Lesson plan

Ratios and fractions

Contents

1. Rationale

Students often get confused with the different ways in which part–part and part–whole relationships can be represented. In this lesson, ratios and fractions are presented together, with a single diagram that represents both the ratio and the fraction. The diagrams provide insight into **mathematical structure** (Key Principle 1) and their use encourages students to see **links between mathematical concepts**, rather than viewing them as separate content. This is important in supporting a **coherent and connected curriculum** (Key Principle 3) and is essential in the FE sector, where there is limited curriculum time.

2. GCSE curriculum

Ratio, proportion and rates of change

R5 divide a given quantity into two parts in a given part–part and part–whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems

R8 relate ratios to fractions

Number

N11 identify and work with fractions in ratio problems

3. Lesson objectives

- Write a ratio as a fraction and vice versa
- Divide a given quantity into parts in a given part–part and part–whole ratio
- Solve multi-step problems using ratios and fractions
- Use representations to provide insight into solving problems

4. Starting points

The lesson assumes that students have some understanding of fractions and ratios. It is likely that they will have experienced a variety of ways of representing part–whole relationships and it is important to **build from what they already know** (Key Principle 2).

5. Research questions

Pedagogic focus

How does the teacher use class discussion to emphasise connections in mathematics?

Maths focus

What evidence do you observe of students using representations to develop understanding of, and connections between, fractions and ratios?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	5	Introduce the context of buying a baguette to share for lunch. Discuss how the baguette can be shared and remind students of the language of fractions and ratios.	Mini whiteboards Slides 2–3
Explore 1	10	Ask the students to work in pairs, matching fraction cards to ratio descriptions of different ways the baguette could be shared.	‘Sharing baguettes’ grid ‘Fractions’ cards Slides 4–5
Explore 2	15	When students are a substantial way through matching ratios and fractions give them some diagram cards to place in the column between the ratio and fraction columns.	‘Diagrams’ cards Scissors Slide 6
Explore 3	15	Once students have completed the ratio–diagram–fraction matches give them a set of description cards to place along the edge of the grid.	‘Descriptions’ cards ‘More sharing’ handout (Extension) Glue, Scissors, A3 paper Slide 7
Discuss	15	Discuss what the diagrams revealed about the relationship between fractions and ratios. Discuss how students adjusted their cards after being given the diagram cards.	Mini whiteboards Glue Slides 8–13
Review	20	With the students, explore a common misconception and discuss how fractions and ratios are linked. Describe part–whole and part–part relationships.	Mini whiteboards Slides 14–15
Practice question	10	Ask students to answer an exam question and after a few minutes discuss their thinking.	Slide 16

7. Teacher guidance

Introduction

Aim	To introduce students to the context
Materials	Mini whiteboards
Slides	Slides 2–3
Time	5 minutes

A key element of teaching for mastery is the **'concrete, pictorial, abstract' (CPA) approach**. This is when students develop their understanding of abstract mathematical concepts and structures, starting with concrete objects, moving on to pictorial representations and then linking to abstract mathematical symbols. This section of the lesson starts with the context of sharing a baguette.

Another fundamental aspect of the mastery approach is **building on existing knowledge** approach (Key Principle 2). When discussing possible ways of sharing the baguette, acknowledge the students' ideas and approaches and explain that in this lesson they will be using what they already know and can do in order to deepen their understanding.

What the students might do and what you might do

Slide 2 Introduce the context of two friends (Amy and Bikram) buying a baguette to share for lunch. Ask students to suggest a possible way that Amy and Bikram could share the baguette.

It is likely that students will assume that Amy and Bikram will share the baguette equally so will get half each. Ask for an alternative way to share the baguette to encourage students to think more deeply about the problem.

The students may want to know exactly how much Amy and Bikram each paid, but at this stage the amounts are irrelevant as this activity is only intended to remind them of concepts they will have encountered previously.

Slide 3 uses both ratio and fractions to describe how the baguette is shared. Discuss the difference between Bikram's terminology (fractional) and Amy's terminology (ratio).

Notice how students engage with the language of fractions and ratios. How easily do they appear to relate fractions to ratios? If they appear to be confused, spend a little time revising the differences between fractions and ratios. Reassure them that the main activity for the lesson is designed to help them understand the different ways of describing how things are shared.

Explore 1

Aim	To explore the relationship between ratios and fractions
Materials	‘Sharing baguettes’ grid and ‘Fractions’ cards, scissors (if the cards have not already been cut out)
Slides	Slides 4–5
Time	10 minutes

In this section of the lesson, students work in pairs to match fraction cards to ratios. It is a deliberate design feature of this task to hand out the fraction cards without the diagram cards at this stage. It is likely that some students will make incorrect matches, and will then realise that they have made mistakes when they are unable to match all the cards. They may not know how to correct their mistakes (cognitive conflict). The diagrams cards, which are intended to help the students resolve their cognitive conflict, are handed out later.

The pair work encourages a **collaborative culture** where students work together and share their understanding and provides opportunities for students to explain their thinking to one another (Key Principle 5). This way of working contributes to a classroom culture in which **everyone believes everyone can succeed** (Key Principle 5).

What the students might do and what you might do

Slide 4 Tell the students that they are going to be working in pairs. This important slide emphasises the ways in which students should work together. Many students may have very little experience of working collaboratively in maths lessons, and it is helpful for them to be given some guidance about how they should collaborate and what is expected of them.

Slide 5 Explain that the class is going to explore different ways that Amy and Bikram could share the baguette. Arrange students into pairs and give each pair a copy of the ‘Sharing baguettes’ grid and the ‘Fractions’ cards. (Students will need scissors to cut the cards up if they haven’t already been cut into cards prior to the lesson.)

Note that some rows in the grid have more than one fraction card and some rows don’t have any. Students are not expected to create cards for rows that have no fraction but do need to complete the blank card F10, which they should place in a row that does not have a fraction card or as an additional card in a row that already has a card matched.

Observe students as they complete this task but try not to correct their mistakes at this stage. Instead, encourage students to explain their reasoning to each other and listen in to their explanations as you circulate. A common misconception that you may observe is assuming that the ratio 1 : 2 is equivalent to the fraction $\frac{1}{2}$.

Explore 2

Aim	To use diagrams to check their understanding
Materials	‘Diagrams’ cards, scissors (if the cards have not already been cut out)
Slides	Slide 6
Time	15 minutes

This section of the lesson uses diagram cards to provide insight into the meaning of both the fraction (part–whole) and the ratio (part–part) and hence the relationship between the fraction and the ratio associated with the diagram.

Both the ratio and the fraction are represented using the same diagram. The use of diagrams provides insight into **mathematical structure** (Key Principle 1) and highlights the connection between the different concepts.

What the students might do and what you might do

Slide 6 When students have matched the majority of the ‘Fractions’ cards, give out the ‘Diagrams’ cards. Ask students to place the cards in the ‘Diagram’ column in the appropriate cell. How do the diagrams support their thinking?

Card D9 is blank and needs to be completed by students.

As students work on the task, support them in working together and encourage them to explain their reasoning. Remind them to take turns. If you notice one partner is placing all the cards or they are not working collaboratively, ask a student to explain a card that has been placed by their partner.

Explore 3

Aim	To explore ways of describing links between fractions and ratios
Materials	‘Descriptions’ cards, A3 paper (optional), ‘More sharing’ handout (extension), scissors (if the cards have not already been cut out)
Slides	Slide 7
Time	15 minutes

In this section of the lesson, students match a set of ‘Description’ cards to the completed ratio–diagram–fraction matches. The description cards support students in **making connections** between fractions and ratios, and reinforce the ways that they are often described in exam questions.

What students might do and what you might do

Slide 7 Once students have completed the ratio–diagram–fraction matches they can be given the set of 'Descriptions' cards.

Students may find it helpful to glue the grid onto an A3 piece of paper so that there is more space to the right of the grid to place the 'Descriptions' cards alongside the appropriate row. Some rows have more than one description card and some rows don't have any. Students are not expected to create cards for rows that have no description but do need to complete the blank card W8. They may choose to complete this card for a row that has no description card or add a further description to a row that already has a description card matched.

Observe students as they complete this activity and identify which descriptions cards will be particularly important to highlight in the whole class discussion. Students often struggle with 'one and a half times as much' (Card W6) and it may be appropriate to withhold this card in some cases.

You could suggest that students photograph their completed task so that they can each keep a copy.

Deepening understanding Students who have completed their matches can use the 'More sharing' handout to explore additional ways in which Amy and Bikram could share the baguette. The blank cards on the handout can also be used to add some additional fractions and/or descriptions to the existing scenarios A to I.

Discuss

Aim	To identify what the representations revealed about the relationship between ratios and fractions
Materials	Mini whiteboards, glue
Slides	Slides 8–13
Time	15 minutes

Discuss the completed task, particularly focussing on those matches that you noticed that pairs struggled with. Make sure that students understand the significance of the order in which values are presented when describing a ratio.

What students might do and what you might do

Arrange a discussion that exposes the development of students' understanding, calling on students whose work you identified during the exploration activities.

Slide 8 Students usually like to know the 'answers', and while this is important it is also important to understand how they arrived at these answers. Ask the students to explain their thinking when they first matched the fraction cards to the ratios. You may like to call on a pair who initially got it wrong and later corrected their work. Did the matched card remain in place or did the introduction of additional card sets cause students to move the card? Why was this?

Establish any **links and connections** that students have made for themselves while working on the task. During the discussion it is important to re-phrase student explanations using the language of parts and whole.

Slides 9–10 ***(optional)*** These slides give the answers for the three Explore tasks and can be used if required.

Slide 11 Arrange a discussion to draw out an understanding of what is the same and what is different about fractions and ratios.

Ask the students what is the total number of parts? What fraction does Amy get? What fraction does Bikram get? What is the ratio in which Amy and Bikram share the baguette? By the end of the discussion, students should understand that, in a ratio, adding gives the total number of parts. In a fraction, the denominator gives the total number of parts.

Slide 12–13 It is important that students understand the significance of the order in which values are presented when describing a ratio.

Slide 12 Present the two scenarios and ask students whether the two ratios are describing the same way of sharing. Give students a couple of minutes to consider this in pairs. Students may want to draw a diagram for each scenario on their mini whiteboards.

Discuss students' diagrams and use the diagram on the slide to establish that 'Amy and Bikram share the baguette in the ratio 1 : 3' is the same as 'Bikram and Amy share the baguette in the ratio 3 : 1'.

Ask students to work out what fraction of the baguette Amy and Bikram get in each case. They should establish that Amy gets $\frac{1}{4}$ of the baguette and Bikram gets $\frac{3}{4}$ of the baguette in both scenarios. Comparing the scenarios by considering the fraction that Amy and Bikram receive in each case highlights the importance of understanding the relationship between ratios and fractions and **making connections between these two areas of mathematics** (Key Principle 3).

Slide 13 The comparison of ratios can be linked back to the 'Sharing baguettes' grid. Did students notice that rows B and H describe the same scenario? Which rows did they choose to place cards F3 and D6 in? Did they place them in the same row or different rows?

Review

Aim	To examine a common misconception
Materials	Mini whiteboards
Slides	Slides 14–15
Time	20 minutes

This section of the lesson ensures that students understand how the numbers in a fraction are linked to the numbers in a ratio, before moving on to working out how much Amy and Bikram should each pay for the baguette.

What the students might do and what you might do

Slide 14 Present the scenario and ask students to determine who is right – Amy or Bikram. Can students draw a diagram to show whether Amy is correct? Encourage students to draw a diagram and refer to the 'Sharing baguettes' grid as appropriate. Some students may have placed card F2 in row D initially and thought the same as Amy. If they corrected this, they may now recognise that Amy is incorrect.

Establish the relationship between the numbers in the fraction and the numbers in the corresponding ratio (e.g. if Amy and Bikram share the baguette in the ratio 2 : 3, Amy gets $\frac{2}{5}$ of the baguette. If Amy gets $\frac{2}{3}$ of the baguette, Amy and Bikram are sharing the baguette in the ratio 2 : 1).

Slide 15 (*optional*) Ask students to calculate how much Amy and Bikram would be paying towards the cost of the baguette (total cost £1.20) for both the fraction $\frac{2}{3}$ and the ratio 2 : 3. This supports students in developing their understanding of ratios and fractions still further.

Deepening understanding Students could be asked to calculate how much Amy and Bikram would need to pay for their share of the baguette in each of the scenarios A–I.

Practice question

Aim	Students apply their knowledge to an exam question
Materials	'Practice question' handout. It is not necessary to print this out: the question can be displayed on the board.
Slides	Slide 16
Time	10 minutes

Students apply what they have learned earlier in the lesson to an exam question. Students may want to just check their solution, but the focus when discussing the practice question should be on students' thinking and to **expose the connections that students have made** between fractions and ratios (Key Principle 3).

What the students might do and what you might do

Slide 16 Notice whether students draw a diagram to support their thinking. How does drawing a diagram demonstrate the links between fractions and ratios?