOLAR could have been adapted better to fit in with the interventions.		
F	Activity 8, were really basic but the scaffolded approach they really enjoyed and gave them a good introduction	one group didn't like it. They couldn't find the answers but they then used the bar models and they realised they could do it	The bar models and ratio tables gave the students a good structure after being out of school for so long after what they had been through.	I was nervous about using these.	I loved the double number line but didn't use it enough. I really want to use it again.	
		Shared bar models, Centicubes and ratio tables in our starters all staff members have been encouraged to use them	the students enjoyed the method for them to hook onto	I would definitely use them at the beginning.	I wish I had more groups to try it on as most of my groups were so low level.	
		The worksheets I let them work through it. They were brilliant. They could see the progression and how you could apply it. Not changing the numbers really helped. Using Variation Theory.	At the beginning of the year they were resistant.	A new teacher who are construction learners. They can visualise much better. You gave me a calculator for a non calc question. But needed to be earlier on.	Having the interventions early on but I got a bit lost I couldn't try everything. More structured	
		Would use the worksheet as a starting point	the students enjoyed the method for them to hook onto		I have really enjoyed doing this hands-on research project. I am testing myself and getting students feedback. It forces to be reflective and	
			The low level learners because of the tag and CAG grading, Ratio tables for them to having it in a grip was really helpful. FS L1. They would draw the tables on the laminated table using 2:1 2:1 2:1 but that couldn't work for 600 so the ratio table worked really well.			

Learner interviews Raw Data

	manipulative			
Learner	Bar modelling	Ratio Table	Centicubes	Double number line
A	The bar model helped me understand how to do Ratio. You can see everything more clearly and work it out quicker.	I found that the [ratio tables] helped to find percentages.		The conversion money questions I didn't get these questions in the exam so [the double number line] will help me next when I do the exams.
B	Using bars and cubes laying out has really helped. I am better at ratios now using the boxes. Not getting confused or annoyed. Feel more confident I don't worry about that now.	I understand the percentages a bit more now.	The cubes helped a bit	
C	Using the bar models to help be understand how to find a fraction of an amount	The massive question we did last lesson, I couldn't do that before, my mind was just blank. I got full marks on that [AO3 Question]	Making the cuboid helped me with the understanding of the shapes.	This method really helped with the do the exchange rate method, it helps you see where you are going with it.
		Using the bar models to help be understand how to find a fraction of an amount	Making the cubes and cuboids was fun and a good understanding of 3d shapes.	You can imagine where you are going with it when realting to holidays
			You had to count the tops there were 8 and sides there were 4 down and then there are 32 cubes altogether. It helped	
			"it helped my understanding of 3d shapes	
D		You're teaching us more about strategies and stuff, how to layout and school. They didn't really help us with that. They should make us do the work.		
E	When you showed a little box, that helps me click.	doing percentage, using the grid and the blocks has definitely made it easier.	Instead of writing it down its not just seeing them, you can move them around. It really helps	
	It feels good to know it when I get it and puts me in a good mood.			
F	I used to just ignore these questions on an exam - now I would be happy trying to have a go at them. I feel more confident and I can get the answer more easily than with other methods. I've still got to remind myself to use bars.		It helped actually show what the question was in a real form.	
G	I'm pleased that I can now answer questions I couldn't before. I feel more confident.			
H	I would normally skip these questions. I needed to keep trying this approach, I wouldn't get it after just a couple of weeks.	I need to do stuff lots of times to drill it in.	By the end, I visualised the blocks in my head and didn't need to draw them.	
I	I think it would have to be when I learnt how to work out an area of a circle using bar models. It surprised me how to do it and I felt smarter then I ever had in maths and it just helped me.	In Theory in my electrical installation course I used ratio tables for the answer because to know a resistance of something or resistivity we have to use a calculator and type in 5 different things that I didn't know about till I learnt it in maths. The ratio tables helped me.	Using Centicubes was useful for volume and ratio.	
J	The bar models made me understand how to break the questions down and build up the answers. The assessment at the end of each topic gives you an opportunity to see how much of the topic you under stand and what parts you need to work on. It also allows you the see the topic in different types of exam questions.	Doing the sheet of random maths questions at the start of each lesson using the ratio tables helps to keep other topics you might not be working on in class anymore fresh in your head and allows to concentrate and get your brain going for the lesson ahead.	It made maths fun and interesting.	
K	They helped with fractions, percentages and ratios to divide the numbers then see the numbers as pictures.	This was useful at the start of the year and I would like to use these more often.	I liked working in groups using these.	I found this quite hard to understand.
L	Understand bar modelling much better as it's been in the starters every week. The repetition has been good	I Use the [ratio tables] with scale factor in carpentry lessons. The visual things have helped with my understanding much more this year.		
M	Bar modelling - able to answer more questions	Ratio tables have helped me to understand percentages better. I will use ratio more in the future because of carpentry		
N	Bar modelling questions I have really enjoyed (pointed to prompts). Visually it helped and the weay it was taught. Now understand it better	Ratio linking to carpentry, gaps in timber: filler.		

Appendix D Informed consent.

Centres for Excellence in Mathematics: Action Research Teaching approach

Research Participant (student) Information

Weston College, it's network partners and the Centres for Excellence in Mathematics (CfEM) are funded by the Government through the Department for Education.

The aim of the work is to improve the teaching of mathematics for GCSE learners and the improved progression of all 16-19 learners. As part of this work your maths teacher is taking part in developing innovative teaching in Maths. This will result in your teacher using a number of carefully designed resources with you over the forthcoming year. The research team will investigate the effectiveness of the approach by gathering some data from you. This will involve you being asked to complete short surveys during the year and your Exam outcome data being used for the data collection.

The research team from your College would very much appreciate your willingness to take part in the research. It has the potential to assist generations to come to experience better teaching in mathematics.

Data Protection

Your data will be tracked but anonymized. We will not be sharing your name, only the progress that you as pupil (x) have made.

If you wish to withdraw from participating in the research, you should tell your teacher now and at any time over the next year and complete the form that is part of the participant privacy notice. If this form is submitted at any time all of your data collected to that point will be destroyed and no further data collected relating to you.

We do hope, however, that you are happy to participate given the potential that the research has to inform us how to improve teaching of maths.

Ethical issues

We will ensure your anonymity at all stages and no survey or GCSE outcomes will be identifiable as belonging to you personally. In the case of any use of video for any purposes additional permissions will be sought from you.

If you would like any further information about the work of the CfEM programme in general or of the research programme in more detail please contact Katheryn Cockerton Katheryn.cockerton@weston.ac.uk

Katheryn Cockerton
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