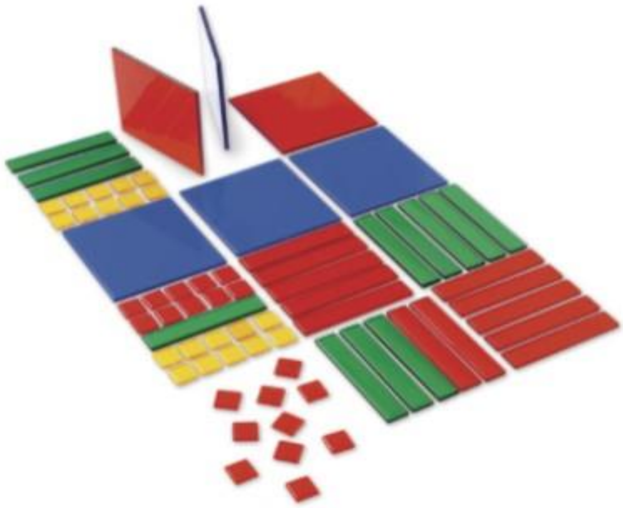

Can Algebra Tiles deepen students' understanding of solving linear equations

Debbie Trueman, Warwickshire College Group, with South & City College Birmingham and Hereford, Ludlow & North Shropshire College

OUR PARTNERS

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About CfEM

Centres for Excellence in Maths (CfEM) is a five-year national improvement programme aimed at delivering sustained improvements in maths outcomes for 16–19-year-olds, up to Level 2, in post-16 settings.

Funded by the Department for Education and delivered by the Education and Training Foundation, the programme is exploring what works for teachers and students, embedding related CPD and good practice, and building networks of maths professionals in colleges.

Summary

The aim of this action research project is to find out whether and how physical Algebra Tiles can deepen students' understanding of solving linear equations. The action research followed two cycles in three colleges across a four month period, exploring how the Algebra Tiles helped learners develop their problem solving. After an exploratory cycle and pilot, data was collected through a baseline test (see appendix one) which was given to the learners at the start of each session in which the tiles were used and then repeated at the start of a subsequent lesson. This cycle of the research ran over a 6 week period. The students overall improved on their baseline tests by 83.9% with more striking progress made by those students who had achieved a grade one or two in the GCSE maths and by those who were taught using a 'shared activity' pedagogy where teachers and students sat at a table and problem solved together.

We critically evaluated our class observations and quantitative data and drew a number of conclusions.

Our Key Findings are:

- Algebra Tiles (AT) are an excellent way of solidifying the abstract concept of zero pairs – and provide Students with a deeper understanding of directed numbers.
- AT are an excellent scaffolding resource, which especially impacted the performance outcomes of lower ability students.
- A 'shared delivery' pedagogy where teachers and learners sit in small groups and work together achieved the best results for all students.
- Observations showed that most students transitioned efficiently from the tiles through pictorial to abstract working.
- Some students gained confidence to explore complex algebraic concepts, e.g. quadratics.
- Spending time to secure understanding of zero pairs, (directed numbers) and collecting like terms will actually save time further along the line.

Our results were good, and our journey to be brave and try something new was impactful for us all! *We hope we will inspire our readers to do the same*.

Contents

Background

In the wake of the Wolf Report (Wolf, 2011), in September 2013 the UK government legislated all 16–18 full time learners who have not achieved grade 4 or higher in GCSE Mathematics to continue studying maths. Whilst the policy continues to be a matter for debate with criticisms from Ofsed and reports of demoralised learners there are others who feel that the policy is achieving its objectives (Fino, 2019).

Indeed gains are being made, 2014 students achieving a grade C or above when resitting GCSE mathematics stood at 35.8% (JQA, 2015), in 2015 that figure stands at 39% TES (2021). However, with research suggesting that students who resat GCSE maths in FE colleges were underperforming in comparison to sixth forms and other providers (Roderio, 2018) there is a clear mandate for development of practice in this context.

About the colleges

Warwickshire College Group (WCG)

WCG has seven sites across Warwickshire and Worcestershire, with over 1000 students resitting GCSE Maths having not achieved the magic grade 4 result. The GCSE Maths course is delivered in 1.5-hour lessons once a week. Students are then directed to spend an hour a week of independent study using a virtual platform.

The students participating in the WCG element of this research are from just one site, Moreton Morrell. The courses the students are studying are; agriculture, animal care, construction, equine and sport. Most students are aged between 16 and 18 years old.

Hereford Ludlow & North Shropshire College (HLNSC)

Established in 1949 HLNSC has six distinctive campuses situated across Herefordshire and Shropshire. Campuses are: Hereford; Holme Lacy; Ludlow; Oswestry; Walford; The Gateway. The college serves ca. 2,500 full-time and 6000 part-time learners (including those on university level courses) and ca. 750 apprentices per annum. The college Mission is : To realise potential and support success.

Karen is based at Hereford Campus. Bonnie at Hereford and Holme Lacy. We both teach GCSE to 16-19 year olds and to adults, although these cohorts sit different papers. Our students range from ungraded to grade 3 when they come to us. We both teach the students in two types of groups: emerging (U to G1) and GCSE mainstream (G2-G3), these courses are delivered over 33 weeks in two hour sessions. There are some groups that are a combination of both groups.

South & City College Birmingham (SCCB)

South and City College has five campuses located in Birmingham and offers FE, HE, and apprenticeship courses. Bournville Campus is based in Longbridge and includes a main campus and a construction centre.

Our maths provision ranges from Entry level functional skills to GCSE resit courses, with Nikita and Mandy teaching predominantly 16-19 year olds. The GCSE resit courses are one year study programmes, with three hours of teacher-led delivery a week. Learners attend two 1.5 hour lessons a week and are expected to complete independent study outside of their allotted timetable.

This academic year, our GCSE entry requirement has been a grade 3, and exemptions were made for grade 2 learners who showed a higher ability during their initial and diagnostic assessments. The students who participated in this research project from Bournville College were aged between 16 and 19 and retaking their GCSE maths qualifications alongside their vocational courses. The cohort from Bournville College participating in this study includes learners enrolled on childcare, health and social care, motor vehicle, and construction courses.

Literature Review

How does the use of physical Algebra Tiles have an impact on, and deepen students' understanding of solving linear equations?

Introduction

The aim of this action research project is to find out whether and how physical Algebra Tiles can deepen students' understanding of solving linear equations. The manipulation of algebraic terms in general is a massive part of the GCSE Maths syllabus. More specifically, students are required to apply their knowledge of algebraic terms to solving linear equations. However, many students struggle to understand and apply the basic principles of this area of maths.

Algebra Tiles were designed to be a concrete representation of the principles and processes associated with algebraic terms and manipulation. Following the concrete-pictorial-abstract (CPA) pedagogic process, the Tiles are a resource that give students experience of the 'concrete' and 'pictorial' and hence the potential to move on to abstract understanding.

Overall, the literature indicates how using algebra tiles can have a great impact on the student's knowledge and understanding of abstract maths concepts. Of particular interest are the views and research already undertaken to find ways to incorporate the use of such tiles effectively.

Effective use of Algebra Tiles

Eaves (2019) makes a very clear point that before using algebra tiles as a manipulative it is important to establish a clear rationale for their use to ensure they are not used just to stir engagement in the lesson,or used as a gimmick. Algebra tiles should be seen as a learning tool. He recommends that students keep records of the process they use so that they can eventually work independently from the tiles. Finally, question the students to ensure they know the structure behind the question rather than learning new mechanisms mindlessly.

The use of algebra tiles was trialled with students with mild learning disabilities in a secondary school in California by Castro (2017). Castro found that the tiles themselves were not directly impacting the learners' understanding but they achieved higher results in the post-intervention assessment. She concluded that algebra tiles have potential and so it is important to continue research in this area.

Their study examined the impact of algebra tiles on students' understanding of the distributive property and evaluating expressions whereas our research focuses on the benefit of algebra tiles to solve equations.

Emeny (2015) believes that the use of algebra tile models can be used to develop the conceptual understanding of topics where bar models are not suitable. However, the use of visual models should be used selectively and used as a bridge between conceptual understanding and abstract reasoning as visual models do not work for all variations of questions.

The use of algebra tiles can be used to demonstrate the concept of directed number: using the four operations; to organise factors and primes, multiples and squares; collecting like terms; substitution; solving equations; expanding brackets; factorising to harder concepts such as completing the square. As with Castro (2017), Emeny has yet to prove that the use of algebra tiles enhances students' conceptual understanding. However, this piece of research does illustrate the importance of zero pairs as a visual to cancel out representations clearly.

Using Algebra Tiles for Linear Equations

Saraswati, Putri & Somakim (2016) set out "to describe how algebra tiles can support students' understanding of linear equations with one variable". The study involved thirty-two 12 and 13 year olds at school in Indonesia, using algebra tiles in their lessons. The data was collected using videos, written work from the students, a pre and post-test, and teacher notes from observations. Hypothetical learning trajectory was then compared with the actual learning process to analyse how effective the algebra tiles were in supporting the understanding of solving linear equations with one variable. The hypothetical learning trajectory included tracking the starting points for the learners, their learning goals, learning activities and also the student's working method whilst completing the task. All this data collection activity was designed to investigate what strategies learners use to solve linear equations with one variable, and whether introducing algebra tiles supported their methods.

In one learning activity, students were given a maths problem which was contextualised to a tiling scenario. After being told how to use algebra tiles and to count how many tiles were used, they were able to:

- demonstrate their thinking with their algebra tiles accurately
- make a shape and effectively apply reducing and balancing concepts to the algebra tiles, and find the unknown variable using this method
- use the removed / cancelling step with subtraction in solving a linear equation when using algebra tiles, which they hadn't been able to do before using the tiles.

The study concluded that the learners were indeed able to use algebra tiles to solve linear equations with one variable. The teachers found that the algebra tiles helped minimise common mistakes and misconceptions associated with linear equations. They also felt that the use of contextualised maths problems had a positive impact on learning.

The main limitation of this project is that the tiles were used for one variable linear equations, it would be useful to apply a similar strategy but with more complex maths problems such as the unknown variable on both sides.

Curtis (2018) edited a collection of research reports into maths teaching and learning. One of these pieces of research was an ongoing study aiming to develop mathematics teachers' knowledge and collaborative practices. The experiences of a Year 10 class in a higher tier GCSE class and a Year 11 foundation tier class at a Welsh secondary school were considered with a view to further developing their understanding of linear and quadratic algebraic expressions.

Most of the teachers in Curtis' (2018) study were in agreement that change needed to take place to promote a deeper level of understanding of the topic. They also felt that there was potential to increase a learner's interest in the topic by introducing additional tools to disturb habitual misconceptions of the area in the subject. They found that some learners, as expected, were reluctant to adapt the new model. However, with the support of the collaborative Teacher Research Group, the teachers persevered.

Manipulative tools were used to represent the given equations and this, together with the learners' sketched workings, enabled the learners to formulate solutions to the problems posed. Curtis' (2018) work identifies the potential to improve and/or change the delivery solutions to historical and stoic algebraic concepts. However, the sample size was small and the study didn't include Further Education learners.

Andrea Wingett (2019) conducted a small study with 10 students who had previously failed the Alegbra 1 class at least once. Her aim was to determine whether or not algebra tiles affect student performance. The teaching approach used was CRA model (Concrete, Representational and Abstract) to teach students through the use of hands on manipulatives.

Five students were shown and guided in their use of algebra tiles over a series of 6 sessions. By lesson 5 they were no longer using tiles to solve problems that they couldn't previously do. Another five students were the control group – they were not given algebra tiles but were given the same activities and problems as the experimental group. Statistically, there was no significant difference between the groups when comparing their pre- and post-intervention assessments. The author concluded that, based on the evidence collected with her students, algebra tiles did not have a significant impact on achievement within algebra.

However, results of the intervention group were higher (though not significantly). Also, this teacher learned that students are initially resistant to the manipulatives, but their attitude changed once they were able to make the connection between how the tiles were a concrete representation of an abstract skill. The researcher also observed that the students who were often unengaged in instruction were better engaged. Students who had a previously negative attitude towards maths, appeared to have a more positive attitude towards maths once the manipulatives were introduced and used.

Wingett's (2019) study is interesting and is worth critically reviewing. It was targeting adding and subtracting polynomials and multiplying binomials rather than linear equations and relied on one source of data from a very small sample. Evidence on confidence and perceived understanding could have been useful. Further, the students could have been given time learning and 'playing' with the tiles first before trying to use them as a tool, though this would have to be outside of class due to the limited time available to deliver a full curriculum. Our students, who are doing courses such as agriculture, forestry, construction and animal care are likely to be a more hands-on cohort than Wingett's sample and therefore potentially more visual learners who would respond well to algebra tiles.

Larbi and Mavis (2016) investigated the efficacy of using algebra tile manipulatives with 56 junior high school students from two towns in the US. Again split into two groups, both were taught the algebra units over a period of four weeks. The experimental group was taught using algebra tile manipulatives whilst the control group was taught using the conventional 'talk and chalk' methods.

Those students who were taught through extensive use of algebra tiles performed significantly better than those who did not use Algebra Tiles. The use of the algebra tiles enabled students in the experimental group to demonstrate more proficient skills in factoring algebraic expression than their counterparts in the control group.

Lessons associated with the use of manipulative materials also enabled students with different learning styles to benefit equally. Drawing on 'AVK learning styles' theory, the author says that this is because concepts are explained (auditory learners), demonstrated for learners to see (visual learners) and allowing learners to manipulative or model concepts themselves (kinesthetic learners). This study prompts the idea for our own research that algebra tiles should be used in teaching the distributive property to all students to enable them discover the discernible pattern in removing brackets from an expression. In doing, this emphasis must be laid on "process" and not "product".

Conclusion

The literature that has been read for this action research project strongly suggests that Algebra Tiles can deepen students' understanding of solving linear equations. The research reviewed above is with small samples of students and not in the FE context. Nevertheless, it indicates that there is a belief amongst at least some mathematics practitioners from around the world that the use of algebra tiles can successfully impact the student's learning and understanding.

The use of new materials to teach and promote learning will always be a debate for educators. The learning journey for many FE maths students has been a negative one to-date, suggesting that continuing the same teaching and learning practices are unlikely to work in FE. The use of new tools, for all or for a few learners, is a positive step if it helps in any way at all to deepen the understanding of the topic given.

The evidence suggests that the use of algebra manipulatives should be carefully considered before being put into practise in the classroom. Rather than being a gimmick, evidence should be produced by students to cement their understanding of algebraic concepts with a view to becoming independent from the need to use the tiles as a means of finding algebraic solutions. Evidence has been collated showing that the use of algebra tiles reduces common mistakes, makes the learning more meaningful as they are used as a bridge from the conceptual to the abstract. It is suggested that once the tiles are used effectively they could be used to help solve linear equations and, potentially, harder algebraic concepts such as expanding, factorising and completing the square.

Methods

Focus and approach

The overall focus of the project is to investigate whether the use of physical Algebra Tiles will have a positive impact on, and deepen students' understanding of solving linear equations. In order to address this question the project needed to understand the impact of the Algebra Tiles on the learners' problem solving abilities. Secondary to this, the teachers' reflections on learning to use this tool allowed us to understand what experiences the teachers had, and what aspects of their practice were developed.

This required a mixed methods approach, requiring quantitative data sets focused on the learners' developing abilities, and qualitative data sets to observe how learners used the tiles to problem solve, and gather teachers' reflections on the resources and the learning in the session in which they were used.

Cycle one

The action research followed two cycles in three colleges across a five month period. The first cycle of research was collaborative and exploratory in nature. Data were not collected during this cycle which spanned late December 2021 to early February 2022, instead it was used to work across the team (staff n.7) to conduct a literature review and develop and pilot both physical and virtual tiles. Teachers shared their experiences and decided upon the best resource to use - physical Algebra Tiles were purchased for everyone. This cycle also proved to be of value as it allowed the group to understand the importance of 'zero pairs' and also the limitations of the tiles, for example, when dealing with division.The importance of understanding 'zero pairs' was also raised in some Literature reviews and in Cycle two there was explicit explanation of how tiles 'cancelled out'. This also helped deal with directed numbers.

Cycle two

The second cycle, the sole data collection cycle commenced by focusing down on just one area of algebra that cycle one had indicated the tiles were most useful with and the team collaboratively developed resources to ensure a uniformity of approach. This was piloted before the fieldwork began to allow the team to develop confidence and competence in the approach before taking the practice into the classroom, and allowed each researcher to scope a workload for data collection of not more than two classes so that the project would not overwhelm them during a difficult period.

The 6 week cycle two began with data being collected from learners through a baseline test (see appendix one) which was given to the students at the start of each session, before the tiles were introduced. The questions were of increasing difficulty.

The same test was then repeated after the intervention and progress measured on a student by student basis.

The intervention involved teachers introducing the tiles and how they can support solving equations either as a front of class exposition, or, where numbers permitted, as a shared (group) activity. E.g. learners sitting around a table together and working in groups.

The resources used during the intervention were prepared questions which were RAG rated (All, Most, Some) in order to allow for scaffolded and differentiated learning. Some were printed on to individual cards whilst others used worksheets but the questions remained uniform. All students worked with their own set of tiles on a whiteboard.

During the intervention, the teacher would observe the learners' use of the tiles for approximately 30 minutes and were asked to record evidence of how the learners were using the tiles, any ‘wow’ or ‘penny drop’ moments (see appendix two) and photos of learners' work were taken.

In order to gain consent the teacher gave verbal notice of the nature of experiment, learners were given assurances that work would be anonymised and they would not be identifiable in any photos. All learners were given the option not to take part and instead complete different learning activities.

The third data point were teachers’ personal reflections (see appendix two) to track the impact on teacher learning. These reflections focused on the process of learning and the use of Algebra Tiles.

Ethics

All assessment data produced through the project was kept on a spreadsheet on password protected computers and all observations, reflections and photos were anonymised and stored on college clouds. All data has a destruction date of September 2022 after publication of the project.

Results

The overall progress made by students

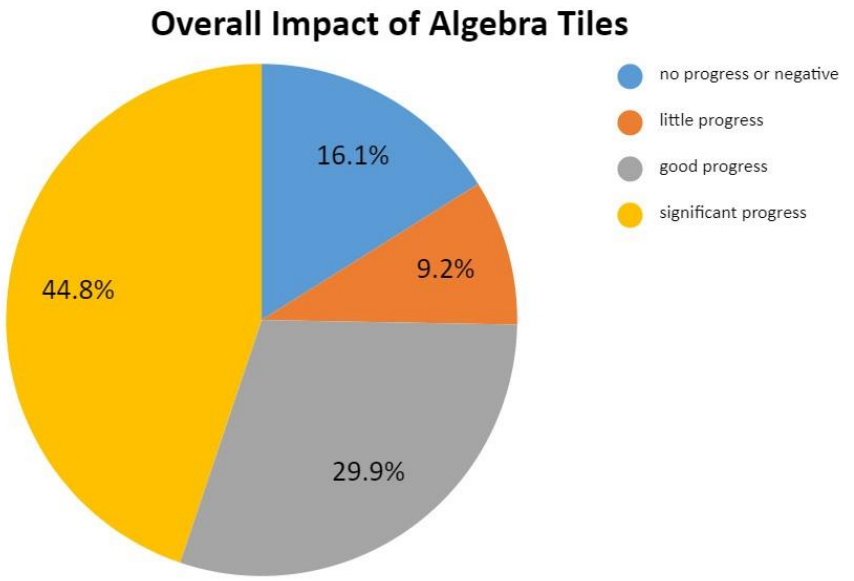


Figure One: Impact of algebra tiles for all groups.

<0 No or negative progress
1-5 Little progress
6-10 Good progress
>10 Significant progress

The overall impact of the intervention was that 83.9% of students improved their score when solving linear equations after the intervention, across all grades. The teachers often has examples of students who had made significant progress:

Teacher N :Case study student
"His score went from 44% to 52% a week after the intervention, he said "he thought the tiles were really useful for him and his understanding improved just when using the RAG rated questions on his own"

The progress made by students seemed to rest upon the physical and visual nature of laying out the algebra, as well as their impermanence boosting students confidence to engage with the problems as the data below suggest [bold is our emphasis]

"AT are really helpful they made me work out algebra ***visually instead of wording*** *which I struggle with but using the tiles today i really enjoyed it made maths fun"* [Student in Teacher D lesson observation]

*"Having the ability to **reset** when they felt they had gone wrong was huge for my class, it gave them the **confidence** to give it a go."* [Teacher B lesson observation]. The importance of the reset was also seen in a reflection by Teacher N: *"The resource was particularly helpful for a learner who requires a scribe during his maths lessons. He appreciated the **low stakes option of the tiles, and said he enjoyed having the freedom to make his own mistakes by just moving the tiles around**. **He was able to rectify any errors much quicker than if he was working on paper and relying on a scribe.**"*

"*The **physical element** of clearing sides and balancing out on the other side for some students was almost transformative in their understanding, with comments like, "Oh my god, I get it!" "I couldn't do this before"*. [Teacher K, Lesson Observation]

One student of Teacher N's expressed a preference for the **"f*eel of the plastic tiles*** *over the handmade laminated ones*" and said they were a good tool.

One student recorded the physical manipulation of the tiles aided memory *"I usually keep **forgetting steps** but t**he tiles are useful in this**"* [Teacher M: Lesson observation]

Teachers' reflected that the physical element of the algebra tiles particularly assisted with understanding the concept of 'zero pairs'.

"All the students particularly enjoyed the concept of zero pairs and always looked out for them." [Teacher M: lesson observation]

"Zero pairs essential to grasp and needed further explanation for some" [Teacher D: lesson observation]

Of the 15 students who did not improve, 7 remained the same and 8 scored less than in the pre test. In the teachers' discussions about these students there was some suggestion that their drop in score may be due to unknown anticedances. For some of these students teachers' annecodes suggested personal/emotional distractions meant their results were not representative of their observed ability:

"*A learner confided in me that they had been removed from the family home and were now residing with grandparents. A further learner described their morning routine as a nightmare due to the length of time spent travelling and the amount of public transport they had to use made them extremely stressed and tired*." [Teacher D: Reflections]

There is evidence that some students did not find the algebra tiles useful, however, we are unable to offer a clear correlation between these opinions and students who did not improve their score:

"One student said, "I am not very fond of the Algebra tiles. Personally for me I found it harder to understand then the normal method how school teach as young children."" [Teacher M2: Lesson observation 2]

Greater progress for lower level students

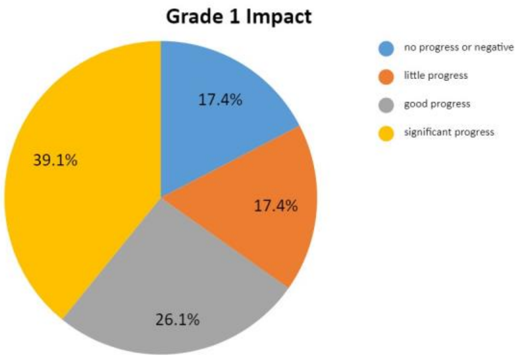


Figure Two: Progress made by students who have achieved a Grade 1 in GCSE maths

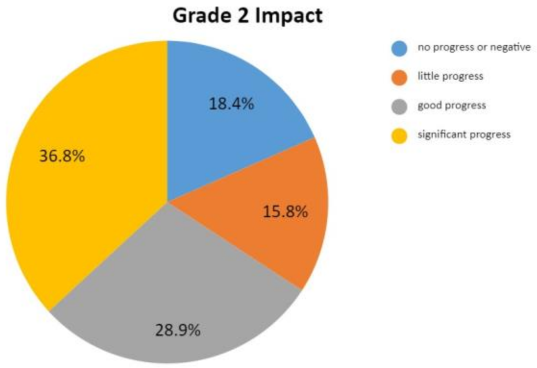


Figure Three: Progress made by students who have achieved a Grade 2 in GCSE maths

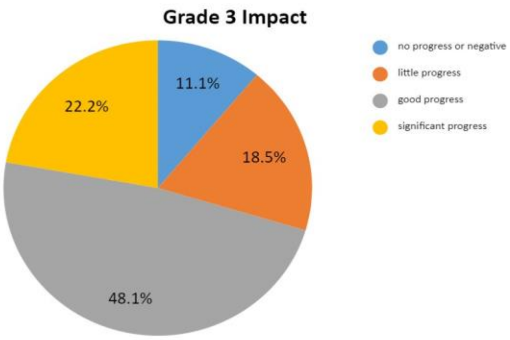


Figure Four: Progress made by students who have achieved a Grade 3 in GCSE maths

The data in figures two, three and four suggests a correlation between the level of students' GCSE grade and the level of progress made with solving linear equations when taught using algebra tiles. Whilst all groups made between 62 and 66% progress at the level of good or better, those learners who initially achieved a grade one or two improved their score by greater than 10 marks were noticeable:

Grade	Percentage of students making significant progress
1	39.1%
2	36.8%
3	22.2%

Teacher reflection on this phenomenon in the observation reports and in the post-intervention reflections were high, suggesting that these differences were noticeable in classroom as well as test performance.

The teachers reflections on why there may be a correlation between improvement in test scores, previous GCSE achievement and the algebra tiles focus on two key variables; the level of support required for the learner and the level of perceived difficulty of the question [emphasis added in bold]:

"*When demonstrating how the algebra tiles worked some students felt they didn't need them as the* ***types of questions I*** *was demonstrating were very straight forward. As I developed this into harder examples more came on board and so when they set off to have a go themselves they were more accepting of their* ***potential support****."* [Teacher B: Lesson observation]

"Ss [students] who were confident with ***easier solving*** *[sic] were reluctant to use tiles (overcomplicate) but saw their value with variable on both sides e.g. 2x+2=3x-1"* [Teacher D: Lesson observation]

"Those with secure methods were reluctant but stuck with them and ***saw benefit on the tougher ones*** *with negative answers"* [Teacher D: Lesson observation]
e.g. $x - 2 = 2x + 1$, when $x = -3$.

"AG didn't think that the tiles were beneficial to him, but he did think they were a good tool – we said that for his level, ***it may have been more useful to look at some more difficult examples using the tiles*** *rather than the linear equations we used."* [Teacher N: Lesson observation 2]

"One of the students had special needs and she ***usually gets distracted quickly if she found anything hard****. However, I saw a smile on her face throughout the session and she worked hard. She seemed completely engaged and learning throughout the session.*" [Teacher M2: Lesson observation 3]

The other area of focus was how secure the problem solving methods that students has previously learned were, although this was not consistent across all observations [emphasis added]:

"The tiles may be beneficial to lower level learners **who have no preconceived ideas** surrounding the topic of Algebra." [Teacher M3: lesson observation]

"Student's who were of higher ability and had achieved strong grade 2 or 3 previously and ***were familiar with the algebraic concepts and had used the notation of x, y etc engaged with the tiles representation much more easily and effectively****."* [Teacher M: Lesson observation]

"The tiles really helped ***better performing students to replace misconceptions****."* [Teacher K: Lesson observation]

The final observations hinged around students' reluctance to use resources they may associate with lower levels of education:

"*There is some resistance to the use of physical manipulatives from some grade 3 students because they may feel judged using them or believe that the use of them will not help them to learn."* [Teacher k : Lesson Observation]

Delivery method

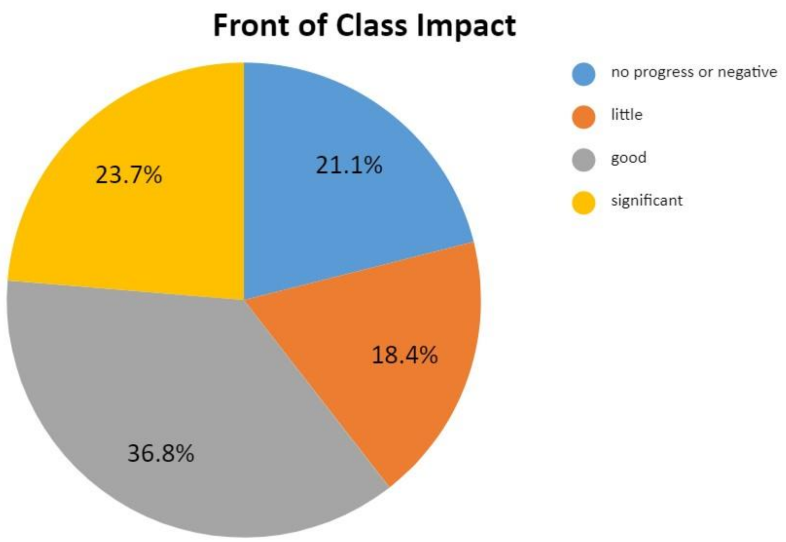


Figure Five: Improvement on baseline tests based on 'front of class' delivery

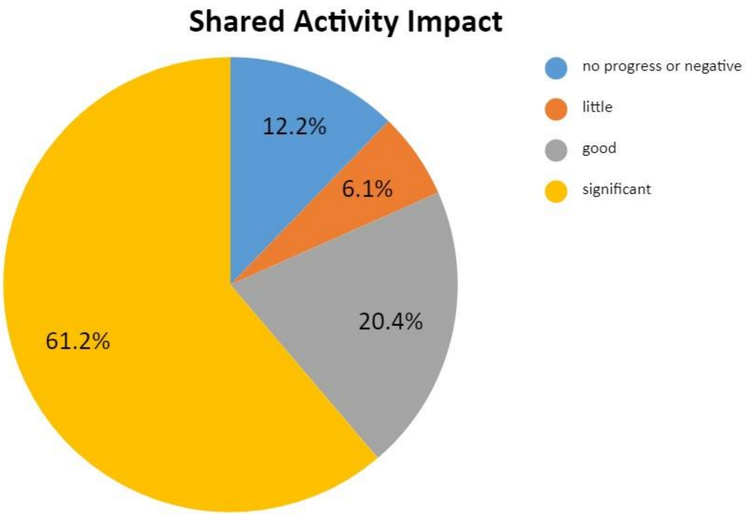


Figure Six: Improvement on baseline tests based on 'shared activity' delivery

The data for delivery strongly indicates that 'shared delivery' facilitates an improvement from the baseline test scores of 10 marks or greater with 61.2% of students who were taught using this method achieving 'significant improvement' in comparison to 23.7% in more teacher-led classes. The 'shared delivery' pedagogy involved the teachers and students sitting together as a group working on linear equations.

Teacher reflections suggested that developing reasoning and facilitating personal sensemaking may have been important actions from 'shared delivery':

*"Showing **why we did certain actions** was so beneficial and having the tiles meant that this could be done in a way that **made sense to them** rather than continuing to be just a process they could see what was actually happening."* [Teacher B observation reflections]

However, other reflections point to the success of this pedagogy being dependent on other considerations such as time, size and grouping:

"They need more time and practice to see the actual learning from the tiles" [Teacher M2: Lesson observation]

"It was a big group found hard to run a lesson and support every student" [Teacher M2: Lesson observation]

"I think this method can work better in small groups of similar ability" [Teacher M2: lesson observation 3]

Deepening and extending understanding

As discussed earlier, all teachers indicated that the physicality of the resource appeared to facilitate the deepening of students' understanding. As mentioned in the discussion of delivery methods teachers were clear this needed to be accompanied with practice time for all the students, but notably those who achieved a lower grade in their GCSE maths.

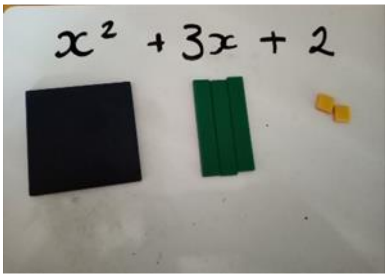
"The concept clicked with TB when working through the virtual IXL tiles, but his understanding improved further when he ***was able to actually set up the equations himself, move the tiles around, and work with zero pairs****."* [Teacher N lesson observation]

"They give you a ***visual representation*** *to help" [Teacher D: Lesson Observation]*

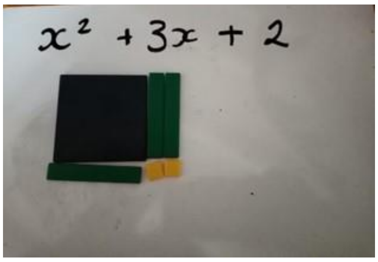
It was clear, if students ***practised t****his enough the tiles improved their understanding.* [Teacher K: Lesson observation]

*"The concept learning and assimilation using concrete manipulatives and progressing onto pictorial and abstract understanding took more than one lesson in majority of the students and the process needed to b****e repeated a few times*** *especially with the low ability learners."* [Teacher M: Lesson observation]

For some students, the opportunity the tiles presented to 'play' and practice maths concepts without committing an answer to paper and being 'wrong' gave them the confidence to to explore more complex algebraic concepts such as quadratics as is explained here by Teacher B:

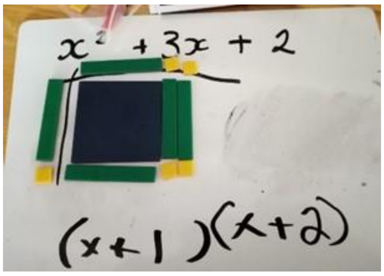


"Quadratics (grade 1 students) this was an unplanned extension when we did our first round of 'playing' with the tiles.



We started with factorising and the idea that you need to always make a rectangle with the pieces we have.

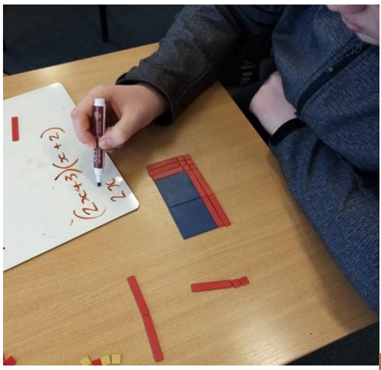
(We had already spent time on what the pieces represented)



From there we looked at the top and side were what would be in the brackets, we put the pieces in to help visualise that part too

Sometimes the students forgot that the red meant negative but I wasn't too bothered by that in the initial stages.

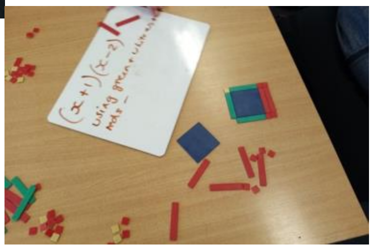
We then looked at expanding brackets.



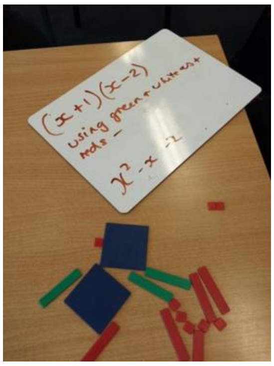
Starting with putting the top and side and then filling up the rectangle.

We took the top and side away at the end so that the pieces didn't get counted (in hindsight we could have drawn the grid again but the student was working on the table for this example)

Then I gave a negative to colours. Still little mistakes in the bottom on the side) and will encourage more use of in



look at and put a reminder for set up (the 1 on the top should be at the grid would have helped which I future as it would help students to progress to diagrams and eventually just the multiplication grid.



The student used zero pairs to help finish this off (unfortunately he had moved his tiles before I got the photo!"

Exams

3 students raised concerns that whilst the manipulatives were useful in class they could not be used in exams [In the observations of Teachers N, M3 and B). However, other students commented that they could try drawing the tiles in their exams and one of Teacher M's students used this method of drawing the tiles on her baseline test.

Teachers' practice development

Teachers' reflection suggests that much like the student using a tool such as algebra tiles requires a development in both the understanding of the pedagogical approaches required to use the tile successfully and in what circumstances it may be appropriate to use the tiles, and this can only be developed through practice. The five key pedagogical approaches that were identified by the teaching team included:

- Introduction of the algebra tile early in the academic year
- Greater time for practice and completion of the tasks for lower level groups
- Splitting the classes into ability levels ● Contextualising questions to aid understanding
- Teachers need time to practice the use of the tiles.

"Having taken part in the action research has given me a fabulous insight into how learning can be made so effective using the concrete-pictorial-abstract approach. I had come across tiles in the past and knew they had the potential but did not know how much. ***The research helped me to identify that and developed my confidence in using them****."* [Teacher M1: Personal reflection]

"I think this method can work better in ***small groups of similar ability***" [Teacher M2: lesson observation 3]

"Given that post 16 is limited in time allocated to maths compared to primary and secondary education – the ***early introduction of the algebra tiles*** *across a variety of topics could help all students to feel more comfortable using them.* ***The use of the tiles when teaching negative numbers would be useful".*** [Teacher K: Lesson observation]

"**I will need to practice further so that I can become "fluent" in their use**, furthermore I will need to demonstrate confidence along with ability when cascading the ideas to other groups along with my peers." [Teacher M2: Lesson observation]

*"I observed that f****amiliarity with the tiles was of significant importance****, hence i****ntroducing tiles at a very early stage of the year t****o learn various other mathematical concepts like directed numbers, simplifying expressions, factorising could make a huge difference in terms of engagement and performance of students with solving equations.*" [Teacher M: Lesson observation]

*"****They need more time and practice*** *to see the actual learning from the tiles"* [Teacher M2: Lesson observation]

"Contextualising the questions for better understanding." [Teacher M: Reflection]

Discussion

The overall positive impact of the use of algebra tiles (83.9%) appears to link to the physical and visual nature of the tiles. This was not an issue that was addressed by the literature review, but does tentatively suggest it is an issue that may require further exploration.

The strong progress made by the students who achieved a grade one or two at GCSE maths begins to address the gap identified by Castro (2017) and Emeny (2015) that there is no 'proof' that the tiles develop students' understanding. Whilst we would not offer our results as such, they are an indication of developing understanding for this set of learners. However, our evidence does suggest that the tiles need careful consideration by the teachers to encourage their use at the right point in learning for each student as the data suggests that the level of scaffolding or support required and the perceived level of difficulty of the question for the student are factors in the tiles usefulness to learners. This is reflected in one of the five key pedagogical approaches being time for teachers to practise as this kind of expertise only develops through this mechanism (Winch, 2010).

All students appeared to benefit from the tactile nature of the tiles, there is tentative evidence of the development and extension of learning attached to the opportunities the tiles gave students to 'play' with the maths concepts without having to commit themselves to a definitive answer on paper. This suggests the opportunity for thinking and reflection is useful to the development of complex maths concepts.

For the lower level learners the tiles provide a different method to engage with the maths problems. For the learners with higher initial GCSE grades the tiles helped them work through misconceptions which supported the findings of Saraswati, Putri & Somakim (2016). Consideration of the type of question framed to the students was further developed by the team with the suggestion made that contextualisation of the question posed was a further development that could develop students' understanding.

Of particular interest to all teachers was the success experienced by the groups learning through a 'shared activity' pedagogy, sitting together and working as a group on the linear equations. This delivery method appeared to develop the reasoning and personal sensemaking of the learning, again suggesting that there is indeed some initial deepening of understanding experienced by students. However, it is clear from the data that not all learners enjoyed learning through the algebra tiles, this reluctance to adapt to the new tool was noted in the literature by Curtis (2018) and Wingett (2019). This recommendation by the team of early adoption of the tiles in class may be a method of gradually overcoming this issue, as well as giving time to develop reasoning and sensemaking.

In summary, whilst an excellent resource, these tools require a thoughtful and structured pedagogical approach from the teacher in order to elicit the most benefit to the learner.

Conclusions and Recommendations

Conclusions

With 83.9% of our learners showing improvement in their baseline test scores, it is clear that the inclusion of algebra tiles was a successful scaffolding tool. The data shows that the manipulatives were particularly valuable for our lower level learners. Learners currently working at a grade 1 and grade 2 GCSE maths level showed an average 37.95% significant progress on their baseline tests, compared to just 22.2% of our grade 3 learners.

The delivery method also affected the outcome of the baseline tests. When the algebra tiles activity was shared, what we have termed the 'shared delivery pedagogy', the peer working aspect strengthened the success of the learning tool, with 61.2% of students who worked with the tiles in a group making significant improvement in their test. When compared to the 23.7% of students who participated in teacher-led delivery showing significant improvement, this evidence demonstrates the positive impact peer work has upon the use of manipulatives.

Further, the data suggests the results discussed above were linked to the ways in which the algebra tiles helped the learners to solidify their understanding of the abstract concept of zero pairs and provide students with a deeper understanding of directed numbers. In reflection, the participating teachers all agreed that spending time to secure understanding of zero pairs, (directed numbers) and collecting like terms were beneficial actions for the learners, allowing them to transition efficiently from the tiles through pictorial to abstract working. Moreover there is data which tentatively indicates development of much higher level mathematical thinking with some students gaining confidence to explore complex algebraic concepts, e.g. quadratics which appears to be related to the students ability to 'play' with the tiles and practice responses without needing to commit themselves to paper and be 'wrong'.

Recommendations

Recommendations for Teacher Action

The data suggests that teachers are likely to get the best results by using algebra tiles with a 'shared delivery' pedagogy. However, as a teaching team the reflections strongly suggested pointed to a further five key pedagogical approaches which are recommended for teacher action:

- Introduction of the algebra tiles early in the academic year
- Greater time for practice and completion of the tasks for lower level groups
- Splitting the classes into ability levels ● Contextualising questions to aid understanding

- Teachers need time to practice the use of the tiles.

Recommendations for Leader Action

Teachers need time to practise how and when to use the algebra tiles, it is clear this pedagogy works most effectively as teachers feel confident with the tiles. We recommend that CPD and coaching is made available to all maths teachers in FE colleges to support the development of these tools.

Ensure timetabling, class size and curriculum organisation allows for an early introduction of the tile into the maths classrooms, into classes of a size that allows teachers to bring togethers learners of a similar ability in order to work in the recommended ‘shared delivery’ pedagogy.

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Appendices

Appendix One - Baseline Test

Baseline Test: Solving Equations

Name:___________________ **Date:**___________________

Q1. Solve $4x = 8$

$x =$ ________________________ **(1)**

Q2. Solve $3x = 12$

$x =$ ________________________ **(1)**

Q3. Solve $x + 5 = 9$

$x =$ ________________________ **(1)**

Q4. Solve $x - 3 = 14$

$x =$ ________________________ **(1)**

Q5. Solve $x - 8 = 5$

$x =$ ________________________ (1)

Q6. Solve $3x + 5 = 20$

$x =$ ________________________ **(2)**

Q7. Solve $5x + 4 = 19$

$x =$ ______________________ **(2)**

Q8. Solve $3x - 8 = 19$

$x =$ ______________________ **(2)**

Q9. Solve $2x + 5 = 1$

$x =$ ______________________ **(2)**

Q10. Solve $4x + 2 = 2x + 12$

$x =$ ______________________ **(3)**

Q11. Solve $3x - 6 = x + 4$

$x =$ ____________________

(3)

Q12. Solve $9x + 1 = 6x + 13$

$x =$ ____________________ **(3)**

Q13. Solve $10x + 3 = 7x + 12$

$x =$ ____________________

(3)

TOTAL MARKS 25

Appendix Two - Observation Sheet

OBSERVATION SHEET

Teacher:	Date:
ARG:	No. of Students:
Current GCSE Level of Group:	
Brief description of the intervention, including resources used, starting points , prior learning & time taken.	
Reflect (using bullet points) what you noticed with regard to the students' engagement and learning during the intervention. Include what you observed about students' knowledge and understanding? (include student quotes & attach any pictures or student work etc.) • •	
What went well with regards to the resources? What worked particularly well with students? What were the 'penny drop' moments? What could be improved? (EBI)	

Appendix Three Intervention Question Bank

ALL	a) $x + 4 = 9$ b) $x - 4 = 3$ c) $4 + x = 7\ x$ d) $+ 5 = 12$ e) $6 + x = 12$	f) $2x = 6$ g) $3x = 12$ h) $5x = 10$ i) $4x = 4$ j) $½\ x = 3$
		Mark /10
MOST	a) $3x + 3 = 12$ b) $3x + 3 = 9$ c) $2x + 3 = 9$ d) $2x + 3 = 7$ e) $2x - 3 = 7$	f) $2x - 3 = 5$ g) $3x - 7 = 8$ h) $2x - 7 = 5$ i) $3x - 2 = 7$ j) $3x - 2 = -8$
		Mark /10
SOME	a) $2x + 1 = x + 4\ 2x$ b) $+ 1 = x + 5$ c) $3x - 1 = x + 3$ d) $3x - 1 = 2x + 3$ e) $2x + 3 = 3x$	f) $2x + 1 = 3x - 4$ g) $2x - 1 = 3x - 5$ h) $2x + 1 = x + 5$ i) $4x + 1 = 3x - 1$ j) $4x - 1 = 3x - 3$
		Mark /10