To increase and deepen learners' conceptual mathematical understanding by using sequences of concrete, pictorial and abstract representations delivered online through virtual manipulatives

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Working in partnership with the Education and Training Foundation to deliver this programme.

About CfEM

Centres for Excellence in Maths (CfEM) is a five-year national improvement programme aimed at delivering sustained improvements in maths outcomes for 16–19-year-olds, up to Level 2, in post-16 settings.

Funded by the Department for Education and delivered by the Education and Training Foundation, the programme is exploring what works for teachers and students, embedding related CPD and good practice, and building networks of maths professionals in colleges.

Summary

This action research project was designed to investigate how a CPA (Concrete, Pictorial and Abstract) approach to teaching and learning can be used with virtual manipulatives, using learnings and experiences from their physical counterparts. The ambition was to find an alternative approach in this way to support teachers during the covid-19 pandemic when physical manipulatives were restricted within the classroom setting and remote teaching commenced. It also meant teachers and students were able to use these tools in the classroom but also outside of the classroom whenever they wanted (removing the requirement of the presence of the physical manipulative in the process). Another issue was overcoming the barriers many colleges across our network faced with lack or limited technological resources for the resit GCSE Math programme delivery. One thing that was for sure is that most students (if not all students) had mobile phone devices, connected to Wi-Fi or the college internet, which often distracted them during the lesson. It was apparent very early on that the use of an app on a mobile phone device as a teaching and learning tool was rare, and this was reflected in the minimal number of apps that we found available, to develop students conceptual understanding in Maths in this way.

Five trial teachers were involved with the process of refining and developing a mobile phone app that would deliver on our combined goal of deepening conceptual understanding in a particular branch of the resit GCSE Math curriculum (in this case FDPR). Approximately 60 students received the intervention over a period of 2/3 weeks. Data was collected through a series of pre and post surveys of both trial teachers and participants receiving the intervention. Review meetings by the action research group drove decisions and developments of the app throughout the year. A product of these discussions and actions can be found in the final trial material found in Appendix B.

Learners responded reasonably well to the intervention by demonstrating improvements in their self-declared confidence and academic self-concept in the areas taught with the virtual manipulative. Time was found to be a significant factor for how CPA methods and manipulatives should be introduced and used in the classroom. This will be an important feature moving forwards with further investigations of other virtual manipulatives available to support teaching and learning of different topics of the GCSE Maths resit curriculum.

Contents

Background

Student attainment outcomes in mathematics are of increasing importance to individuals, colleges and society as successive governments seek to ensure that the UK workforce has enough quantitative skills for an increasingly data-driven and technology-rich future. There is a growing expectation that young people continue their mathematics education beyond school into colleges. In 2015, the UK government applied the Condition of Funding for Further Education (FE) colleges, so all students who fell short of a grade 4 GCSE at the age of 16 are now required to retake their GCSE or work in improving their mathematics alongside their vocational courses and A Levels.

Nationally, less than 1 in 5 students achieve grade 4, post 16. Furthermore, the more times students attempt the GCSE exam (they may retake the qualification twice a year), the less likely they are to pass. To break this cycle of 'failure' for approximately 80% of students each year, the Department for Education have funded a multi-million pound 'Centres for Excellence in Mathematics' programme until March 2023 which is being managed by the Education and Training Foundation, a not-for-profit organisation who supports teachers and leaders across the Further Education sector. Christ the King Sixth Form College is one of the 21 Centres for Excellence in Maths and they have been innovating, developing new and exciting ways to teach the fundamental mathematical concepts in the classroom. Christ the King Sixth Form College and their network partners have been investigating the use of virtual manipulatives as an alternative to physical manipulatives due to the restrictions put in place in using resources in the classroom during the global Covid-19 pandemic.

This project was designed, implemented, and delivered through teachers and students across Southeast London and Greater London (Lewisham, Sidcup, Bexley and Greenwich).

Literature Review

The aim of this literature review is to first explore the history, definitions, and differences between virtual and physical manipulatives. It will investigate the growing need and development of technology within the classroom as a pedagogical tool and explore the physical hardware needed for such developments. It is important to outline what is currently used in the classroom (with both physical and virtual manipulatives) and to investigate some of the impacts and findings from these. The review will also explore strategies that help embed the use of manipulatives in the classroom, particularly for maths, and the ever-popular Singaporean approach to manipulatives using Concrete, Pictorial and Abstract representations.

History and definition of Virtual Manipulative

The history of the term 'virtual manipulatives' arise in the late 1990's. Resnick et al. (1998) had a goal to use virtual manipulatives to "*embed computational and communications capabilities in traditional children's toys. By using traditional toys as a starting point, we hope to take advantage of children's deep familiarity with (and deep passion for) these objects*" (p. 282). Dorward and Heal (1999) were funded by the National Science Foundation and created the National Library of Virtual Manipulatives (NLVM) which is a collection of Java-based applets for K-12 mathematics teaching and learning. In the UK, K-12 is equivalent to the education of 4 –16-year-old students.

Moyer, Bolyard and Spikell (2002) define a virtual manipulative as "*an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge*" (p. 373). They have also been defined as "*computer-based renditions of common mathematics manipulatives and tools*" (Dorward, 2002, p. 329). Virtual manipulatives are often dynamic (ability to be manipulated on a screen of a device) visual/pictorial representations of physical manipulatives such as multi-link cubes, fraction towers, Cuisenaire rods, geometric solids, algebra tiles or geoboards. Examples of computer-based work which do not fit the definitions proposed by the two authors above include: filling in a form or worksheet on a screen; or simply answering questions on an app in the presence of a virtual object. Throughout history mathematicians, and people involved in mathematical activities, have used several tools, such as sliding rules, compass, calculators and recently computers. These "tools" help with understanding and problem solving but also facilitate deepening and extending the mathematics and its relation to our world. Computers made life easier for mathematics educators and people doing mathematics with the help of several software packages capable of word-processing and making difficult mathematical calculations and drawings. After computers became ubiquitous and affordable, attention soon shifted from "learning to use computers to do math" to "using computers as an aid in a math lesson". Earlier applications considered the computer as another medium to display and test the content material in the form of programmed instruction (Skinner, 1954).

Changing world of technology, the need to develop Virtual Manipulatives

At the time in 2002 when Moyer et al. defined this "*new class of manipulatives*", it was described as a manipulation using a computer mouse. Today, virtual manipulatives are used through a multitude of devices (e.g., tablets, laptops, phones, whiteboards) and are more commonly manipulated using a stylus, finger, laser, touch pad or a traditional mouse.

Desktop or laptops computers are no longer the primary way students access the internet and 95% of 16-24-year-olds in the UK own a smartphone (Statista, 2019). Young people are using smartphones with integrated Wi-Fi and internet access to communicate more, play games, and consume more content. However, many of the virtual manipulatives already developed are ‘web apps’ and cannot be used on a smartphone (or if they can be displayed, users experience a loss of function, and the app cannot be used). Another issue is the ‘offline’ availability of apps; enabling students and learners to access content outside of the classroom without the need for Wi-Fi/internet connectivity.

Moyer-Packenham, Bolyard, & Spikell (2002) predicted the evolutionary change towards technology in the classroom and that it was “*just around the corner*”. Moyer went onto to state that “*Virtual manipulatives may very well be the most appropriate mathematics tool for the next generation*”. Dorward & Heal (1999) go as far as to say that they also foster as much engagement as the identical physical manipulative (PM) they were constructed and designed from.

Moyer-Packenham & Westenskow (2013) identified that a feature of virtual manipulatives (VMs) was motivation:

“This feature of the virtual manipulatives impacts student learning through students’ affective responses (i.e., VMs were enjoyable), student interest (i.e., VMs maintain students’ attention), and student engagement (i.e., students persist longer at mathematical tasks).” (p. 44)

Moyer concluded that “*virtual manipulatives, as well as VM/PM combined, have unique embodiments that have positive impacts on student achievement in mathematics”.*

Clements & McMillen (1996) compared the benefits of virtual over physical manipulatives:

“*Paradoxically, research indicates that computer representations may even be more manageable, ‘clean,’ flexible, and extensible than their physical counterparts… computer manipulatives were just as meaningful and easier to use for learning*”. (p. 49)

Clements & McMillen (1996) also suggested that virtual manipulatives may reinforce the sensory-concrete links*, “so, computer manipulatives can help students build on their physical experiences, tying them tightly to symbolic representations. In this way, computers help students link Sensory-Concrete and abstract knowledge so they can build Integrated-Concrete knowledge*” (p. 55).

Physical Manipulatives: what is currently used in the classroom?

Manipulatives are physical objects that can be used as representations or models of mathematical concepts to develop understanding in the user, allowing them to solve problems and gain access to abstract ways of thinking previously unavailable. Modern examples include Dienes (base-ten) blocks, algebra tiles, Unifix Cubes, Cuisenaire rods, number lines, fraction pieces, pattern blocks, Numicon, and geometric solids, however manipulatives have been around for a while: Plato refers to Egyptians using manipulatives with their student would-be scribes.

The usage of manipulatives in classrooms, especially in the younger years, have long been recommended by educators (The National Council of Teachers of Mathematics, 1989). Manipulatives have often been more praised than used (Johnston-Wilder & Mason, 2004) although their popularity seems to be increasing, exemplified by the prevalence of Singapore mathematics and the current CPD opportunities for teachers. Hart (1993) found that teachers often use manipulatives for “fun lessons” and subvert the value of them as aids to mathematical thinking. Moyer-Packenham (2001) showed teacher “*beliefs about how students learn mathematics may influence how and why they use manipulatives as they do*”.

They found that some teachers had decided whether to use physical manipulatives in the classroom based on the behaviour of the group, with some teachers indicating they were concerned about maintaining 'control' of their groups.

To be of use to a student, manipulatives must allow the user to extract the mathematical structure (Johnston-Wilder & Mason, 2004). Durmus & Karakirik (2006) recommend that a physical manipulative needs to be "*simplistic [in] design"* and which enables easy manipulation. Laski, Jordan, Daoust, & Murray (2015) add to this that manipulatives should not have distracting or irrelevant features. Counter to this however, Mason & Watson (2019) note that *"materials that create some confusions to be resolved seemed to be more effective for learning than materials that present no problems"* (p. 23). Mathematical structure can be extracted and developed by an iterative process of using the manipulative, and working abstractly or symbolically (Laski et al, 2015) – cycling or switching between Bruner's (1966) stages of enactive-iconic-symbolic or CPA, rather than a one-way directive process towards abstraction (Mason & Watson, 2019).

Manipulatives on their own, however, do not automatically teach students mathematical concepts. In fact, it may be that students require a certain level of conceptual (all be it informal) understanding if they are to access what it is the manipulative is being used to teach – even though the manipulative is often considered a concrete representation. In 1964, John Holt (cited in Johnston-Wilder & Mason, 2004) showed that only students who already understood base and place value could effectively use blocks to solve problems.

The way in which manipulatives are used by teachers and students undoubtably influences their efficacy. Laski et al (2015) argue, from a Montessori perspective, that links between the manipulative and the mathematical concept should be clearly explained. Durmus & Karakirik argue that students "*should be given an opportunity to play with manipulatives*" and that just a "*demonstration by a teacher is not sufficient to realize their full potential."* Similarly, Wheatly, writing in 1992 (cited in Johnston-Wilder & Mason, 2004), notes that to "show" a student a mathematical concept using manipulatives is still based on the abstract first concept of learning. Ultimately, students must become fluent and comfortable in their use of using a manipulative so that they use it naturally and automatically as a problem-solving tool (Moyer-Packenham 2001).

In a meta-analysis of studies using manipulatives Suydam & Higgins (1976) conclude that if employed properly, lessons using manipulatives will produce greater mathematical achievement than lessons in which manipulative materials are not used, and gave the following suggestions on appropriate use of manipulatives:

1. Manipulative materials should be used frequently in a total mathematics program in a way consistent with the goals of the program.
2. Manipulative materials should be used in conjunction with other aids, including pictures, diagrams, textbooks, films, and similar materials.
3. Manipulative materials should be used in ways appropriate to mathematics content, and mathematics content should be adjusted to capitalize on manipulative approaches.
4. Manipulative materials should be used in conjunction with exploratory and inductive approaches.
5. The simplest possible materials should be employed.
6. Manipulative materials should be used with programs that encourage results to be recorded symbolically.

Heddens (2005) argue that using manipulative materials in teaching mathematics will help students learn:

- to relate real world situations to mathematics symbolism.
- to work together cooperatively in solving problems.
- to discuss mathematical ideas and concepts.
- to verbalise their mathematics thinking.
- to make presentations in front of a large group.
- that there are many ways to solve problems.
- that mathematics problems can be symbolised in many ways.
- that they can solve mathematics problems without just following teachers' directions.

Although a lot of research concludes that physical manipulatives are beneficial for teaching and learning mathematical concepts, Clements & McMillen (1996) proposed that using manipulatives does not always guarantee conceptual understanding. In one study, students not using manipulatives outperformed students using manipulatives on a test of transfer (Fennema, 1972). Arguably, this study was carried out on 7–8-year-olds where new ideas can be more easily ‘added’ to their cognitive structures. It is often the failure to link their action with manipulatives to describing the actions (Clements & McMillen, 1996). Hart (1993), in a study in with 8–13-year-olds, found that the process of formalisation through concrete experiences often failed and suggests that often the failure of manipulatives to improve students conceptual understanding may be due to lack or ineffectiveness of “bridging activities” linking concrete and formalisation stages.

CPA (Concrete Pictorial Abstract)

Multiple representations are regarded as particularly significant for students' conceptual understanding (Charalambour & Pitta-Pantazi, 2007). However, research shows that when students ask for help and show their teacher a calculation representation, some teachers struggle to link the calculation to the representations (e.g a bar model) and that struggle may be linked to subject knowledge (Dreher, Kuntze, & Lerman, 2016). Multiple representations, such as diagrams, graphical displays and symbolic expressions are important to convey the various aspects of the same mathematical concept.

However, representations, no matter how concrete they are, often do not serve the purpose of clarifying concepts if they are perceived as an end-product rather than as a tool to interpret the reality. Durmus & Karakirik (2006) comment that there are two different approaches in using models in a learning environment. Firstly, “*Learning to model*”, which they describe as teachers teaching learners how to model the reality. It is often found with this approach that learners have to have “*a significant understanding of the underlying objects of the model and could be regarded as the end product of an educational process rather than being used certain while concepts are trying to be conveyed”.* Contrary to this, *“Learning with Models”,* is described as to encourage learners to solve problems using and with the support of ready-made models:

“Learners are expected to see the relationships between objects in the model and expected to construct mathematical concepts through “mathematical abstraction”. This approach advocates creating specific models, activities and manipulatives, which is the main focus of this presentation, for every area of mathematics”

To date, there is a consensus amongst researchers that one of the predominant factors contributing to the complexities of teaching and learning fractions lies in the fact that fractions comprise a multifaceted construct.

The Concrete Pictorial Abstract (CPA) framework has stemmed from the enactive-iconic-symbolic modes of representations as conceived in ‘Theory of Instruction' (Bruner , 1966).

Acquisition of knowledge begins when the learner’s experience from the action undertaken (concrete representation) is translated into images (pictorial representation). With the extensive exposure to both the concrete and pictorial representations, the learner starts to make links to the different concepts with the use of symbols (abstract representation).

In line with this theory, Singapore Ministry of Education in early 1980s tasked The Primary Mathematics Project team led by Dr Kho et al to come up with instructional teaching materials. In their curriculum documents (Singapore Ministry of Education, 2007; Singapore Ministry of Education, 2012) ‘Concrete’ also refers to the concrete experiences, in addition to the concrete manipulatives as initially explained by Bruner of his enactive mode.

There is no definitive way in how these modes are carried out or how long a learner should remain in each mode of the teaching process. Although there is a tendency to stay longer in the enactive mode or the concrete representation, the teacher’s ultimate aim must be to get the low achieving students fluent in the symbolic stage. There is also the tendency to move quickly through the enactive and iconic stages. In doing this, the learners may not be equipped with the required understanding of the modes to fall back on when their procedural methods fail, especially when learners are met with challenging problems (Hoong, Kin, & Pien, 2015). Teachers must be the judge on how to adapt the CPA levels to suit their students’ needs (Gattegno, 1987). To ensure a smooth transition, the teacher must be equipped with the adequate understanding of the CPA approaches.

Teachers play a major role in deciding what concrete model to use from a selection available to them, facilitate in making connections between the external representations (concrete models or pictorial models) and thus guide the learner to create an internal representation in their memory (Goldin & Kaput, 1996). Goldin & Kaput (1996) throw light on the fact that the learner might fail to recognise and make connections between external representations (say for example, fraction tiles) they have experienced when this concept was first introduced to the external representation (say for example, fraction discs) the teacher uses during the recollection phase. This failure to make connections is widely seen among the post 16 GCSE learners; a lot of thought needs to be put in deciding what concrete models must be used with the low-attaining learner.

With the introduction of Singapore Math, Singapore Bar modelling has become a popular pictorial representation in analyzing and solving the arithmetic and algebraic word problems. In Singapore bar modelling, the learner is first taught to recognise the problem type (part-to-whole, comparison model or before-after model) draw the bars, label the known and unknown quantities and then find the solution (Ban Har, 2010). There is over reliance on formal conventions in terms of which bar model to use and how it should be labelled.

Another popular take on the pictorial representation is Realistic Mathematical Education (RME) originating from the Dutch Education System (Dickinson, Eade, Gough, Hough, & Solomon, 2020). Bar models of this framework include the fraction bar, percentage bar, ratio table and double number line (bars flattened to form number lines). In RME approaches, the learner is taught to use informal strategies in contextual situations. Progression occurs when they start to see similarities in the 'model of' the situation and generalize it to apply to more complex situations, thus making it a 'model for' problem solving (Hough, Gough, & Solomon, 2019). The study throws light on the challenges the Singapore Bar Modelling method pose in teaching problem solving strategies to low attaining learners. When the context of the problem does not suggest bar model, the low-attaining learner, who has a weak number sense and those who struggle with conventions, find it difficult to use Singapore bar model methods.

Virtual Manipulatives Studies: what is currently used through online based programs?

Technology is rapidly growing in all aspects of modern societies, and education is no exception. Mobile learning has also emerged as a new technological achievement and educational trend that provides both educators and learners with ample opportunities (Ilci, 2014). Ilci (2014) examined the levels of mobile learning readiness and mobile learning acceptance in pre-service teachers in the Faculty of Education at Middle East Technical University. The results suggested that the levels of mobile learning readiness and mobile learning acceptance among pre-service teachers were moderate. The term “mobile learning” is still developing day by day and its exact meaning is still unclear. Despite the ambiguity, there are some keywords to explain this concept. Traxler (2007) points out some keywords, such as personal, spontaneous, situated, private and portable to explain mobile learning.

There is currently a range of virtual manipulatives available online, some are free and some are not. For example, Mathsbot: https://mathsbot.com/manipulativeMenu offers a wide variety of free manipulatives such as Dienes blocks, bar models, Cuisenaire rods and double-sided counters [Image A]. They work well on a larger screen but are not optimised for mobile phones; there is a loss of functionality when these virtual manipulatives are accessed through a mobile device. The main reason for this is because the manipulatives are designed to be used through a device with a larger screen such as an iPad or laptop and is accessed via a web platform and is not a downloadable app.

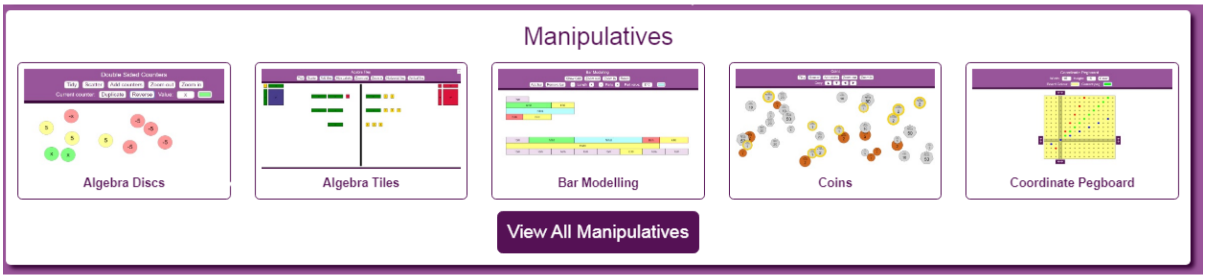


Image A: take from https://mathsbot.com/manipulativeMenu (Mathsbot, n.d.)

Mathigon (Polypad – Virtual Manipulatives – Mathigon) also offers virtual manipulatives within its Polypad. This is also available on a phone app but there is not a specific bar model functionality [Image B]. Unlike Mathsbot, Mathigon is available as a mobile app through iOS and Android devices without loss of any functionality. The Polypad contains options of working with Polygons, Number Tiles, Number bars, Number lines, Fraction Bars, Fraction Circles, Algebra Tiles and much more. As this Centre for Excellence of Maths Action Research was first interested in the use of virtual manipulatives to support the teaching and learning approach to fractions, decimals, percentages and ratios, the uses of the Polypad were explored. It was found that the fraction bars, although useful to use pictorially to give a representation of a fraction, there did not seem to be the functionality to manipulate the bar into a desired number of parts, to change and select the number of parts being shaded or to resize the bars. The fraction bars did not seem to draw upon building the students conceptual understanding of a fraction or indeed to use the virtual manipulative as a tool for solving problems. There was little interchangeability between fraction, decimals, percentages and ratio/proportion using the fractions bars within the polypad.

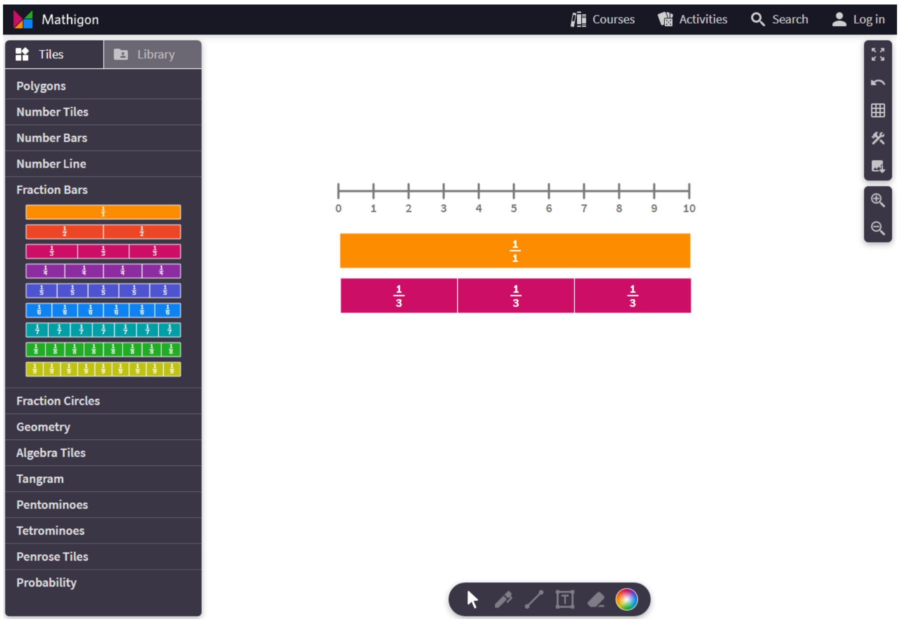


Image B: screenshot taken from https://mathigon.org/polypad (Mathigon, n.d.)

Specific phone apps include Fraction Strips, Relational Rods and Algebra Tiles by Mathies [Image C], which do meet the needs and functionality required for this action research and are free to download. There is the ability to manipulate a fraction bar or a relational rod, splitting a given bar into further sections to visualise equivalent fractions and proportionality (also useful for addition and subtraction) and functionality for combined fractions/decimal/ratios. The screen does seem to have extra functions which are not necessary, make the screen busy and not very intuitive for the user experience [Image D – screenshots of apps]. This would seem to agree with Laski et al (2015) in that virtual manipulatives must not have irrelevant or distracting features.

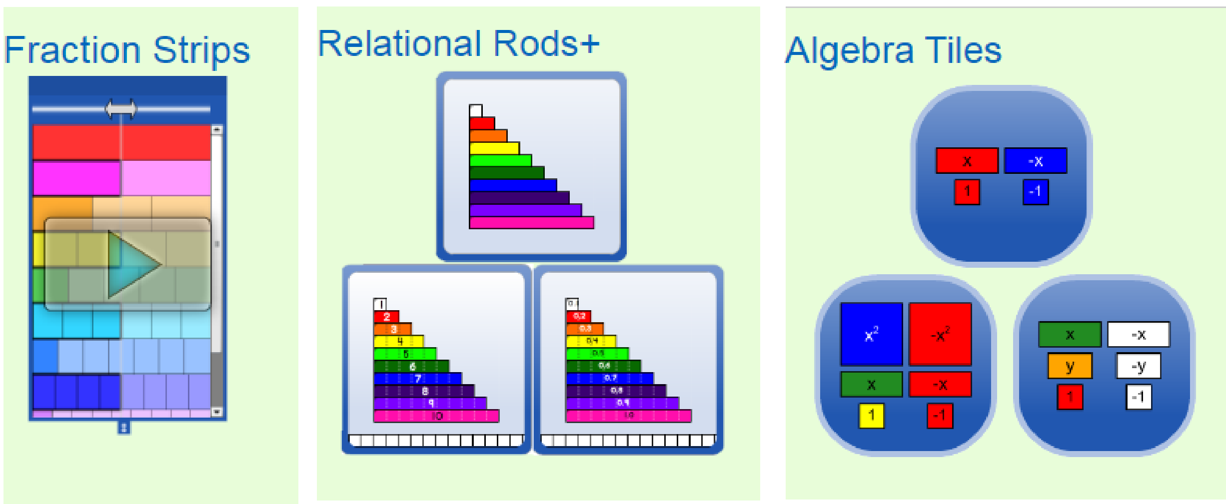


Image C: screenshot taken from https://mathies.ca/apps.php#gsc.tab=0 (Mathies Apps, n.d.)

Fraction Strips Overview	Relational Rods + Overview	Algebra Tiles Overview
Represent fractions by dragging pieces from the fraction tower into the workspace. Pieces can be placed in a line to form a train. Manipulate the pieces and trains to compare and order fractions or to model fraction operations. Opened files contain all the tool steps performed; use undo and redo to review these steps	Represent, compare, order, and operate on whole numbers, fractions or decimals by dragging rods from the tower into the workspace and manipulating them. The colour scheme of the rods can be customized, including to a traditional and a primary palette which are similar to physical manipulatives. Use the annotation feature and imported images to communicate solutions. **Relational Rods** - rectangles of various lengths where each rod can be evenly subdivided by the smallest square rod. **Whole Number Rods** - rectangles of lengths 1 to 10 with numeric labels and dashed divisions on each rod. **Decimal Number Rods** - rectangles of lengths 0.1 to 1.0 with numeric labels and dashed divisions on each rod.	Represent and model operations with integers and polynomials. Tiles representing 1, x, x², y, y² and xy along with their opposites can be dragged into the workspace from the scrollable selection panel at the left. Once in the workspace they can be moved, copied, re-oriented, or negated individually or in groups. The tiles can be configured to match the colours of the physical manipulatives commonly available in classrooms. This virtual algebra tile tool also includes 1, x, and y line segments. Unlike the physical tiles, the value of x and y can be adjusted.

Image D: Screen shots of Mathies apps

Further searching revealed an app called ‘Manipulatives’ which has a good selection (although no bar model) but costs $19.99. There certainly appears to be a gap in the market for well-thought-out math manipulative mobile phone application along with a sequence of lesson plans for delivery. This action research aims to develop such an app along with a structured sequence of lesson plans and materials for delivery alongside the app using the understanding and research gained from this literature review.

Methods

Participants

For this action research project, all five participating action research teachers agreed to target one resit GCSE Maths class at their site for the planned intervention. Consequently, this meant that a minimum of 40 learners would receive the intervention and be able to feedback any findings from their experiences. Learners would span across Southeast London and Greater London (Lewisham, Sidcup, Bexley and Greenwich) and would be across an age range of 16–18-year-olds as well as adult learners. This intentionally gave the project a unique comparison and insight into how different age groups of learners responded to the intervention and whether these differences/similarities could be explained.

Procedure

A range of both qualitative and quantitative data would be used along the yearlong journey of the project. Initially, it was important to have an insight into the types of learners who would be receiving the intervention (e.g., age, number of attempts at the qualification, main study programme, experience with technology, confidence etc). This was conducted through a pre-Survey completed online via Microsoft Forms survey (Appendix A).

The action research group used the Literature Review and their own experiences of using physical manipulatives to design a sequence of lessons to support the delivery of the virtual manipulatives app in the classroom (Appendix B). This was segmented into three distinct but interleaving units, each of which were allocated approximately 3 hours of lesson time.

Feedback from teachers was discussed and shared amongst the action research group during the delivery of the intervention and noted as qualitative observation evidence towards the findings.

Students were also asked to produce written reflections at the end of each lesson on how they had found using the manipulative and how their thinking had changed. Students were then asked to complete post-Survey, again via Microsoft Forms (Appendix C).

The key principles and objectives of the project are as follows:

Research Objectives:

1. To explore the research literature and available technologies, and colleagues' current practices and innovative thinking on CPA
2. If appropriate, to develop or refine an App that delivers the required manipulatives to learners learning outside of the college
3. Use the Focused 15 SOL produced by Grimsby CfEM to develop the CPA models/apps specifically targeting these key topics.
4. To analyse in what ways a CPA approach enables students to grasp and understand key skills in topics and why.
5. To compare different teacher and learner cohorts and explain these differences (for example, grade 1 vs 2 vs 3, creative vocational programmes vs not, age of learner)
6. To share results and, if possible, effective approaches, with GCSE math re-sit teachers locally and nationally.

Findings

To analyse our findings, we will first look back at each of Action Research objectives and discuss the data and evidence collected and if they provide any insight towards this key principle. This section will also seek to link any suggestions from the Literature Review that support these findings or whether the findings suggest something different.

Key Principle 2

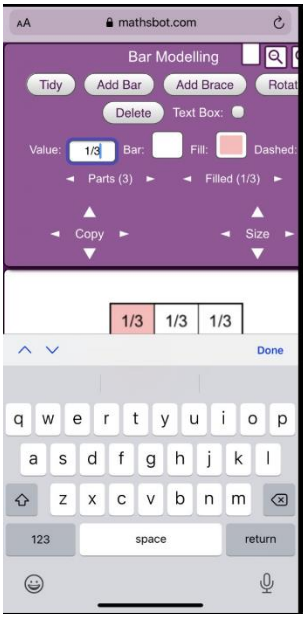
If appropriate, to develop or refine an App that delivers the required manipulatives to learners learning outside of the college

As an action research group, it was first appropriate to think carefully about what apps/software there was currently available on the market that was free or low cost to use with students and on what device. The app would need to support and facilitate the manipulative in a virtual space to improve the conceptual understanding we wanted our learners to have in Fractions, Decimals, Percentages and Ratios, and be readily available on mobile phone devices (Android and/or iOS). It was clear from the research that computer or virtual manipulatives can be '*just as meaningful and easier to use for learning'* that their physical equivalents (Clements & McMillen, 1996).

Teachers play a vital role in which concrete model to use to help facilitate a learner's progression and understanding (Goldin & Kaput, 1996) and this feedback by them was important to the decisions made during the developmental year of this project. Using the experience, knowledge and CfEM community, it was apparent that the use of Bar Models would be the most applicable use of concrete manipulative for this intervention.

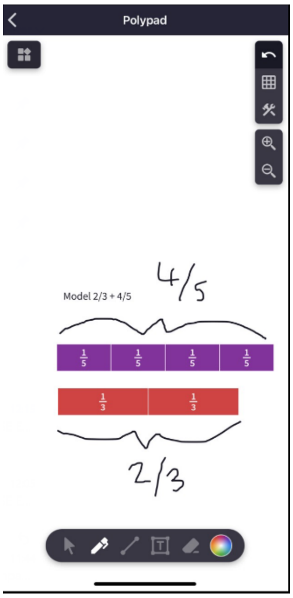
Learning about the importance of multiple representations playing an important role in the CPA model (Charalambour & Pitta-Pantazi, 2007) and in particular '*Learning with Models*' (Durmus & Karakirik 2006), it was almost immediately apparent that the app should be designed as a learning tool rather than as a self-directed learning package (moving from one question to another integrated within the software, using a virtual manipulative in the process of doing so).

Each action research teacher trialled every virtual manipulative where access was available, where it was revealed that functionality on the desired platform (in this case a mobile phone) was a huge issue and barrier to overcome.



Mathsbot.com (which many GCSE Math teachers across the national are familiar with and use to model in classrooms) was fit for purpose, had easy functionality and demonstrated models that built on conceptual understanding, but had technical problems when accessing it through a mobile phone device. For example, once a bar model was created on the platform (see left), manipulating it or selecting the bar itself would automatically reveal the keypad which would hide the majority of the screen visible for the user (this was made worse in landscape view) and would keep reappearing even after minising the keypad option.

Mathbot.com, would be a great option for this intervention for those with direct access in college and out of college to a laptop and internet connectivity. This was something the project wanted to avoid due to the lack of IT equipment/hardware available for the resit GCSE Maths course and for students working from home (where the only use of technology is their mobile phones).

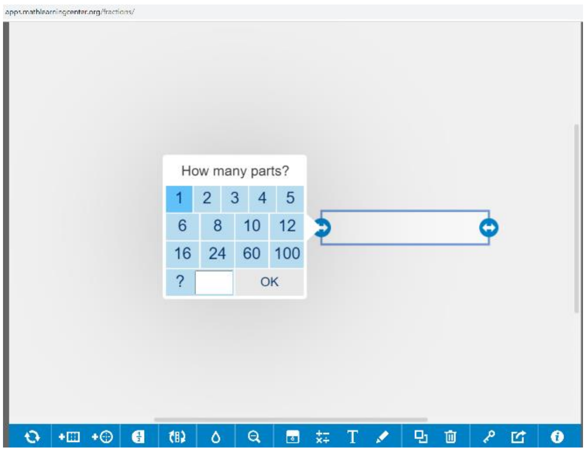


Action research teachers enjoyed the simplicity of use and features in the **Mathigon Polypad** app which was free to download on android and iOS devices. This was something that appealed to our target audience.

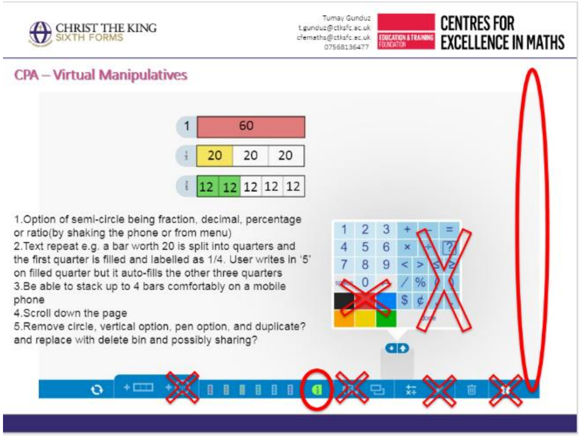
In additon to this, annotations were easy to add to the fraction bars as well as the ability to split a selected bar into individual parts.

However, what the polypad did not offer was the functionality to increase or decrease the number of parts once a bar was selected (e.g. changing one whole bar into a bar with five parts). To do this one must select the whole bar separately to a bar with five parts. This caused some issues when the screen size was already limited and the multiple bars needed on one screen to be able to compare.

Mathigon did have a range of virtual manipulatives apps within its Polypad feature which would be appropriate when exploring algebra (algebra tiles) and probability (virtual dice/spinners) in the future.



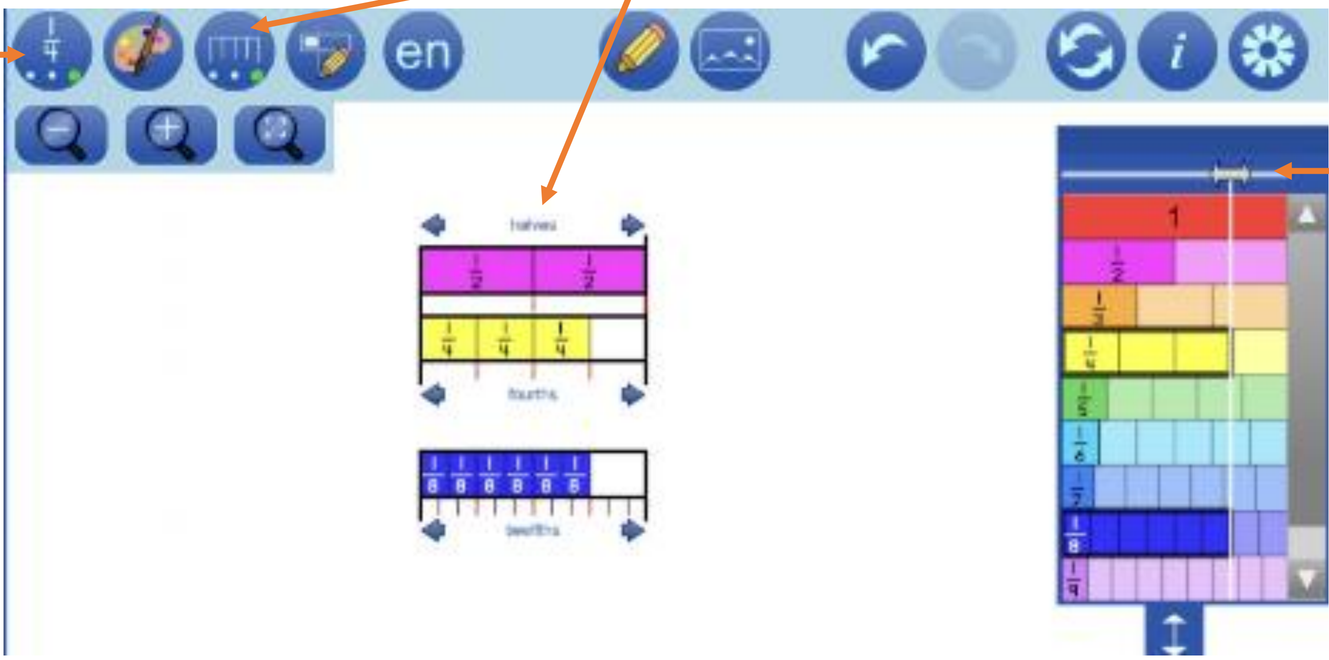
The **Maths Learning Center** fraction app (https://apps.mathlearningcenter.org/fractions/) came closest to the features the intervention would require (e.g. easy manipulation, good user interface and functionality, ability to split a given bar into a number of parts) could be used for comparisons between Fractions, Decimals, Percentages and Ratios. However, again, the MLC fraction app was only available on iPad or full web page browser.



The action research group decided to speak directly to the developers at MLC and came very close to agreeing contracts to transform their web-based virtual manipulative into a mobile app platform but with some alterations to the user interface (see left). For example, only bar models needed and not fraction circles, vertical only and not horizontal bars and a scroll down function within the app, were a few changes we recommended.

Unfortuately, due to the complexcities of working across countries and on a short time frame of what is normally expected to build an app, we were unable to materialise this by the end of the academic year.

It was agreed that the use of the **Mathies Fraction Strip app**, seemed most applicable, user-friendly, accessible and appropriate to support the intentions of the research. Comparisons between equivalent fractions was a functionality that was easily demonstrated and could be explored by the user through the use of the <u>divisor selection</u>.



This enabled users to compare fractions without the need of having multiple bars on the screen at once (flicking between the arrows to change the fraction divisors from halves, to thirds, to fourth etc).

The <u>left hand side menu bar</u> also had the opportunity for students to compare equiavlent fractions as they made their selection.

<u>Labelling</u> could be swicthed on or off dependent on the requirements of the learners.

Of the learners taking part in the intervention and who had completed the pre-survey (37), 44% had said they had used a virtual manipulative before. When exploring this in more detail and probing which virtual manipulative, responses of VLE platforms (e.g. matshwatch, Hegarty) were common. These would not fit the definition defined and outlined by Moyer, Bolyard and Spikell (2002) of a virtual manipulative.

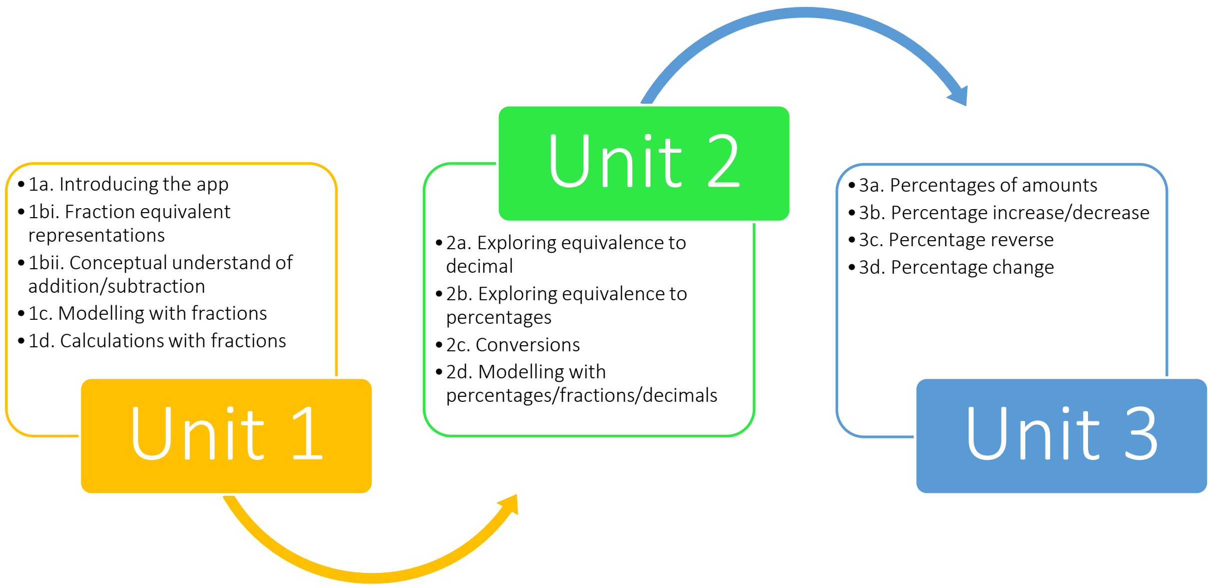
Key Principle 4

To analyse in what ways a CPA approach enables students to grasp and understand key skills in topics and why.

This key principle was mainly explored in the literature review. The learnings from that piece of research such as necessary approaches and pedagogy in CPA, enabled the action research group to think more carefully on the design of the sequence of units that would support the use of the virtual manipulative selected.

The enactive-iconic-symbolic modes of representations as conceived in 'Theory of Instruction' (Bruner , 1966) was an important factor in recognising that the amount of time spent in the concrete framework (enactive-iconic) was not prescriptive and in fact teachers must be the judge on how to adapt the CPA levels to suit their students' needs (Gattegno, 1987). This is particularly true for a low achieving resit GCSE Math learner, where the teachers' ultimate aim must be to get learners fluent in the abstract stage (symbolic). Moving through the enactive-iconic-symbolic stage too slowly or quickly could mean students may not be equipped with the required understanding of modes to fall back on when their procedural methods fail.

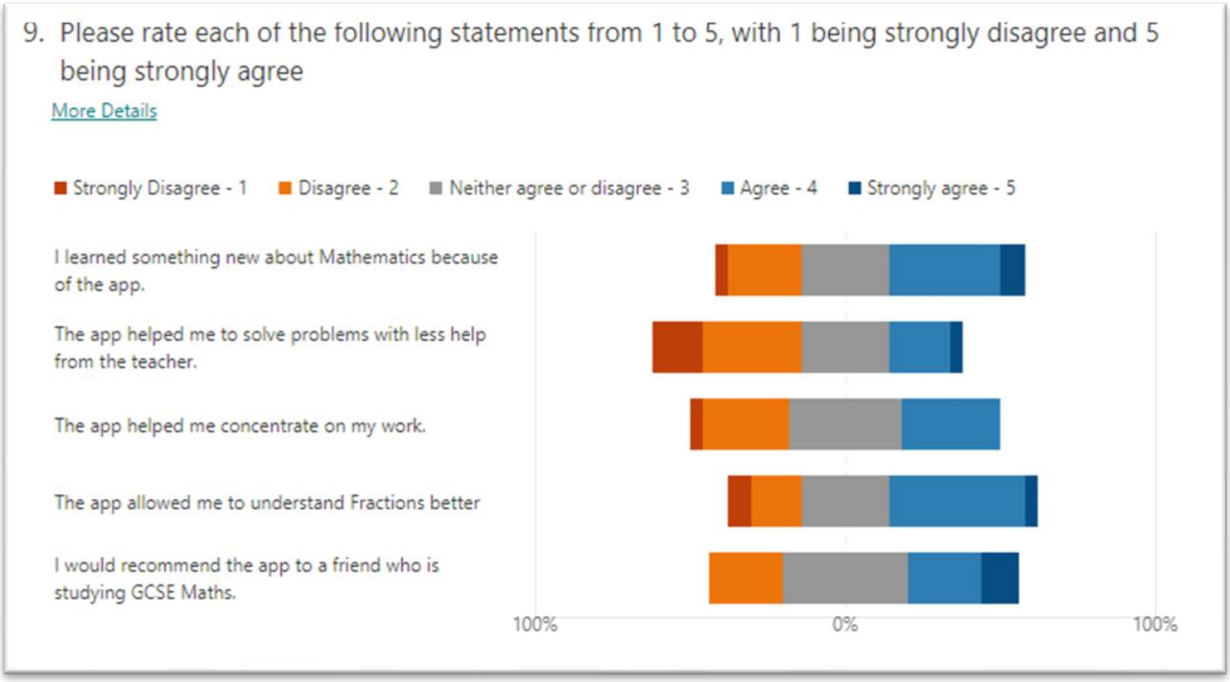
The design of the CPA approach for the purposes of this intervention came in the form of three units each of which required an approximate three hour delivery time (full lesson of units found in Appendix B).



Key findings of student response when using a virtual manipulative to support CPA delivery of Fractions, Decimals and Percentages

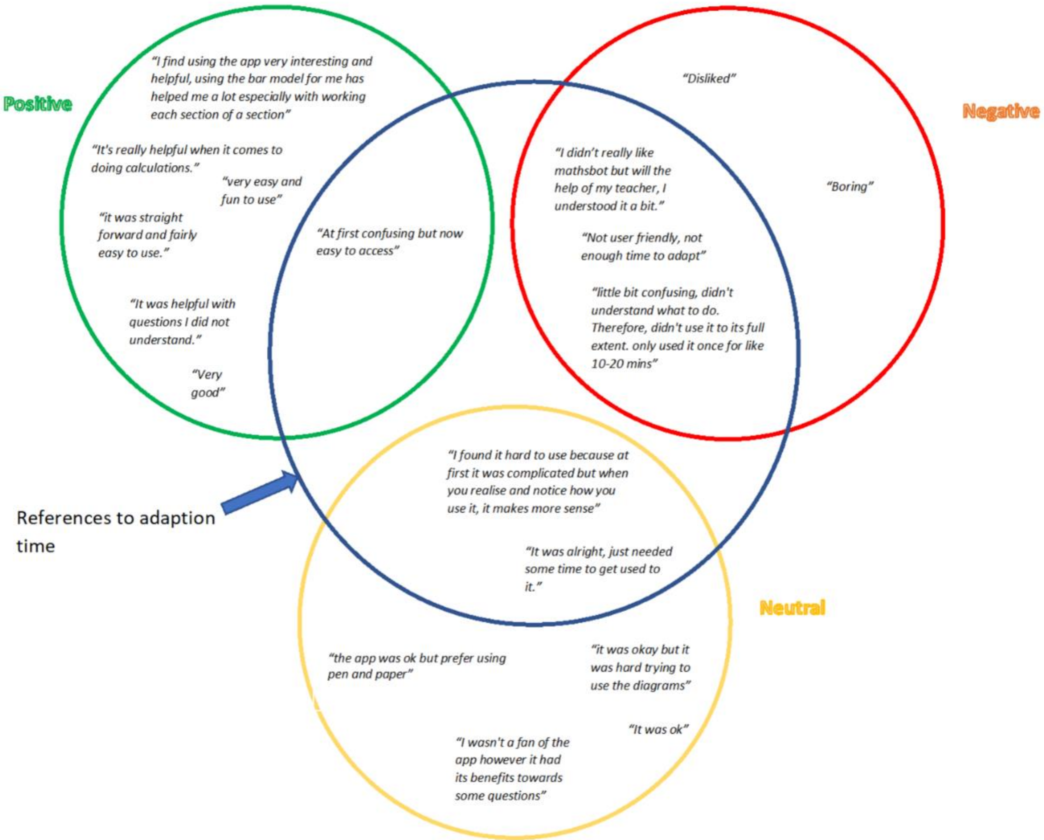
Through the post-survey results (of which 25 learners completed) it could be suggested that more learners than not were positive that the app had helped their learning, understanding, and said they would recommend the app to a friend.

Overall, there were approximately equal numbers of learners who said the app helped with concentration, than those who did not. This finding was also revealed through the trial teacher feedback in which it was suggested that some learners found the use of their mobile phone a distraction in class, but that these were typically students who had been distracted by their mobiles previously. It is important to factor in the covid restrictions during the intervention delivery, in which teachers were unable to circulate their classrooms or approach learners one to one in close proximity to be able to monitor what was on their mobile phone screen. A solution to this problem was not found but monitored and noted in the future recommendations of this project.



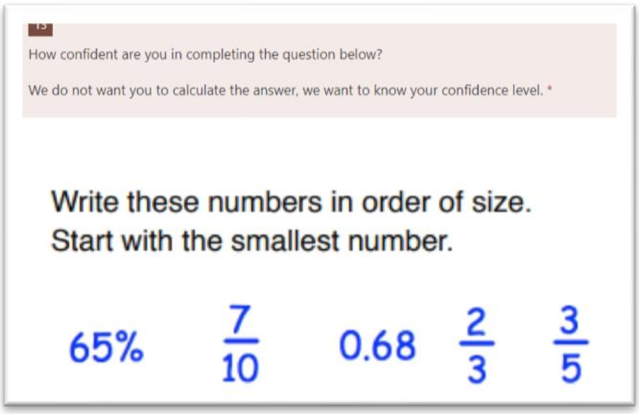
A finding, which was not expected by the action research group, was that more students disagreed that the app helped them solve problems with less help from the teacher. This goes against some of the findings in the literature in which manipulatives help students to become more independent problem solvers (Suydam & Higgins 1976). The difference between our results and those presented in the literature review may be due to students needing more time to adapt to the app and for it to become a true problem-solving tool for them. It was agreed by the trial teachers that the tools (whether physical or virtual) needed to be incorporated throughout the delivery of the GCSE Maths course and not as stand-alone activities. Interleaving and revisiting them between topics was felt as an important factor for students to adjust and familiarise themselves with the conceptual understanding they can offer.

Of the learners responses to the post-survey, the following Venn-diagram demonstrated between learners reference to adapting to the virtual manipulative over time. It was interesting that, of those who commented negatively about the use of the virtual manipulative app, the most common theme was in reference to time and adapting to the app.



Neutral responses consisted of learners liking the app but prefering alternate and more traditional methods such as "*pen and paper*". Positive responses suggested that the use of virtual manipulatives had some influence over the learners enjoyment towards the topic/activity as well as its application and use towards some questions.

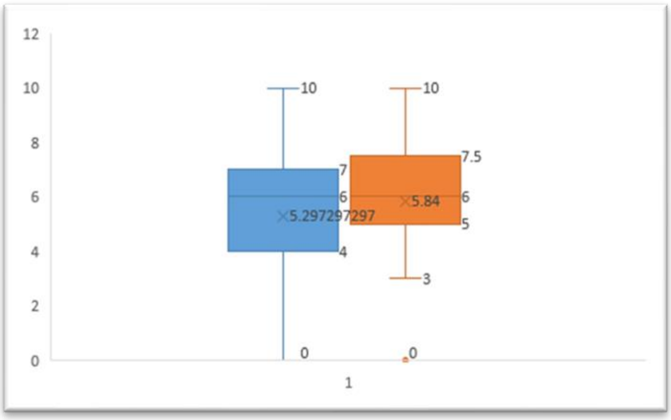
A response to using manipulatives or the CPA approach with learners that had not presented itself during the literature review was students' academic self-confidence and concept.


How confident are you in completing the question below?

We do not want you to calculate the answer, we want to know your confidence level. *

Write these numbers in order of size.
Start with the smallest number.

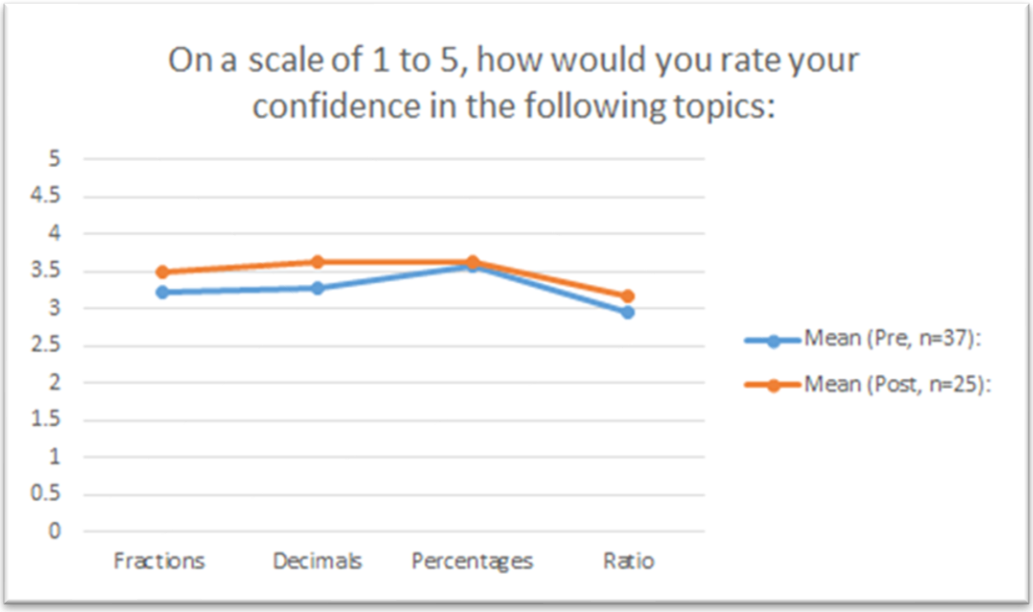
65% $\frac{7}{10}$ 0.68 $\frac{2}{3}$ $\frac{3}{5}$



-Post Survey comparison: box-plot showing pre (blue) and post (orange) in confidence in refernece to question 13 in survey (Appendix A and C)

Students' self-declared confidence levels to solve a problem converting between decimals, fractions and percentages were higher on average (median and mean) and had a lower interquartile range after the lessons. This would seem to indicate that he virtual manipulative or the designed unit itself had a positive impact on a students self-declared confidence or concept. These results are not conclusive or guaranteed as less students completed the post questionnaire (n = 25) than the pre questionnaire (n=37).

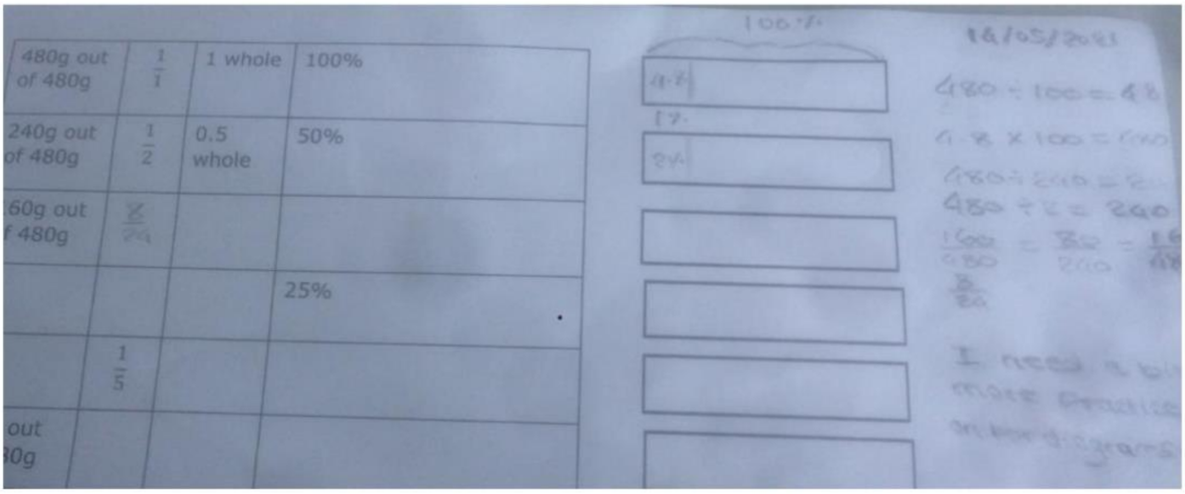
This trend of growing self-declared confidence was also seen in response to how students rated their confidence in the units individually (Fraction, Decimals, Percenatges, Ratio) before and after the intervention.



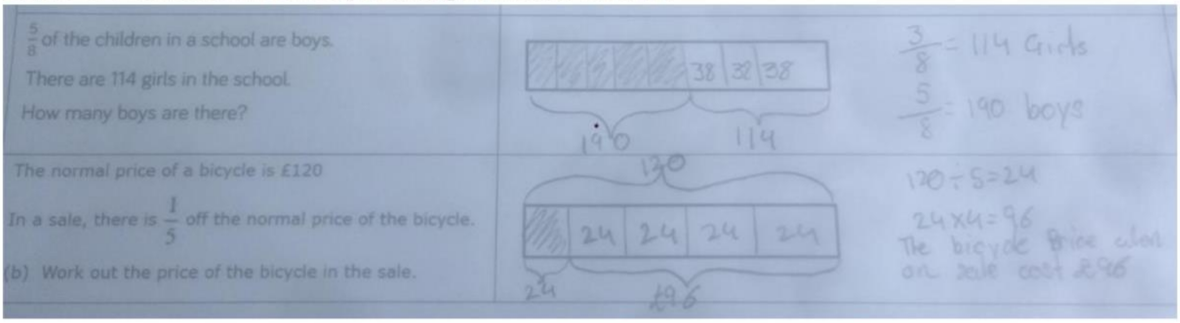
Interestingly, Unit 3 which related to percentages, was not delivered to any of the participants due to the time limitations the action research group experienced during Spring/Summer 2021. This seems to be evident from the findings above in which the confidence of percentages pre and post survey did not change. Again, it is worth noting the difference in numbers of those who took the survey pre and post.

Moving towards more abstract models, students demonstrated a range of preferences in how they approached typical GCSE Maths questions in the FDP (Fractions, Decimals, Percentages) topic (see images below).

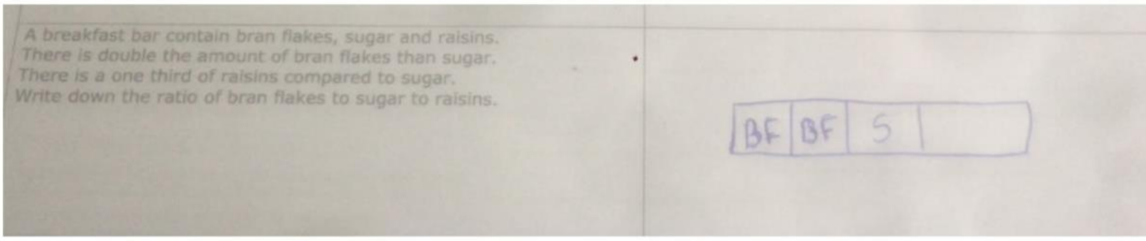
Grade 1 student attempting some divisions after teaching of how bar models can be used to convert fractions into percentages:



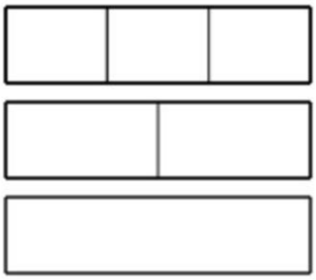
Grade 3/4 student successfully building some bar models:



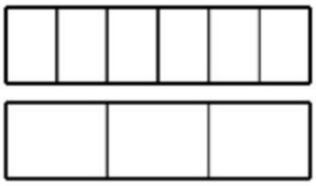
Grade 3 student attempting to build a bar model, but having given up half way through:



Another students attempt for this problem:



Attempt 1



Attempt 2

Conclusions and Recommendations

Conclusions

This action research project has provided teachers within the post-16 GCSE Maths resit sector an opportunity to explore how a CPA approach can be used with a virtual manipulative. It is clear that if accessibility to laptops or computers are possible in or out of the classroom, there are a healthy number of platforms, such as Mathsbot and Math Learning Centre, to choose from and use, for free. These platforms can be classified as virtual manipulatives and meet the needs of building conceptual understanding for resit GCSE Maths learners studying the relationship and models of fractions, decimals, percentages and ratios.

There is, however, a huge and worrying gap in the market from our findings for a virtual manipulative of this kind accessible through a mobile phone device. It may be assumed not necessary, but given that desktops or laptops computers are no longer the primary way students access the internet and 95% of 16-24-year olds in the UK own a smartphone (Statista, 2019), young people are using smartphones with integrated Wi-Fi and internet access to communicate more, play games and consume more content. This change in consumer behaviour towards technology will ultimately need to change the way in which we access such virtual manipulatives in the classroom with limited technological resources (Moyer-Packenham, Bolyard, & Spikell 2002).

Learners seem to respond better to apps that are user friendly and easy to navigate. Of those who found this more challenging, a time reference and adapting to the app was a commonly spoken of as a way of overcoming this.

Of the topics taught in conjunction with the virtual manipulative chosen in this study, learners self-declared confidence and academic self-concept seemed to increase pre and post intervention.

Recommendations

- The type of physical manipulative used for a particular topic should first be explored before entering the world of virtual manipulatives.
- Teachers using a CPA model should consider conceptual understanding over fluency skills when developing the enactive-iconic-symbolic modes of lesson planning.
- To understand there are no set criteria of recommendations for time spent in each mode by a teacher (Gattegno, 1987) but that it is the teachers ultimate aim to move students into a symbolic/abstract mode, developing fluency at this point in time.
- It is critical that students are given time to ‘play’ and explore the app or virtual manipulative before formally introducing it alongside mathematical concepts/instructions.
- CPA models (physical or virtual) should not be used as stand-alone activities at individual points of the curriculum but as a tool to embed within the entire course, offering an opportunity for student to build links and adapt to the model for it to become embedded learning.
- To be aware of how mobile phone devices can be a distraction for some learners within the classroom setting and to monitor this by circulating the classroom or using behaviour for learning expectations before introducing the app to a group.

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Appendices

Appendix A

Centre for Excellence Math - APP Questionnaire (PRE)

This questionnaire is to gather data and information on the learners who have participated in the Centre for Excellence Math Action Research Project (App - Virtual Manipulatives).

* Required

1

Please read the following statements before beginning:

- I understand the purpose of the research project and my involvement in it.

- I understand that I may withdraw from the research project at any stage and that this will not affect my status now or in the future.

- I understand that while information gained during the study may be published, I will not be identified and my personal results will remain confidential.

- I understand that I may contact the researcher or supervisor if I require further information about the research. *

○ Please tick here to give you understand and agree to participate in this questionnaire

About you

This section is to gather some information about you

2

Which college do you attend? *

○ Christ the King Sixth Form College Emmanuel

○ Christ the King sixth Form College St Marys

○ Learning Enterprise College Bexley

○ Shooters Hill College

3

What is the highest GCSE Maths grade you have already achieved?

○ U

○ 1

○ 2

○ 3

○ 4

○ 5

○ 6

○ 7

○ 8

○ 9

4

How old are you? *

○ 16

○ 17

○ 18

○ 19+

5

How many times have you sat an actual GCSE Math paper? (including year 11)

○ Never sat a formal exam

○ Once

○ Two times

○ Three times

○ Four or more times

6

What is your main study programme or employment? (e.g. BTEC Business, A Levels, Construction etc) *

Enter your answer

The app

This section is about the app

7

Have you used any Mathematical Apps before?

- ◯ Yes
- ◯ No

8

If you answered yes to the previous question, which maths app have you used?

Enter your answer

8

Have you used any of the following manipulatives before? *

- ☐ Cuisenaire Rods
- ☐ Fraction Wall
- ☐ Unit Box
- ☐ Counters
- ☐ None
- ☐ Other

9

If you chose 'Other' in the previous question, please name them.

Enter your answer

10

On a scale of 1 to 5, how would you rate your confidence on the following topics: *

	Not confident at all 1	2	3	4	Very confident 5
Fractions	○	○	○	○	○
Decimals	○	○	○	○	○
Percentages	○	○	○	○	○
Ratio	○	○	○	○	○

11

How confident are you in completing the question below?

We do not want you to calculate the answer, we want to know your confidence level. *

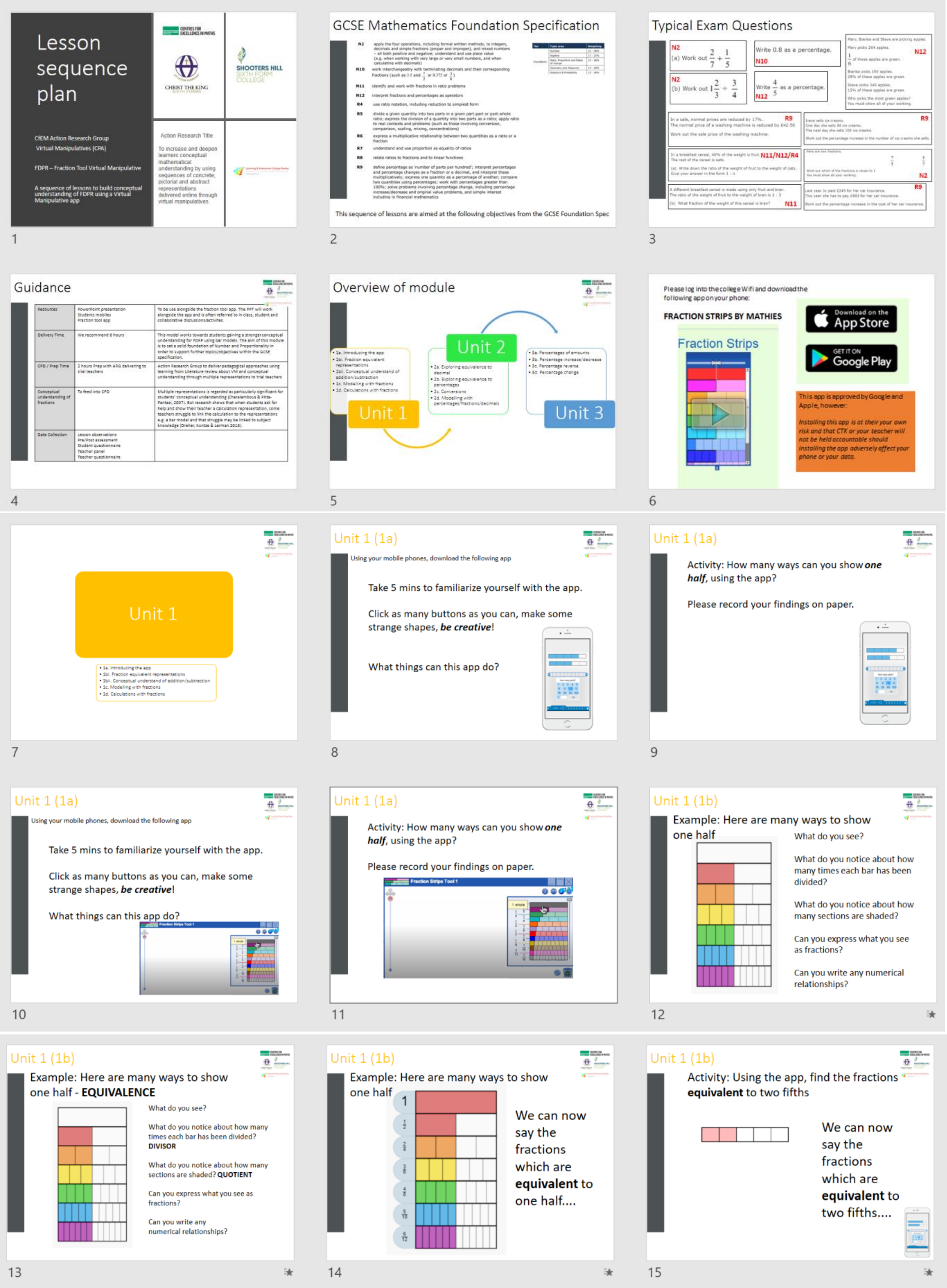
Write these numbers in order of size.
Start with the smallest number.

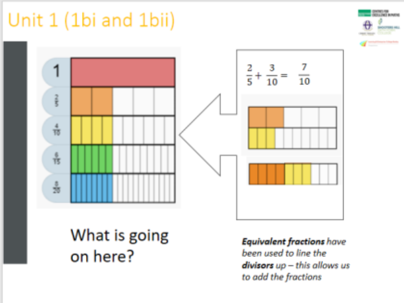
65% $\quad \frac{7}{10} \quad 0.68 \quad \frac{2}{3} \quad \frac{3}{5}$

0	1	2	3	4	5	6	7	8	9	10

Not at all confident — Extremely confident

Appendix B


Lesson sequence plan
CfEM Action Research Group
Virtual Manipulatives (CPA)
FDPR – Fraction Tool Virtual Manipulative
A sequence of lessons to build conceptual understanding of FDPR using a Virtual Manipulative app
Action Research Title
To increase and deepen learners conceptual mathematical understanding by using sequences of concrete, pictorial and abstract representations delivered online through virtual manipulatives
SHOOTERS HILL SIXTH FORM COLLEGE
1
GCSE Mathematics Foundation Specification
This sequence of lessons are aimed at the following objectives from the GCSE Foundation Spec
2
Typical Exam Questions
3
Guidance
4
Overview of module
Unit 1
Unit 2
Unit 3
5
Please log into the college Wifi and download the following app on your phone:
FRACTION STRIPS BY MATHIES
Fraction Strips
Download on the App Store
GET IT ON Google Play
This app is approved by Google and Apple, however:
Installing this app is at their your own risk and that CTK or your teacher will not be held accountable should installing the app adversely affect your phone or your data.
6
Unit 1
7
Unit 1 (1a)
Using your mobile phones, download the following app
Take 5 mins to familiarize yourself with the app.
Click as many buttons as you can, make some strange shapes, be creative!
What things can this app do?
8
Unit 1 (1a)
Activity: How many ways can you show one half, using the app?
Please record your findings on paper.
9
Unit 1 (1a)
Using your mobile phones, download the following app
Take 5 mins to familiarize yourself with the app.
Click as many buttons as you can, make some strange shapes, be creative!
What things can this app do?
10
Unit 1 (1a)
Activity: How many ways can you show one half, using the app?
Please record your findings on paper.
11
Unit 1 (1b)
Example: Here are many ways to show one half
What do you see?
What do you notice about how many times each bar has been divided?
What do you notice about how many sections are shaded?
Can you express what you see as fractions?
Can you write any numerical relationships?
12
Unit 1 (1b)
Example: Here are many ways to show one half - EQUIVALENCE
What do you see?
What do you notice about how many times each bar has been divided? DIVISOR
What do you notice about how many sections are shaded? QUOTIENT
Can you express what you see as fractions?
Can you write any numerical relationships?
13
Unit 1 (1b)
Example: Here are many ways to show one half
We can now say the fractions which are equivalent to one half....
14
Unit 1 (1b)
Activity: Using the app, find the fractions equivalent to two fifths
We can now say the fractions which are equivalent to two fifths....
15



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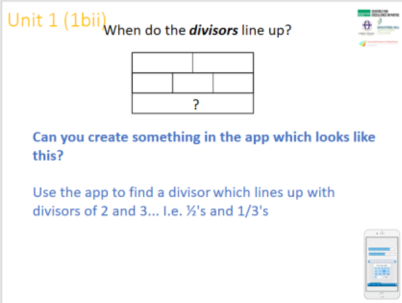

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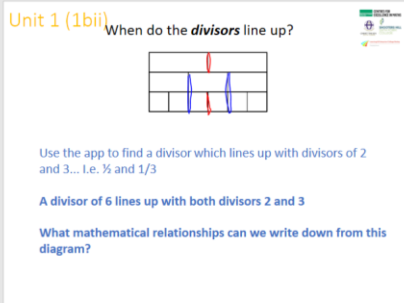

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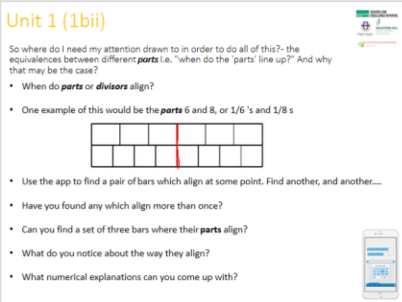

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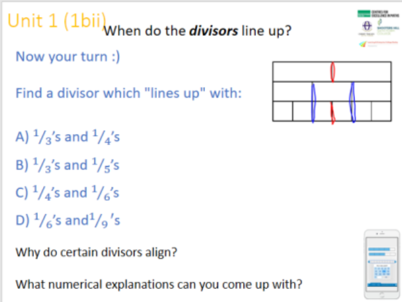

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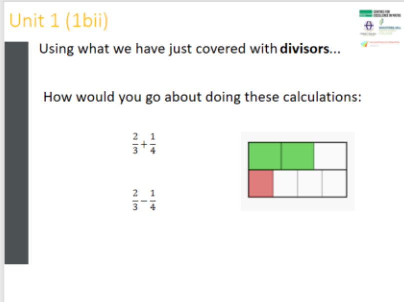

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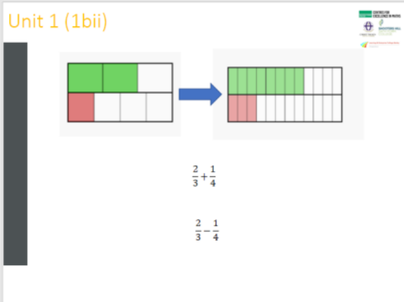

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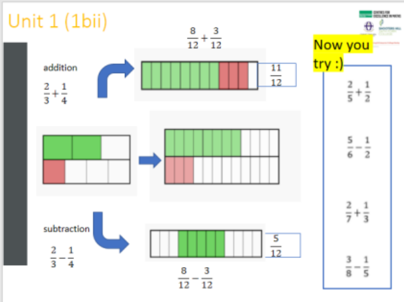

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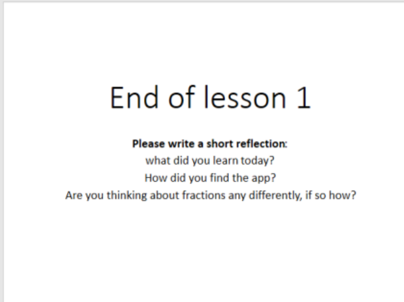

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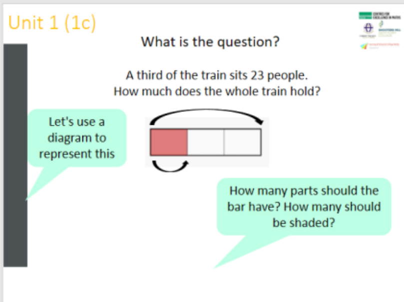

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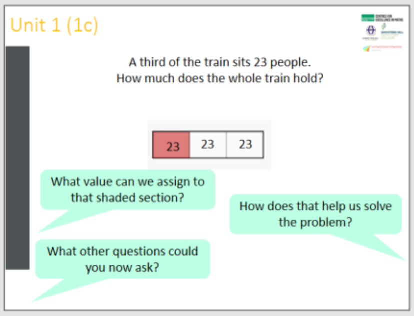

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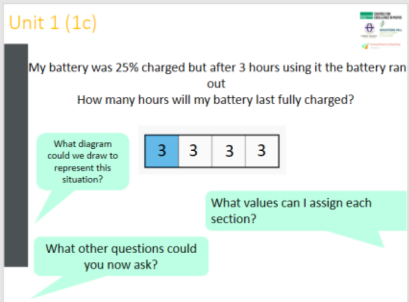

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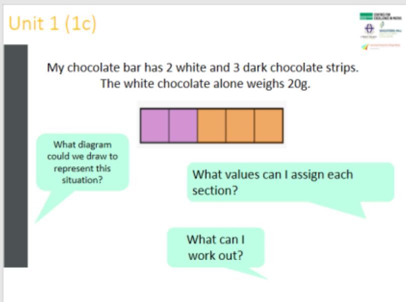

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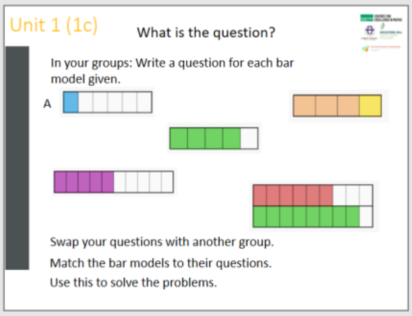

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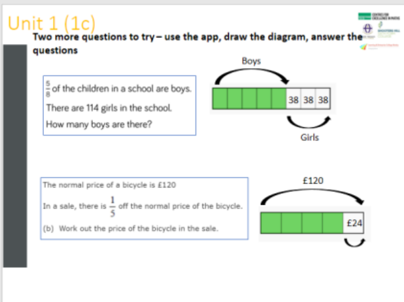

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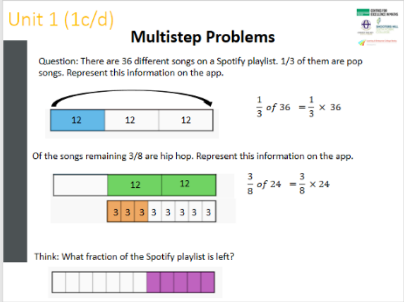

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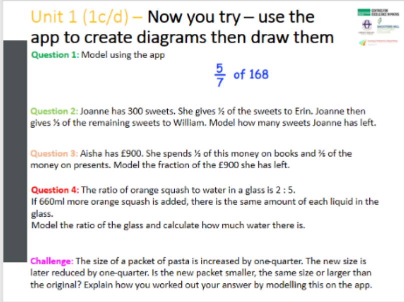

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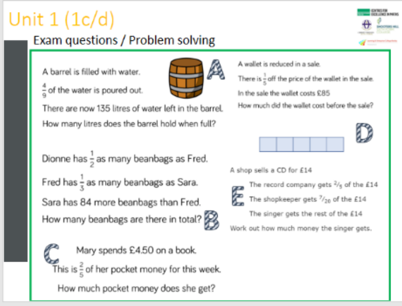

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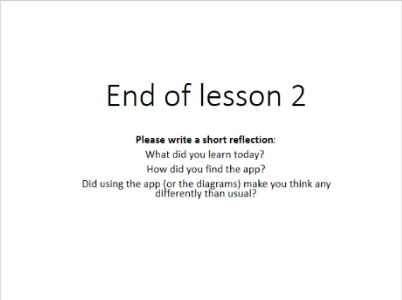

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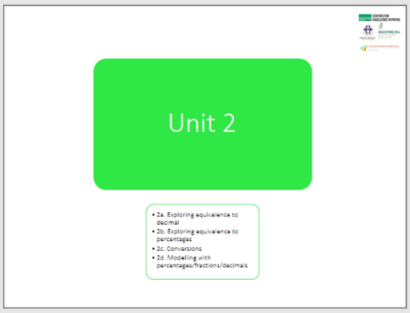

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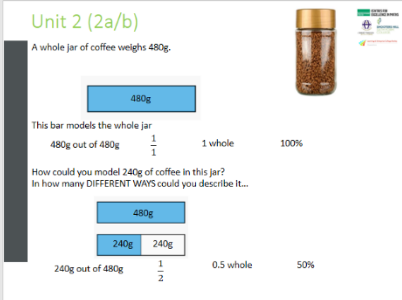

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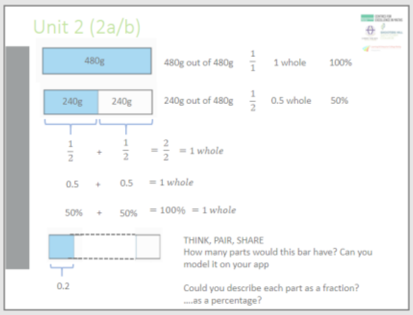

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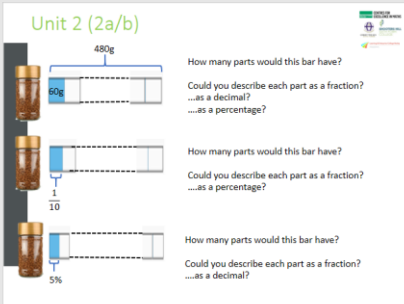

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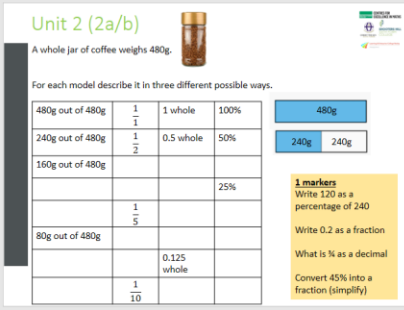

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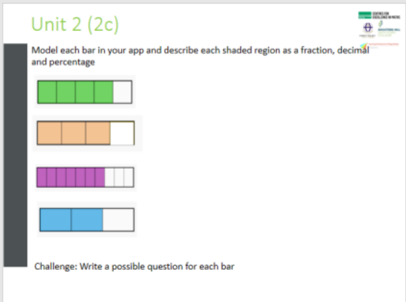

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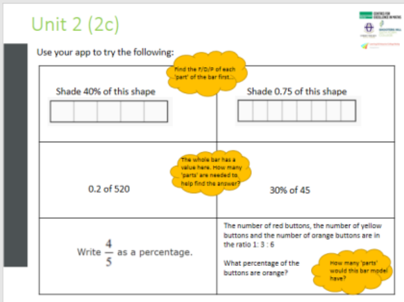

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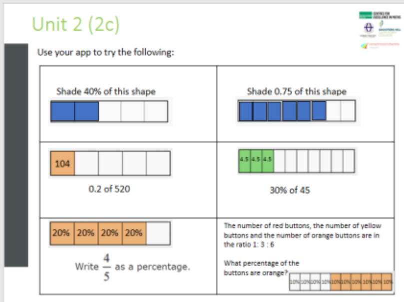

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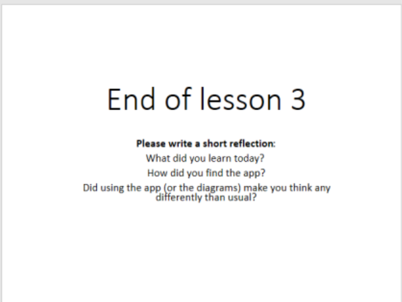

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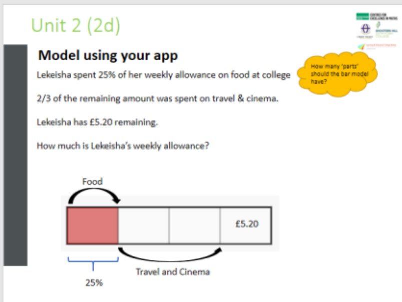

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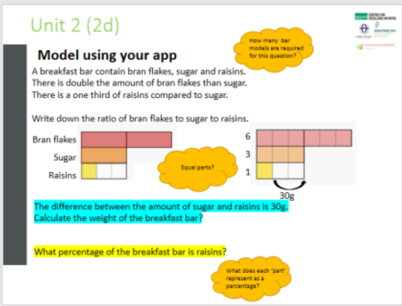

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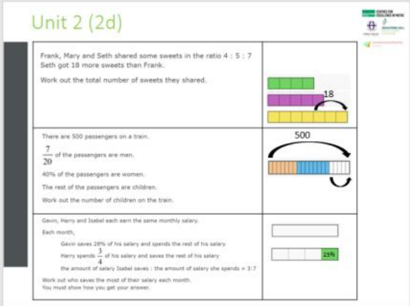

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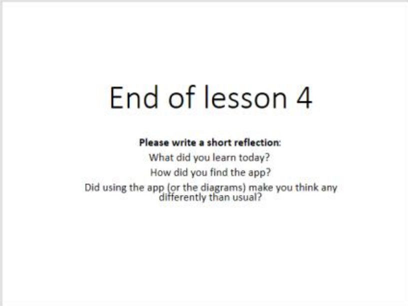

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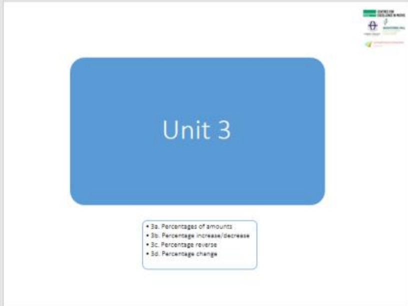

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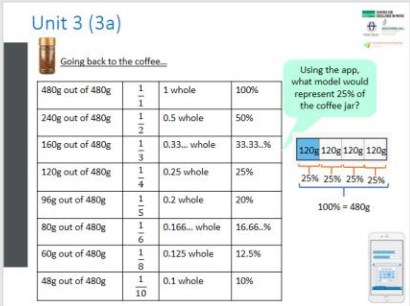

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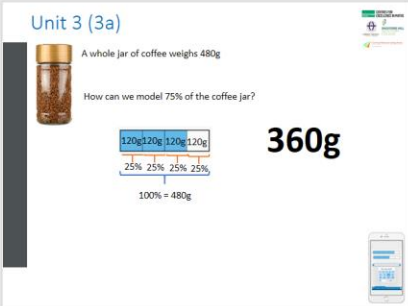

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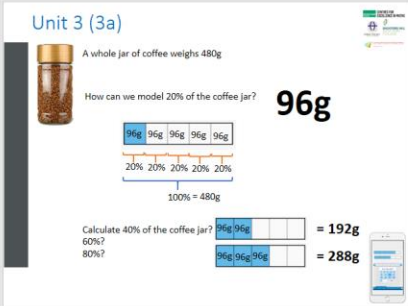

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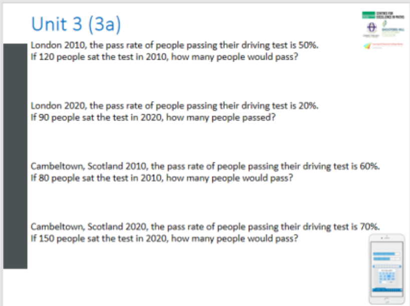

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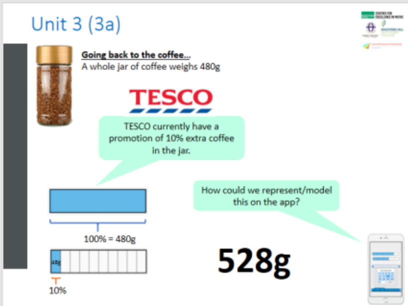

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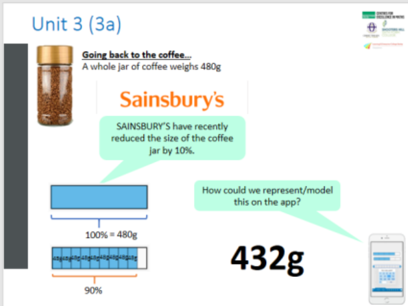

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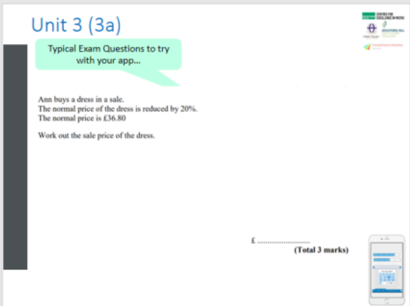

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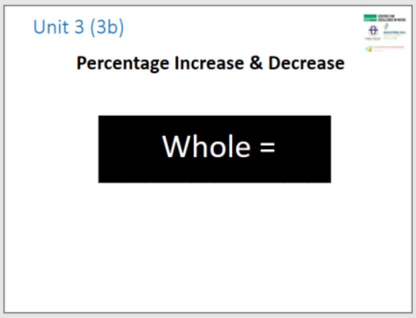

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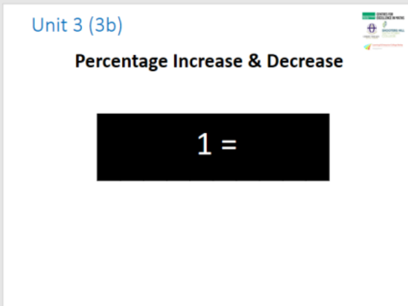

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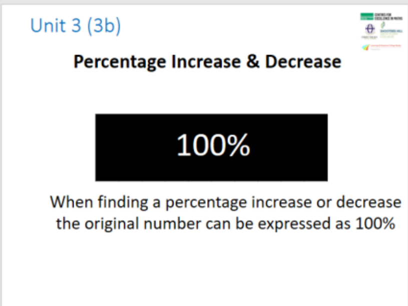

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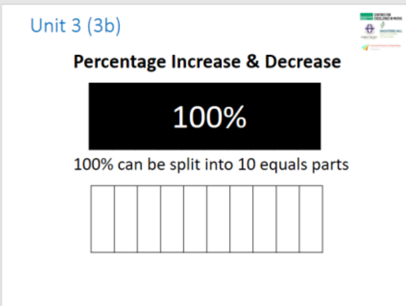

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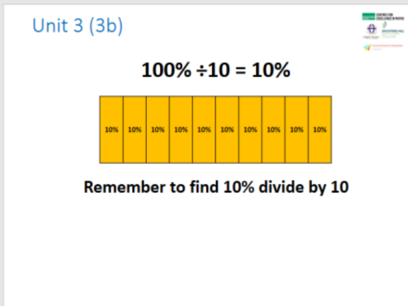

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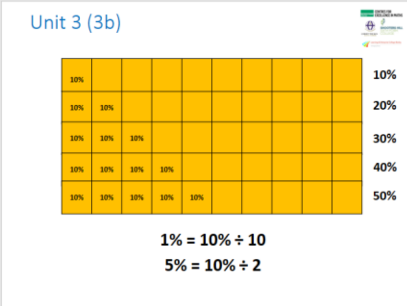

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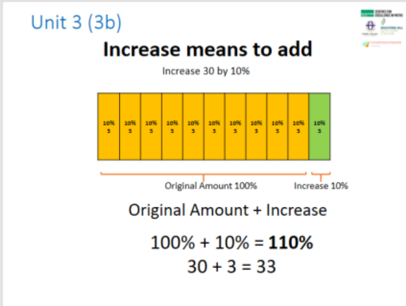

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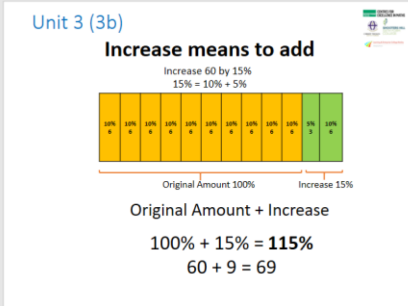

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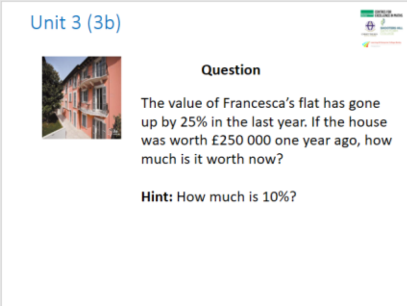

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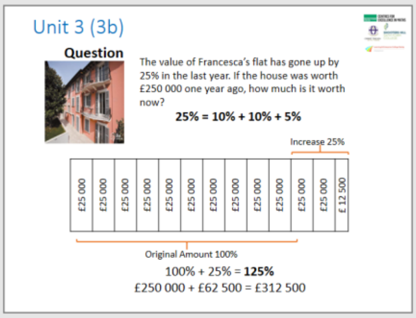

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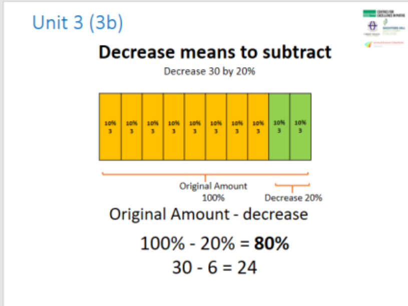

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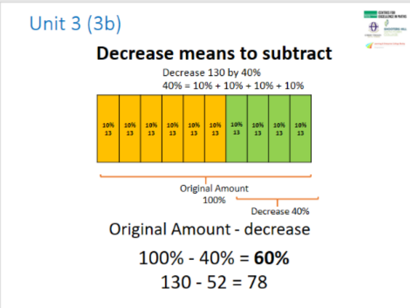

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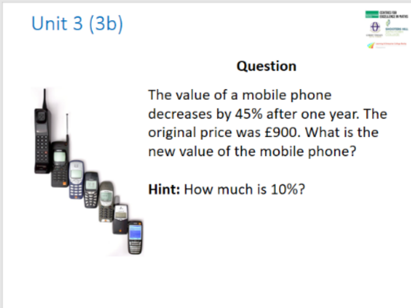

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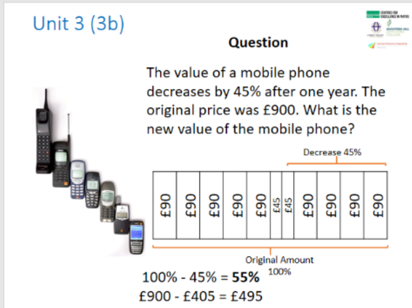

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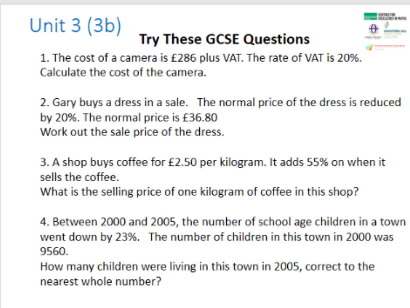

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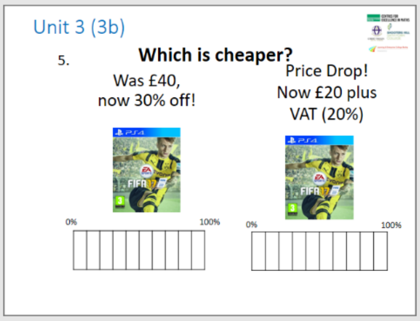

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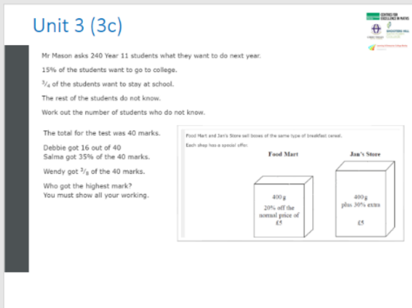

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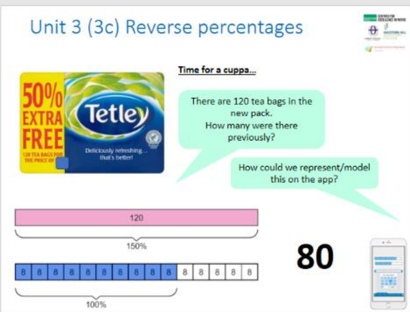

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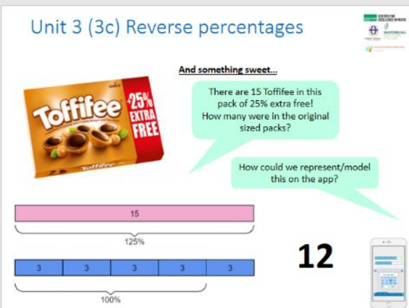

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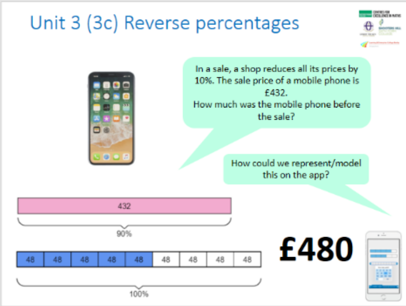
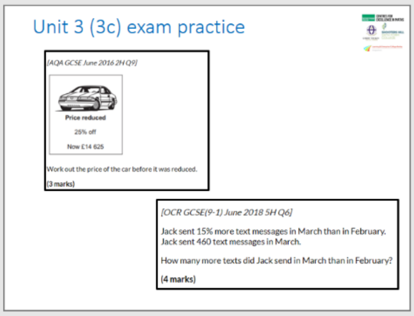
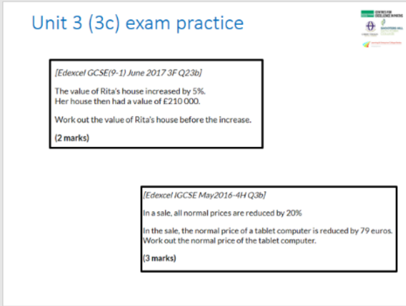

Unit 3 (3c) Reverse percentages
In a sale, a shop reduces all its prices by 10%. The sale price of a mobile phone is £432.
How much was the mobile phone before the sale?
How could we represent/model this on the app?
432
90%
48 48 48 48 48 48 48 48 48
100%
£480

76


Unit 3 (3c) exam practice
[AQA GCSE June 2016 2H Q9]
Price reduced
25% off
Now £14 625
Work out the price of the car before it was reduced.
(3 marks)
[OCR GCSE(9-1) June 2018 5H Q6]
Jack sent 15% more text messages in March than in February.
Jack sent 460 text messages in March.
How many more texts did Jack send in March than in February?
(4 marks)

77


Unit 3 (3c) exam practice
[Edexcel GCSE(9-1) June 2017 3F Q23b]
The value of Rita's house increased by 5%.
Her house then had a value of £210 000.
Work out the value of Rita's house before the increase.
(2 marks)
[Edexcel IGCSE May2016-4H Q3b]
In a sale, all normal prices are reduced by 20%
In the sale, the normal price of a tablet computer is reduced by 79 euros.
Work out the normal price of the tablet computer.
(3 marks)

78

Appendix C

Centre for Excellence Math - APP Questionnaire (POST)

This questionnaire is to gather data and information on the learners who have participated in the Centre for Excellence Math Action Research Project (App - Virtual Manipulatives).

Hi Tumay, when you submit this form, the owner will be able to see your name and email address.

* Required

1

Please read the following statements before beginning:

- I understand the purpose of the research project and my involvement in it.

- I understand that I may withdraw from the research project at any stage and that this will not affect my status now or in the future.

- I understand that while information gained during the study may be published, I will not be identified and my personal results will remain confidential.

- I understand that I may contact the researcher or supervisor if I require further information about the research. *

○ Please tick here to give you understand and agree to participate in this questionnaire

About you

This section is to gather some information about you

2

Which college do you attend? *

○ Christ the King Sixth Form College Emmanuel

○ Christ the King sixth Form College St Marys

○ Learning Enterprise College Bexley

○ Shooters Hill College

3

How old are you? *

○ 16

○ 17

○ 18

○ 19

○ 19+

4

What is the highest GCSE Maths grade you have already achieved? *

○ U

○ 1

○ 2

○ 3

○ 4

○ 5

○ 6

○ 7

○ 8

○ 9

5

How many times have you sat an actual GCSE Math paper? (including year 11) *

- ◯ Never sat a formal exam
- ◯ Once
- ◯ Twice
- ◯ Three times
- ◯ Four or more times

6

What is your main study programme or employment? (e.g. BTEC Business, A Levels, Construction etc) *

Enter your answer

The app

This section is about the app

7

What was the name of the Math app that you used in class? *

Enter your answer

8

In a few sentences, please explain how you found using the app? *

Enter your answer

9

Please rate each of the following statements from 1 to 5, with 1 being strongly disagree and 5 being strongly agree *

	Strongly Disagree - 1	Disagree - 2	Neither agree or disagree - 3	Agree - 4	Strongly agree - 5
I learned something new about Mathematics because of the app.	○	○	○	○	○
The app helped me to solve problems with less help from the teacher.	○	○	○	○	○
The app helped me concentrate on my work.	○	○	○	○	○
The app allowed me to understand Fractions better	○	○	○	○	○
I would recommend the app to a friend who is studying GCSE Maths.	○	○	○	○	○

10

On a scale of 1 to 5, how would you rate your confidence in the following topics: *

	Not confident at all - 1	2	3	4	Very confident - 5
Fractions	○	○	○	○	○
Decimals	○	○	○	○	○
Percentages	○	○	○	○	○
Ratios	○	○	○	○	○

After using the app, how confident are you in completing the question below?

We do not want you to calculate the answer, we want to know your confidence level. *

Write these numbers in order of size.
Start with the smallest number.

65% $\frac{7}{10}$ 0.68 $\frac{2}{3}$ $\frac{3}{5}$

0	1	2	3	4	5	6	7	8	9	10

Not at all confident — Extremely confident