



**CENTRES FOR
EXCELLENCE IN MATHS**



Responsive teaching techniques to address misconceptions through peer explanations

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About CfEM

Centres for Excellence in Maths (CfEM) is a five-year national improvement programme aimed at delivering sustained improvements in maths outcomes for 16–19-year-olds, up to Level 2, in post-16 settings.

Funded by the Department for Education and delivered by the Education and Training Foundation, the programme is exploring what works for teachers and students, embedding related CPD and good practice, and building networks of maths professionals in colleges.

Summary

Mastery teaching calls for the teacher to check knowledge is secure before moving on. But what if it's not?

It is essential to learning that we respond to the information students give us in assessments to build their understanding. In this study we investigated a responsive teaching approach that allows teachers to respond to misconceptions that persist after teaching without needing to fully reteach the whole topic or making time to do 1-1 feedback and target setting with every student.

We work with GCSE Maths resit students aged 16-19 who are trying to improve from a grade 3 to a grade 4. We have recently introduced a system of end of unit checks using the Diagnostic Questions website. We experimented with some different approaches to discussion and review of students test results.

We found that a 30-minute discussion-based review can be an effective way to address the most common misconceptions, and that the most effective discussions involve

- Paired students
- Teaching students how to give feedback
- Giving feedback on anonymous work
- Leveraging that discussion into more advanced work on the same topic

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Background

Introduction

In order to improve student achievement in GCSE Maths resits in a sixth form college context we have introduced a mastery learning scheme of work which focuses on particularly key topics for success and teaches them in depth through longer units of work, including multiple representations. Towards the end of each unit there is a regular low stakes assessment known as an end of unit check. This formative assessment is designed as a diagnostic tool to elicit the evidence needed to allow us to identify any remaining misconceptions and respond to meet the needs of all students.

Background

Our college and cohort

“Working within a safe, welcoming and stimulating environment, which embraces diversity and promotes respect, we help students fulfil their academic potential and become thinking, questioning and caring members of society.”

Leyton Sixth Form College has about 2000 students, mostly aged 16-19 and studying full time at level 3. Around 60% of students are doing A-levels and 40% are on vocational programmes such as BTEC. We also offer BTEC and ESOL courses at Level 1 and 2 to enable students to access further learning through progression at the college. Around 600 plus students go on to university each year from both A Level and Vocational courses.

Local Government data shows that “Waltham Forest is currently ranked 82nd most deprived borough nationally according to the 2019 Index of Multiple Deprivation (an improvement from 35th in the 2015 edition, and 15th most deprived in the 2010 edition) (Waltham Forest Borough Council, 2021).”

We have been a member of the Centres for Excellence in Mathematics (CfEM) since October 2018. We participated in the CfEM research project National trials for Mastery with the University of Nottingham in 2019-2020.

Our learners and our goals for GCSE Maths

Since it became mandatory for students who had not achieved a “pass” (C or 4) at GCSE to resit, the GCSE resit programme has grown from around 200 students to around 600 students. We offer GCSE Maths to all students who have not yet achieved a grade 4. We split this cohort into two courses, one for students who have a grade 3 and are working towards a grade 4, and one for students who have less than grade 3 with the goal of achieving a grade 3 and progressing to the next level alongside the other courses they are doing the following year. The value added on these courses is excellent, and overall students on the 3-4 level do better than the national benchmark for success in GCSE resit Maths, but we are ambitious for more of our students to pass GCSE Maths before they leave college.

The table below shows the grade progression for students at LSC from when they entered to when they left in 2021. Note that this progression has often been achieved over a process of 2 or 3 years while students complete their other courses at LSC.

Finishers summer 2021

Entry Grade	No. of students at this grade	Numbers achieving grade in 2020-21							Percentage of students who dropped back at least one grade	Percentage of students who stayed at the same grade level	Percentage of students who progress by at least one grade
		X/ U	1	2	3	4	5	6+			
3	169	0	0	0	40	113	13	0	0.00%	23.67%	76.33%
2	103	0	0	19	31	51	2	0	0.00%	18.45%	81.55%
1	37	0	12	14	6	5	0	0	0.00%	32.43%	67.57%
U/ NONE	24	1	2	1	9	8	3	0	-	4.17%	95.83%
TOTAL	333	1	14	34	86	177	18	3	Progress Score = 1.0926		
** 1 STUDENT ENTERED WITH GRADE 4 ALREADY ACHIEVED											

GCSE resit and mastery learning

Smith (2019) recognises that GCSE resit pass rates are nationally and historically very low, even though the November paper is norm referenced to the June cohort so in theory pass rates could be much higher. She identifies problems with motivation & engagement. According to her students do not choose to resit and they are not interested in “more of the same”.

Mastery learning is an approach aimed at improving student engagement and outcomes inspired by the work of Benjamin Bloom and good practice seen in countries such as Singapore and China. The Education and Training Foundation are supporting many action research projects on implementing this powerful approach in FE and sixth form colleges as part of their Centres for Excellence in Mathematics project, including our project last year, “Using double number lines and bar modelling to teach the GCSE maths curriculum based on the Mastery approach.”

Guskey (2005) presents a history of mastery learning, setting out how Benjamin Bloom devised a series of instructional units, each taking a week or two, followed by a brief formative assessment which gives students feedback on their learning – what they’ve learned well and what they need to learn better, and that this formative assessment is paired with corrective activities to address individual problems or extension/enrichment work for those who have demonstrated that they have learned the key points. This is the model we’re using for the mastery scheme of work for GCSE resit at Leyton. (See appendix.)

As part of the mastery learning approach, we have introduced regular low stakes assessments (end of unit checks) In contrast to the high stakes key assessments we use for tracking students’ progress. The scores from these are not recorded centrally, reported to parents or used to estimate students’ grades. Students are not expected to do significant revision for them. They are designed to elicit evidence which can be used by the student and the teacher to progress their own learning. Because these end of unit checks are topic based, students may choose to record their own score as a way to prioritise revision for high stakes assessment later. They are given a tool to do this but the use of it is optional. At the start of our project, we surveyed teachers’ use of these end of unit assessments and found significant variations in practice. Most teachers considered that the assessments were giving them useful information without being too burdensome to mark, but the ways in which they have responded to this information were varied and sometimes limited. One of the challenges is that students demonstrate different needs in the tests. Some require a focus on fluency: they have persistent misconceptions or simply do not yet have the technical skills needed to, for example, calculate a percentage of an amount. Others have demonstrated

that they are able to complete this task if it is clear what they are to do, but they need more depth, by which we mean experience of scenarios and problems in which they need to form a strategy and select suitable mathematical tools to solve it. This added cognitive load may lead to temporary forgetting of recent learning on the technical skill, so overlearning may be useful.

The aim of this project is to develop strategies that are responsive to these different needs and further the learning of all students through a process of action research.

Literature Review

In 2021, the Centre for Excellence in Maths ran a Maths Teacher Development Group session on the theme of Developing Responsive Teaching to Meet Learner Needs. There was discussion on what responsive teaching means, a variety of understandings being identified, but fundamentally it means meeting learner needs through adapting teaching based on prior knowledge/starting points of learners. Part of the training gave time for reflection on the purpose of responsive teaching – “why do you use a responsive approach...?” what possible reasons could there be to; engage students, make maths relevant to students, build on prior knowledge, make adaptations that help the lesson to meet learner needs, or to make other general connections. The focus on these varied purposes was useful to allow us to clarify what we mean by responsive teaching and what we want to achieve.

Our action research project seeks to implement responsive teaching through gauging the level of understanding that a learner has in a mathematical topic and addressing misconceptions that arise to clarify/deepen their understanding to a level of “mastery”.

The conversation naturally moved into how we as teachers acquire this understanding, - “What are the assessment methods that could be used for responsive teaching without it being a formal test?” Some responses included; quizzes, questioning, mini white boards, entry & exit tickets.

Experience of a project from Grimsby Institute was shared where they implemented the Mastery Loop implicitly in their lesson planning: Assess prior knowledge, teach to fill gaps, assessment of grasp, move on OR approach in a different way before assessing grasp again. They found that it became natural to pre-empt potential misconceptions to be able to deal with them in class. There were various benefits including, learners feeling more engaged due to feeling valued through a tailored approach. One of the challenges was increased tiredness of staff as it was exhausting to work in this way. An important question we can ask then is: Can there be a more natural responsive teaching method that doesn't require huge energy?

Wiliam (2014) says “rather than a focus on data-driven decision-making, the emphasis is on decision-driven data-collection” – the purpose of these end of unit checks is to inform the teachers decisions on what to do next and the use of carefully chosen distractors (wrong answers) makes it easier to draw inferences about where students misconceptions remain and plan to address them. Although Wiliam makes the case for students engaging in self-regulated learning, at Leyton Sixth Form College we preferred a more teacher led approach because it is easier to make sure that students are addressing the issues that will make a difference, have the resources they need and follow through to complete the necessary practice if they are supported in this by activities that take place in the allocated class time with help and guidance from their peers and their teachers.

Smith (2019) has had significant impact with the 5 Rs approach – Recall, Routine, Re-vision, Repeat, Ready. In the “Re-vision” section of her lesson structure she engages students using a hook with a real-life context and then spends 15 minutes eliciting prior knowledge and misconceptions from the class through discussion. She then secures the key skills for the topic before going on to practice in the Repeat section. We recognise this as excellent practice and the potential of this kind of dynamic formative assessment with its ready link to transfer into context but understand also that this is heavily dependent on the skill of the teacher – it would be difficult to implement this consistently across the cohort with teachers at varying levels of experience and confidence, so we prefer an option where the teacher can get the data in one lesson and spend some non-contact time thinking about how to respond effectively in the next lesson.

In his book “Reflect, Expect, Check, Explain” Barton (2020) explains the meaning, purpose and methods of formative assessment. He cites this definition (Cowie and Bell, 1999): ‘the process used by teachers and students to recognise and respond to student learning in order to enhance that learning, during the learning.’ Barton says his favourite method of formative assessment is Diagnostic Questions. These are multiple-choice questions, with responses that are designed to bring out common misconceptions. The reasons Barton gives for why such questions are of high pedagogic value include: quick and easy to collect student responses, high level of student engagement, promotes deeper thinking by students, not least because they are forced to become better at explaining why an answer is incorrect.

Sweiry (2019) shares his expertise in the design of multiple-choice questions for summative assessments with AQA, and it’s interesting that although they were intended for use as part of summative assessment Sweiry and the team at AQA gave significant thought to identifying misconceptions in the design of their questions. The difficulty in writing Multiple choice questions (MCQs) is having plausible distractors (wrong answers) and the ability to assess higher order skills. When writing MCQs, an important question to ask yourself is “What do you want to assess in the question?” and you should already have misconceptions in mind. There also should be no ambiguity with the question and the correct answer. All distractors need to be wrong! Another point made was that if it is easy to eliminate the wrong answers then you are not assessing what they know.

Our project is not about the design of diagnostic questions – a lot of work has already been done on this – but well- designed questions are important to elicit the evidence on which we will respond. Without evidence which quickly indicates specific misconceptions it’s hard to plan a targeted response.

Bell et al (1993) worked on developing a “diagnostic teaching pedagogy” It was found that explicitly addressing misconceptions during teaching improved achievement and long-term retention. In this specific project Bell et al developed some reflective activities to promote metacognitive activity in which students learn about their own learning process, including some of the intervention strategies we plan to use: Students reflecting on learning difficulties and misconceptions, Students teaching students, Students conducting small group discussions. We will be using the result of the multiple-choice questions in the diagnostic tests as a starting point to keep the discussion of possible misconceptions focused.

Evidence from the report of the SSAT (2018) Embedding Formative Assessment project implementing the strategies formulated by Wiliam & Leahy in 140 secondary schools was that “The additional progress made by children in the lowest third for prior attainment was greater than that made by children in the highest third” – although this wasn’t specifically for GCSE Maths or English, we feel this evidence is very encouraging for students who are resitting GCSE Maths because they are in the lowest third for this subject. In particular the

schools in the study report that the vast majority of students consider that the feedback they get is useful and helps them to make progress, and teachers and senior leaders observe that students are more aware of the specific skills or knowledge they need to improve and acting on the feedback they get to refine and improve their work. This would represent a significant improvement in confidence and motivation for our students if we can achieve it.

According to Wiliam & Leahy (2015) as cited by Jones (2021), activating students as resources for each other, when implemented properly, “can substantially increase student achievement, both for those who get help from their peers, and peers who provide the help.” This meets our goal of furthering the learning of all students. Jones goes on to elaborate that part of implementing this properly is creating a culture of support and understanding within the classroom but also offering substantial guidance and modelling to students on how they should offer feedback and advice to each other. In our follow up activities, we need to offer students significant guidance on what they should focus on and what sort of points they need to make in their discussions.

A comprehensive review of the available evidence on feedback and an analysis of practical teaching strategies is made by Fletcher-Wood (2018, pp. 96-121). Effects identified in the available research are highly variable and from this he concludes that feedback is powerful but problematic; given correctly it can improve student performance but given poorly students may give up, reject all feedback or choose an easier goal.

Fletcher-Woods’ view is that feedback must start with the preceding problems or misconceptions and tasks should be designed with feedback in mind to enable a more focused teacher response. Another critical component is the choice of feedback with lower attainers making most improvement from task specific, directive feedback but as their understanding of a topic grows they can move to more general feedback applicable to a range of tasks e.g. can a diagram be drawn, has the original problem been answered?

For feedback to be effective it is crucial that students engage with it in the right way. They must first understand the feedback given; it should be focused, clear and concise. Students must act upon the feedback, a useful test being whether it is “more work for the recipient than the donor” Wiliam, (2017 p. 129). Teachers should check that students have understood the feedback, allow them to make corrections followed up with similar problems for them to complete.

“Students’ emotional responses affect how they react to feedback” (p. 105). Fletcher-Wood highlights the importance of avoiding giving students grades, never hinting students should stop trying and avoiding social comparison i.e. comparing themselves with their peers. Building trust is very important, for example in one study students receiving assurance in the form of a note “I’m giving you these comments because I have very high expectations and I know you can reach them” (Yeager et al., 2014, p.809) were dramatically more likely to redraft their work and gained far better marks. Fletcher-Wood advocates discussing emotional responses to feedback with students and creating a “culture in which students are accustomed to receiving feedback and recognise its value” (p.106).

Foster, 2015 begins from a starting point of developing fluency within learners and makes the point that... Procedural fluency involves knowing when and how to apply a procedure and being able to perform it “accurately, efficiently, and flexibly” (NCTM, 2014, p.1).

The study set about attempting to develop an awareness and honesty of the students’ own confidence when completing maths work. Pupils whose confidence and competence are strongly correlated are said to be “well-calibrated”.

They developed a confidence assessment (CA), where students state alongside each of their answers a confidence level expressing how certain they are. The research looked at 345 school mathematics pupils in five different secondary schools in England, and how they responded to the use of a CA instrument designed to incentivise the eliciting of truthful confidence ratings.

The research saw that pupils readily understood the negative marking aspect of the CA process and their facility correlated with their mean confidence with $r=.546$, $N=336$, $p<.001$, indicating that pupils were generally well calibrated. Their comments indicated that the vast majority were positive about the CA approach, even though it was very different to how assessment is carried out usually in school. Some pupils felt that CA promoted deeper thinking, increased their confidence and had a potential role to play in classroom formative assessment.

An aim of our action research is to improve mathematical fluency in our students. The research seen here by Colin Foster acknowledges a correlation/relationship between fluency & confidence in one's own maths. The aim of this study was to move students towards being better "calibrated" to aid teachers in having a valid reflection of the competence of their students, which feeds into the larger picture of fluency. One of the ways we could measure the effectiveness of our responsive teaching is to measure if we are progressing our students to move towards being well-calibrated. Gauging an understanding of their confidence against their results will give us a picture of their calibration and validity of fluency level.

Conclusion

In conclusion there is evidence that the following responsive strategies which are the focus of our action research will further students learning, building fluency in performing mathematical skills and confidence in applying them to solving problems

- Eliciting evidence of misconceptions from students (based on the work of Barton)
- Addressing those misconceptions explicitly (Bell) through whole class discussion (Smith 5Rs)
- Activating students as resources for each other through discussion in small groups (Wiliam)
- Over-teaching and peer teaching (Fletcher-Wood)
- Being aware of students likely emotional responses to feedback and managing them for better learning outcomes (Yeager et al)

Methods

The information that we were responding to came from multiple-choice end of unit tests, where the choices were designed to reveal misconceptions. (For example, an option for the area of a rectangle of dimensions 2 by 3 could be 10, because that would reveal that the student has used the misconceived idea of adding all the lengths.) The extra dimension to the test, beyond simply picking an answer, was an explanation box, where the student was encouraged to give a reason for their choice. The quality of output there was highly variable, including whether the student would even write anything at all.

In the subsequent lesson, after teachers had marked the tests and identified the questions where their own class had shown they still had persistent misconceptions, we conducted a peer discussion activity aimed at activating the students as a resource for each other to secure increased mastery from the unit, ready to move on. The design of this discussion was refined over the course of the project based on teacher reflections and observation.

We chose the topic, 'Area & Volume', for Cycle 1. We gave the students their own marked work immediately, sat the students in mixed groups of 3 or 4 and asked the successful students to explain their answer to each of the questions. Where most people had the question right, we had planned to make them explain the misconception, but they were extremely reluctant to do so. Teachers observed their own students and later reflected on the conversations they had heard.

We chose 'Indices including roots & standard form' for Cycle 2. To get students to engage in actively discussing misconceptions, we moved to mixed pairs and asked students to discuss anonymous answers, one right and one wrong, then we shared examples of good feedback and discussed why it was good. We did this for two focus questions and then students were given their marked work and asked to give themselves feedback. We had a system of peer observation in cycle 2 to try and gather more qualitative data about students' conversations when their own teacher was not at the table.

We chose 'Sequences & graphs' for Cycle 3. We stuck with mixed pairs and gave students wrong answers to just one question to discuss and write feedback, then gave them a related but harder task (problem solving exam question OR going on to teach a related topic) In cycle 3 one of our teachers did all the peer observations to improve the comparability of the data gathered across different classes.

For Cycle 4, rather than doing any intervention, we gathered quantitative and qualitative data from mocks and practice papers and a shadow test to see if the students had retained what they had learned in the past cycles of teaching and learning.

Results and Discussion

At the start of the project, we surveyed the teachers in the department about whether they had used the End of Unit tests and how they had responded to what the results told them about their students understanding.

9 teachers responded to the survey on use of EoU checks.

All of them had used the checks at least some of the time and most of them used the check every time. Only one teacher said they did not find them useful: he considered the mark as a measure of ability rather than looking at the misconceptions revealed. Most teachers were more selective about following up on the results of the tests, with only one teacher always following up and two teachers never following up (beyond marking & returning the tests)

This shows that supplying teachers with the tests – even tests that are designed to reveal misconceptions – is not enough to ensure they will respond to that information. Without the response, these are just more tests.

We went on to ask more open questions to find out more about how people responded when they did and why they might not always do so.

Making time for individual follow up work – most teachers are solving this by working as a class to review either all the questions or the ones most students got wrong. One teacher does it by providing written feedback on the test to replace a verbal 1-1. Others are taking 20-30 min at the end of a lesson to do 1-1s while other students work on their homework assignment.

In a time pressured course like GCSE resit these solutions are not sustainable in terms of classroom time, classroom management or long-term teacher workload and so it is not surprising that teachers were not doing these consistently enough to form part of a regular working practice.

We did however identify some good practice that we wanted to build on and adopt more widely, and these were built into our activity design for each of the cycles.

Tracking: some teachers were getting students to update their mark in their personal progression charts which allowed students to identify their strengths in maths. This was leading to student engagement and ownership of learning and a desire to engage further with the process of correcting their mistakes. We did go on to do this as a department but it's not a focus of this project.

Addressing misconceptions and giving follow up questions. This furthers the students learning and ensures all students are secure on the key concepts. Students are engaged by the opportunity to make corrections and improve their score on the personal progress chart.

Asking students to share their answers and explain their methods as a small group before going through the questions with the class. Activating the students as resources for each other in this way is good for the learning of both those who got full marks and those who did not, and it relieves the pressure on teachers to do 1-1 explanations.

Encouraging students to **consider one of the answers they know to be wrong** and think about the misconception it represents. Why is it wrong? How would someone get that? How should they fix it? This kind of reasoning is sometimes examined in GCSE papers, so it is both valuable as a learning exercise and for assessment.

Impact of the interventions, during and after.

During the interventions, we collected qualitative data from teacher reflections and peer observations which informed our activity design in the next cycle.

Following our interventions, we wanted to investigate the impact in the longer term.

Did students retain what they learned in the intervention, or have they gone back to prior misconceptions?

Did students in the intervention groups do better than those who weren't on the intervention topics in the March mock?

We examined these questions using 3 tools:

Students work on relevant questions in the March mock. The whole cohort did this mock, so we were able to compare the results from students in the intervention groups with those who were not. This took place before Cycle 3, so we only have this data for Cycle 1 & 2

Students work on practice papers in class as part of exam preparation in May. This was collected by teachers of the intervention groups only. As it was not in test conditions, this allowed for some conversation.

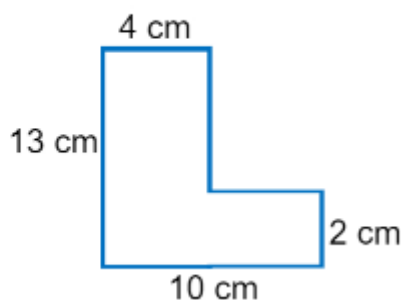
Shadow tests. We observed that exam questions require a level of problem-solving skill and often combine more than one mathematical demand, so it can be hard to tell why a student has chosen not to answer. A shadow test was created with similar questions to the original questions from the End of Unit checks where there had been significant levels of misconceptions which we tried to address in the intervention. 38 students across the intervention classes did the shadow test.

Cycle 1 Area & Volume

Extract from teacher reflection on the discussion of this question

1. What is the perimeter of this hexagon?

- A) 64cm
- B) 46cm
- C) 72cm
- D) 29cm



'Explanations' were mechanical, simply reciting the steps in their calculation. For example, for the hexagon perimeter question, the student 'explained' the numbers that he added: "I added eight and then three and then ..."

This list of numbers in his 'explanation' included those that one had to deduce; so the student didn't even explain something that was ripe for explanation - how did you know the numbers that are not shown?

Even when I prompted with 'Why did you do that?', the response was a blank look.

This strong dependence on specific numbers rather than their roles was a concern from most teachers' observations of their students' peer to small group explanations. We chose to address this in Cycle 2 by teaching students about the qualities of good feedback based on one of the examples given in Jones (2021) and then praising their first attempts at written feedback on these criteria

Kind: *the feedback recognises what went well. This helps people to engage with what you're saying*

Specific: *the feedback connects to a specific problem so the person can see what needs to change*

Helpful: *the feedback tells them how to deal with not only this scenario, but could be useful when they are tackling a similar problem in future*

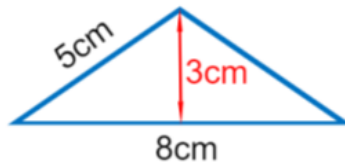
We also observed that students were very reluctant to discuss their own misconceptions and this is why in Cycle 2 and 3 we prepared anonymous work for them to give feedback on.

We observed 2 prevalent misconceptions on this End of Unit check: using the slope height instead of the perpendicular height when calculating the area of a triangle, and failing to find a missing height when calculating areas & perimeters of compound shapes. We examined how effective the group discussion was by giving a shadow test question on these and also by looking at student work on practice papers.

Original End of Unit Questions

2. What is the area of this triangle?

- A. 12 cm^2
- B. 40 cm^2
- C. 20 cm^2
- D. 24 cm^2

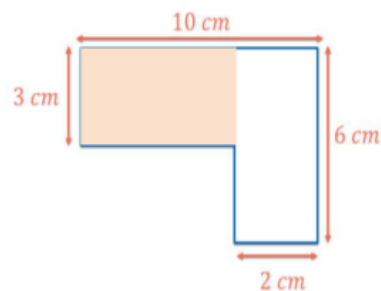


How confident are you about your answer? 😊 😐 😞

Explanation / working:

3. What is the area of the shaded section of this shape?

- A: 30 cm^2
- B: 18 cm^2
- C: 24 cm^2
- D: 15 cm^2



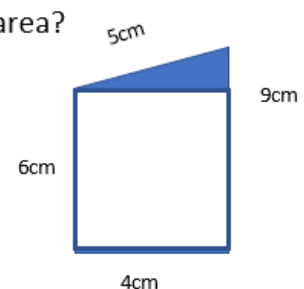
How confident are you about your answer? 😊 😐 😞

Explanation / working:

Shadow Question

5) What is the shaded area?

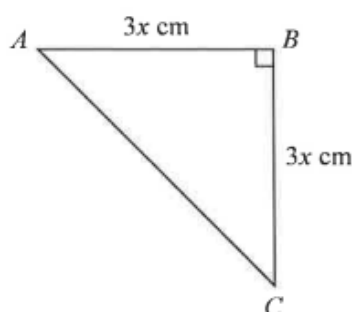
- A: 24 cm^2
- B: 12 cm^2
- C: 6 cm^2
- D: 7.5 cm^2
- E: 18 cm^2



Disappointingly, only around 1 in 3 students got this shadow question right. The most popular misconception remaining was B, which suggests there are still a good number of students who forgot to halve after multiplying height $\times$ base, but that they did recognise the need to find the perpendicular height to get the area of the triangle. Only a few students used the full height (E), suggesting that they do know they need to find the missing sides, but some did use the slant height instead of the base (D)

Q17 May Practice papers – we collected students work on this from 33 students in the intervention classes.

17 ABC is an isosceles right-angled triangle.



The area of the triangle is 162 cm^2

Work out the value of x .

Knowing how to calculate the area of the triangle is critical to being able to solve this problem, but there are added complications:

- Working backwards from the area
- Neither side is known
- Once found, the side length needs further processing to become x

strategy	Correct area of triangle formula	Incorrect area of triangle formula	Total
Set up equation & solve	13 students Mean 2.69 marks	3 students Mean 0.67 marks	16 students
Trial some values of x	3 students Mean 2 marks		3 students
Other	1 student 1 mark	1 student 0 marks	2 students
No attempt	12 students Mean 0 marks		12 students
Total	17 students Mean 2.47 marks	16 students Mean 0.13 marks	33 students total

In summary: 17 students demonstrated that they knew and could use the formula for the area of a triangle correctly, and 4 students demonstrated that they could not. It's not clear why the students who did not attempt the question did not do so. It could be because they don't know how to get the area of a triangle, or it could be the algebra content.

The majority of students who did attempt this question were confident using an algebraic approach and were able to recall and use the area of triangle formula correctly, which demonstrates significant progress from the original End of Unit test on Area & Volume where this was an issue after teaching but before the intervention.

Cycle 2 Indices & Standard Form

Original EoU question

2)

Simplify
 $(2p^3)^4$

A

$8p^7$

B

$8p^{12}$

C

$16p^{12}$

D

$16p^7$

About 1 in 5 students got the right answer (C). The most popular answer was B, showing that students had multiplied the powers as they should but had multiplied the coefficient of 2 by the power on the bracket rather than raise it to the power of 4. In future we might look at rewriting this question so that the power on the bracket is not a power of the coefficient.

Shadow Question

1) Simplify $(5d^4)^3$

A: $15d^7$

B: $5d^7$

C: $5d^{12}$

D: $125d^{12}$

Around 1 in 3 students got the right answer (D), which is progress, although not as much as we might like. Very few answered A so we have addressed that misconception effectively, however around 1 in 3 students answered C which shows that there are still persistent misconceptions around the effect of the power on the coefficient inside the bracket.

Original EoU question

$$\frac{3^{-7}}{3^{-3}} \equiv$$

Most students seemed to know that they should subtract the powers. We thought that they were unsuccessful because of the negative numbers.

A

3^4

B

3^{-10}

C

3^{-4}

D

$\frac{7}{3^3}$

In the cycle 2 discussion we observed a pair of students having a conversation about this question. They had been given two pieces of anonymous work, shown here:

Which student has got Q3 right?
 For the other student, give some feedback. What went well? What should they change?
 Test your explanation with a question of your own.

STUDENT A	STUDENT B
$\frac{3^{-7}}{3^{-3}}$ Two negatives make a positive so $7 + 3 = 10$ which gives you the answer 3^{-10}	$\frac{3^{-7}}{3^{-3}}$ When you minus the powers you get $-7 - -3 = -4$ so 3^{-4}

Extract from peer observation:

There was an argument among a pair of students as one person felt that both were correct methods.

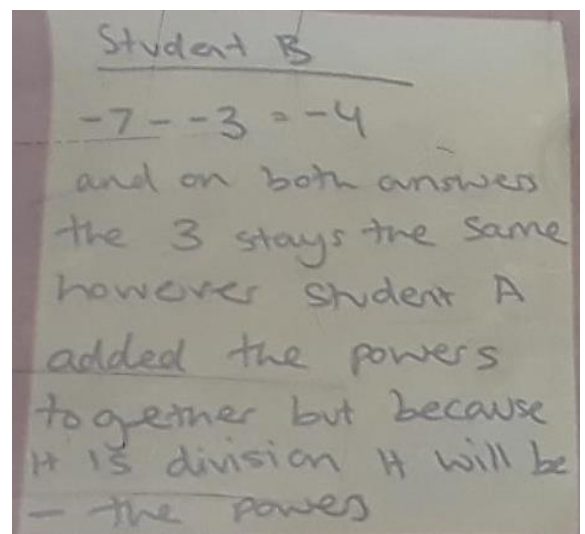
One student then used a calculator to verify his position to check $-7 - -3 = -4$.

"indices you take away, but 2 negatives make a positive"

"the student thinks -7 and -3 is $7 + 3$ which is wrong"

"when you are dividing you have to subtract the powers" – reference to the index law.

They then went on to write some feedback on the anonymous work, shown in this picture:



We also saw negative numbers as an issue on the May practice papers. The table shows the responses of 39 students from the intervention groups on a 1 – mark indices question

(b) Simplify $(m^{-2})^{-3}$

Marks	+ or - powers	Multiplied powers	Other	No attempt	Total
0	11	10	3	4	28
1		11			11

It's pleasing to see that the majority of students had retained the index law and multiplied the powers to get 6 (positive or negative) - more than tried any other method. However, a similar number of students got the correct sign as the incorrect sign at the end (+6). The problem here is with manipulating negative numbers and not the rule of indices being used.

Just over a quarter of the students have used addition of powers to some extent and misunderstood the bracket effect (-1, 1, 5, -5). This shows a twofold misconception, inability to manipulate negative numbers correctly as well as inability to remember the rule for indices with brackets.

This one-mark question relied on the student both knowing the correct rule and being able to manipulate negative numbers.

Shadow Question

2) Calculate $-9 - -7$

We therefore chose to follow up with a shadow question about subtracting negative numbers.

A: 2

B: -2

C: -16

D: 16

The great majority of students got this right, so they have made some progress with negative numbers even if they are still challenging in the context of indices.

Student voice from Cycle 2

Following on from the discussion in cycle 2 students were given a booklet of practice exam questions on the topic. Afterwards, 30 students from the intervention groups were surveyed about whether they thought the discussion had helped them to do the exam questions and what they thought they had learned. The overwhelming majority (27/30) said they thought it

had been helpful to them and many said they'd like to do more of it. Here some of the things they said about the process:

It helped because it allowed me to see different students' perspectives on tackling the same set of questions

I was able to see specifically where I needed to improve my errors for next time

... once I am able to explain my answer, I can understand my working out

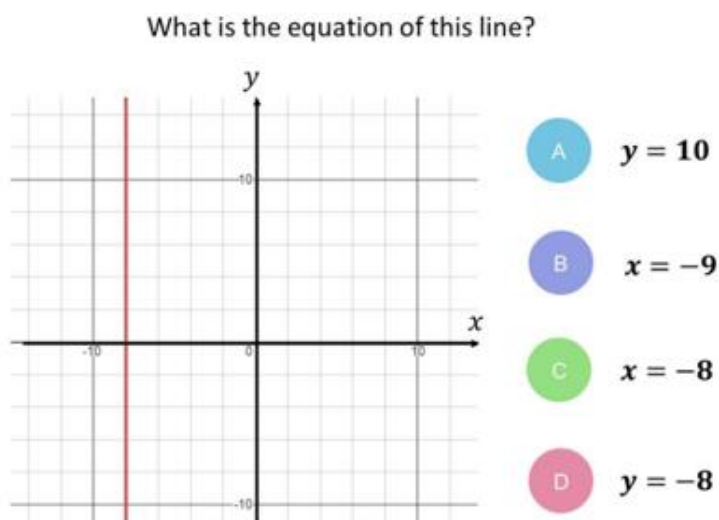
I got feedback on my answers I looked at the feedback and understood what I had got wrong and where I had made my mistakes

Cycle 3 Sequences & Graphs

In this cycle one teacher did all the peer observations so that she could compare common themes of student and teacher behaviour across all the intervention classes. She observed these features:

- Students are much more willing to discuss anonymous work
- There were lots more procedural comments and abstraction than in cycle 1, but still plenty of explanations that were mostly just numbers
- Misconceptions are still remarkably persistent even if students are told the work they are looking at is wrong or shown the correct answer
- Very few students did attempt to create their own follow up questions, even if they were explicitly directed to do so
- Most students wrote some notes or explanations from the feedback given in the discussion activity, but not all, and the quality of what they wrote was variable
- There was an obvious immediate improvement in skills within the lesson, allowing students to be successful on the follow up activity
- The follow up activities the teachers did were varied despite the agreed plan. Some did more challenging exam questions on the reviewed topic and some progressed to a related topic
- One teacher reviewed equations of straight lines and then progressed into teaching reflections. The observer was really impressed with how smoothly the students transferred the reviewed skill into the new context.

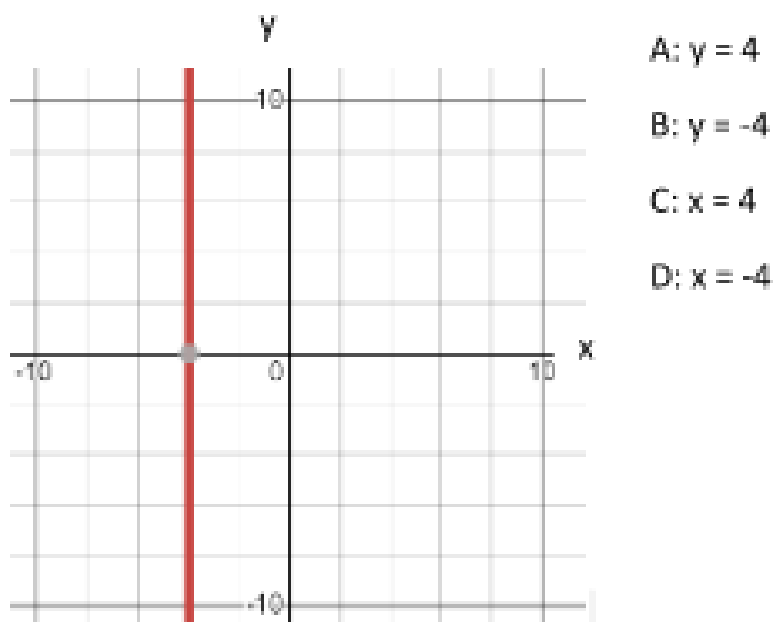
Original EoU Question on Graphs



Nobody got this right. We noticed that most students chose A or D. This may be because they are used to equations of lines starting with $y =$ or because the line is parallel to the y axis.

Shadow Question

What is the equation of this line?



About half the students got this right on the shadow test, which is really good progress, although around a quarter of the students answered B which suggests they have retained their misconception. Very few people answered A so they have at least recognised where the value on the axis is negative.

Sequences (March Q20 - before intervention, May Q4 after intervention)

We had 2 pieces of diagnostic information for this because we had done the March mock before teaching the topic.

This was the question on the March mock

20 The first five terms of an arithmetic sequence are

1 4 7 10 13

Write down an expression, in terms of n , for the n th term of this sequence.

Around 1 in 3 students got this right. The single most popular wrong answer was $n+3$. We then went ahead and taught sequences and did the EoU check afterwards.

Original EoU question

What is the n th term of the sequence 5, 7, 9, 11, ...?

A $2n - 3$

C $3n - 2$

B $2n + 3$

D $3n + 2$

Around half the students got the right answer, B. The most common misconception was getting the common difference and the intercept the wrong way round (D) In future we should change this question to include the response $n+2$ as this was a common misconception shown in the mock before this unit was taught.

Shadow question

3) What is the n th term of this sequence 11, 16, 21, 26

A: $5n - 6$

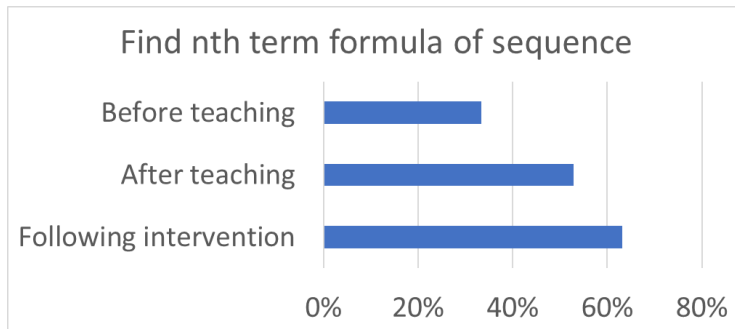
B: $5n + 6$

C: $n + 5$

D: $6n + 5$

Just over 60% of the students got this right on the shadow test. The most popular wrong answers were C (retaining the common misconception from before teaching) and D (the misconception we saw on the EoU check.) This does suggest some progress.

This incremental progress from prior learning through teaching and response is shown in this graph.



Percentages have been used to make it easier to compare results from differently sized datasets, even though they are all relatively small. It's notable that the improvement caused by experienced teachers spending around 2 hours teaching sequences in the classroom is only a bit bigger than the improvement caused by taking 20-30 minutes to respond to the misconceptions shown on a 10-minute test.

Conclusions and Recommendations

Conclusions

Students engaged well with this process of reviewing tests and saw the value in addressing persistent misconceptions. Building metacognitive skills like learning from your mistakes is important for long term success but might not show over the timescale of this project.

Preparing anonymous work is essential if students are going to directly discuss misconceptions. They do then recognise that misconception in their own work when it is returned to them.

The workload for teachers in marking these assessments and preparing anonymous work for students was manageable - in total around 30 minutes per unit per class - and would be sustainable over a longer period, unlike other methods of response which teachers had told us about in the initial survey. The resulting classroom activity was effective and worked as a review for students who had missed part of the unit as well.

Recommendations

Students do not automatically know how to give feedback so in the early stages of implementation it is important to share examples of good feedback for everyone to learn from. This should be anonymous to reduce embarrassment. Once students are giving better explanations this stage can be omitted to streamline the use of class time. Key features of good feedback are that it is **kind** (recognises what was good about the work) **specific** (connects with the misconception in the work done) and **helpful** (could be used when tackling a question with different numbers.) In doing this it helps if the teacher clarifies that learning to give good feedback is one of the lesson objectives and a transferable skill.

Doing these review discussions lead to improvements in fluency, but this is not automatically translated into confidence or skill when solving more complex problems. The teacher needs to build on the repaired knowledge and then introduce more challenging materials in class.

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