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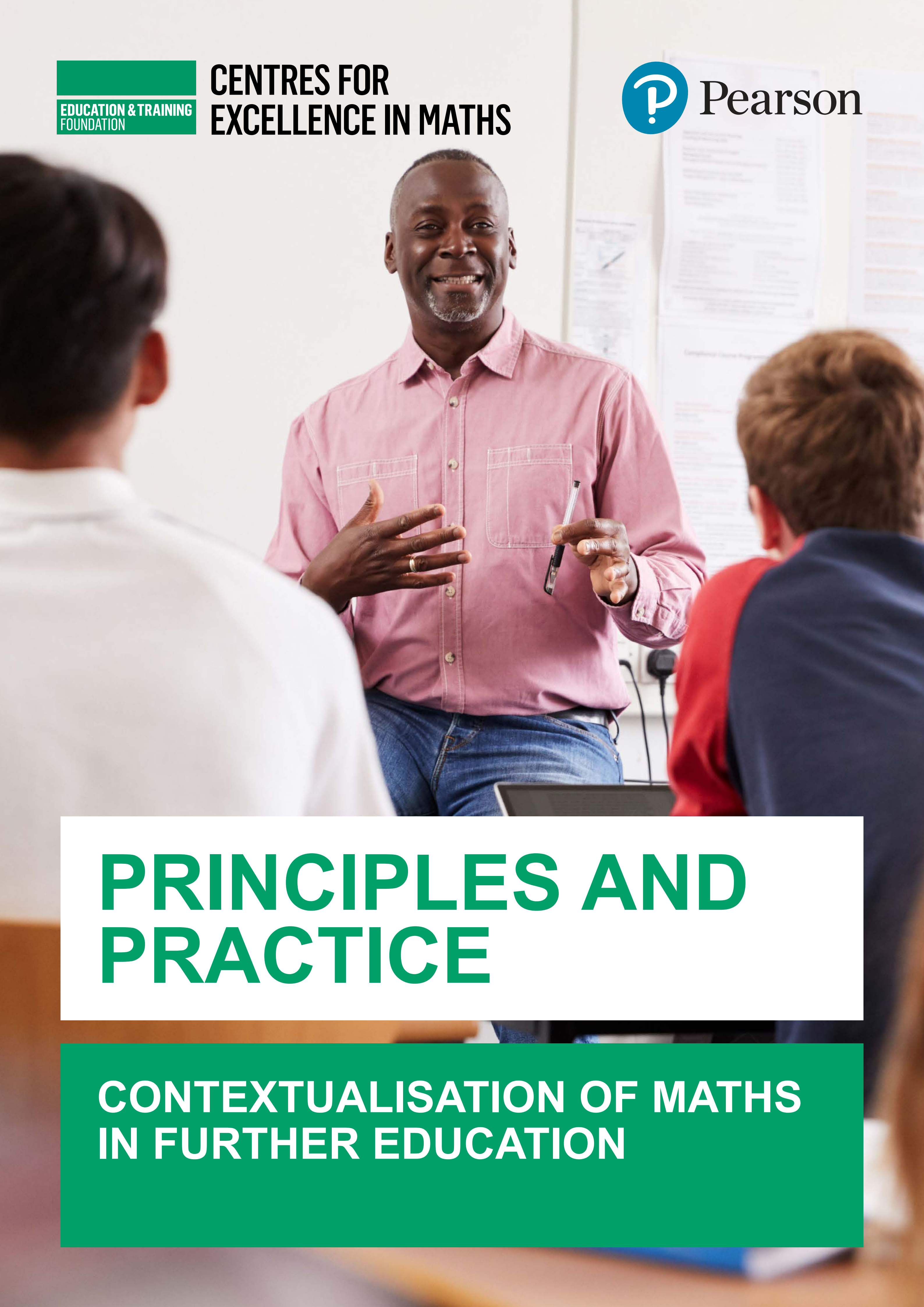

PRINCIPLES AND PRACTICE

CONTEXTUALISATION OF MATHS IN FURTHER EDUCATION

Contents

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Introduction

Programme overview

The Centres for Excellence in Maths (CfEM) programme, funded by the Department for Education (DfE), is designed to deliver a step change in maths teaching up to Level 2 in post-16 settings. A range of expert delivery partners and Centres for Excellence across the country are working together to design and develop evidence-based teaching approaches in four themes:

- an adapted mastery approach for the post-16 sector
- contextualisation – relating maths to real-world situations
- motivating and engaging learners
- using technology and data for maths teaching.

Centres for Excellence and delivery partners

The Centres for Excellence are 21 providers in the Further Education (FE) sector selected to drive innovation and improvement in both their institution and beyond. They are each establishing a network of ten or more partners to share practice across the FE sector. **You can see the 21 centres on the ETF website**.

The Education and Training Foundation (ETF), the national workforce development body for the Further Education and Training sector, is managing and leading the programme, working in partnership with a range of expert partners:

- **Association of Colleges (AoC)**
- **Behavioural Insights Team (BIT)**
- **Eedi**
- **Pearson**
- **PET-Xi**
- **touchconsulting**
- **University of Nottingham**
- **White Rose Maths**.

Trials and action research

An important aspect of the programme is to increase the evidence base of what works in maths teaching in the sector. The Centres for Excellence and the University of Nottingham are working together to carry out a range of trials and action research projects looking at aspects of the four themes. The focus of inquiry will change through the course of the programme.

The trials are supported by classroom resources, a professional development programme and collaborative networks. In 2019–20, the national trials are focusing on GCSE resit courses, with the key mathematical concepts outlined below ensuring that they are focused on improving attainment between GCSE Maths grades 3 and 4.

Key mathematical concepts

The CfEM programme has focused in on a small number of key mathematical concepts. These concepts aim to provide the maximum potential improvement for students. The University of Nottingham has used anonymised exam data from Pearson Edexcel to identify the concepts that might have the most impact on attainment. Data from diagnostic questions are then used to show teachers where to target their teaching on these key concepts by highlighting common knowledge gaps and misconceptions.

The key mathematical concepts are the same across the four themes:

- working with and understanding number
- multiplicative reasoning
- fractions, decimals and percentages
- basic algebra
- measure (area and volume).

Contextualisation

In the contextualisation theme, the focus for the key concepts is on providing insight into mathematical structure through use of context, deepening understanding in a few areas that can support student progress across a range of different situations.

Handbooks

There is a Handbook for each of the four themes. The Handbooks are evidence-based guides for teachers on current research and good practice. They can be used by any post-16 maths teacher looking for evidence-based approaches to teaching. Each Handbook outlines research for the theme, explains why it is important to maths teaching in the post-16 sector, and exemplifies how you might consider developing your teaching to reflect some of what we know from this research.

How to use the Handbooks

Key principles

Each Handbook is structured around key principles. These key principles have been created to reflect both the crucial points from the research and areas where there is the potential to make the biggest difference to teaching. Developed by the delivery partners and teachers from the Centres for Excellence, these key principles provide focus for each theme and allow consistency across the themes and outputs for the programme. They are not meant to restrict how you apply the themes to your teaching. Instead, they are intended to describe how well-informed approaches might apply in each theme and support you in changing your practice in line with these approaches.

Find out more

There are lots of links between key principles and between the different themes. 'Find out more' boxes highlight these links.

Key terms and ideas

The most important terms and ideas in sections are highlighted in separate explanatory boxes.

Further reading

The Further reading sections at the end of the Handbooks give you the opportunity to dig deeper into the research. These references are cited throughout the Handbooks, and particularly important documents can be found through hyperlinks.

Contextualisation

What is contextualisation?

Contextualisation in maths involves making direct links between maths and real-life situations and scenarios, or situations that students can imagine could occur. This gives students insight into the maths they are learning by using it in contexts they recognise and understand.

What research is there into contextualisation?

Research has investigated how context and maths interrelate in approaches to teaching and learning. The areas of **mathematical modelling** and applying maths have also been investigated. Much of this research has been carried out globally, in part in response to the Organisation for Economic Co-operation and Development (OECD) Programme for International Student Assessment (PISA) studies.[1] These studies focus on applying maths to solve problems.

The research spans all age groups, focusing on primary, secondary and tertiary sectors. Although some research has been carried out about contextualising maths in support of vocational learning, little, if any, has been conducted in the context of post-16 general maths courses at Level 2.

Key term

A **mathematical model** is a representation of a real-life problem using mathematical language.

Find out more

Models are explained further in Key principle 2: Staying in context.

What contexts are effective for teaching maths?

Contextualisation is an essential element of maths courses that aim to prepare students for effective and successful engagement in daily life, workplace settings, and other activities.[1]

Appropriate contexts provide meaning to abstract mathematical content and show students the usefulness and applicability of maths for the world outside of the classroom. There are several studies investigating different types of context. These show that 'real' or 'realistic' contexts should be used, as opposed to 'fictitious' contexts that students consider confirm their view that maths does not have uses in real life.[2, 3, 4, 5]

Find out more

Contexts are explained further in Key principle 1: Choosing contexts carefully.

Why is contextualisation important?

The results of the research are important for both students and teachers. Many students entering the FE sector have negative attitudes to maths, influenced by negative prior learning experiences at school. When students use maths in real and realistic contexts, there are three main benefits.

1. Relevance and motivation

Contexts provide meaning for mathematical content. This motivates students by showing how the maths they are learning can benefit them in current and future life and work opportunities, helping them to see maths as a useful subject.[6, 7] This is especially relevant in the FE setting where students need to be able to engage with problems in vocational settings.

2. Bridging access to abstract concepts

Contexts provide a bridge into abstract mathematical concepts and content. They give meaning to unfamiliar concepts and provide an anchor for visualising abstract mathematical relationships.

3. Supporting problem solving and reasoning

Contextualisation facilitates engagement with the types of concrete resources and problem solving that students will encounter in real-life and workplace situations. Engagement with contexts has the potential to improve students' capacity to engage with their reasoning and problem-solving skills.[5, 9]

Challenges of contextualisation

Classroom maths and realistic maths

Contextualisation is not a simple process. Classroom maths, maths used in everyday life and maths used in the workplace can be very different and it can be difficult to transfer mathematical knowledge from school or college to other contexts.[10]

The Realistic Mathematics Education (RME) approach, originating from the Freudenthal Institute in the Netherlands, is a possible solution to this. In the UK, Manchester Metropolitan University (MMU) investigated the impact of the RME approach on achievement and attitudes in post-16 GCSE Maths resit classes.* [5, 11, 12, 13]

It can be difficult to connect abstract mathematical ideas to life outside of the classroom

The research showed that student attitudes and beliefs about the value of maths and their problem-solving skills can be improved by:

- using contexts that draw on initial understanding of a realistic situation, or a situation that students can imagine could occur
- using tasks that have a high degree of realism while also providing a natural opening to understanding important mathematical ideas
- engaging with maths both in general real-world situations and in situations relevant to specific vocations.[14, 15]

Language and comprehension skills

For students, contextual situations need effective literacy, comprehension and interpretation skills. You can support your students by explicitly detailing key vocabulary and by modelling metacognitive strategies for how to interpret and filter relevant information, access relevant mathematical content, work systematically, communicate thinking, and reason, justify and critique their own and others' thinking and models.[16]

Find out more

Metacognitive strategies are explained further in Key principle 3: Staying in context.

*This project provided complete data sets for only a small number of students, and there were concerns from students and teachers about the pace of the approach. These concerns and other recommendations from the evaluation should be taken into account for future interventions.

Key principles

As mentioned in the Introduction, this Handbook is structured around the following three key principles.

Key principle 1: Choosing contexts carefully

1. ***Careful selection of contexts that best highlight the relationship between mathematical structure and context***

Contexts should provide insight into, not obscure, the maths. Students should not get too involved in the context. This requires careful task design and close monitoring of students as they solve problems.

Key principle 2: Staying in context

2. ***Ensuring students spend more time exploring how context and maths are interrelated***

Students should spend time thinking about how context and maths are interrelated. This provides opportunities to make connections and gain understanding of important concepts. They should spend time building from context to abstract reasoning.

Key principle 3: Active learning

3. ***Ensuring students engage in a range of different ways of working on maths, including those that involve peer collaboration***

Lessons should involve students in a range of different activities that require active collaboration with others. This enables them to make connections between contextual situations and maths, focus on mathematical structure and supports the development of conceptual understanding.

Key principle 1: Choosing contexts carefully

The importance of contexts

The use of contexts in maths is not new, but effective approaches use contexts as more than just interesting topic introductions. The **Realistic Mathematics Education (RME)** approach introduced previously is particularly effective in supporting students through a mathematical journey from contextualised to abstract understanding. It has demonstrated positive outcomes for students in both international and UK contexts. In the context of post-16 GCSE Maths resit classes, Manchester Metropolitan University trialled an RME approach in a study funded by the Nuffield Foundation.[11, 12, 13, 14]

Key term

Realistic Mathematics Education (RME) prioritises use of context and model-building as the main way of making sense of fundamental mathematical concepts. This enables students to visualise and internalise mathematical processes without resorting to rules and procedures which have no meaning.

Choosing contexts

When using the RME approach to contextualisation, it is important to choose contexts that are realisable and relevant. The contexts should provide a need for maths and give the relevant ideas a recognisable mathematical structure. The contexts should provide insight into the maths without obscuring it. Students should not get too involved in the contexts to the detriment of understanding the mathematical concepts.

Good contexts are ones that can be used[17]:

- to construct new maths as well as to apply previously learned maths
- both as the starting point and the source for learning maths[8]
- to encourage students to develop mathematical strategies and models which are helpful in a range of other contexts.

What the research shows

Types of context

There are several studies investigating different types of context.[1, 2, 3, 4]

- 'Real' contexts – directly reflect real-life settings and practices. These are essential for vocational courses where a primary aim is to explore real-world workplace practices.
- 'Realistic' contexts – show a simplified version of a real-life context, but still bear a reasonable resemblance to the real scenario. Real and realistic contexts are useful for maths courses such as Functional Skills and GCSE Maths, as they enable students to access abstract concepts and demonstrate useful applications of maths.
- 'Fictitious' contexts – bear little resemblance to real life.[19] The consensus is that these are not useful for engaging and motivating students in maths learning. Instead, they perpetuate the idea that maths is not useful and that situations have to be invented to demonstrate its value.

> **When used appropriately, contexts provide meaning to abstract mathematical content**
>
> University of Nottingham[18]

Approaches to using contexts in maths

The best contexts are simple and straightforward ones that show the maths clearly. The structure of the context should map to the structure of the maths. Some contexts are better than others for this and different context types can be used according to purpose.[1, 20] There are two main approaches, with different context types as part of each.

1. Contexts that give priority to mathematical structures. The aim of these contexts is to help students access the maths. This approach is better suited for preparing for GCSE Maths than in vocational contexts.

- Context-free teaching emphasises formal and abstract mathematical structures.
- Modified contexts are a version of an authentic context, modified to make it easier to align the context and the mathematical structure.
- Artificial contexts are invented contexts that may not be completely appropriate to real life but are designed to fit a particular mathematical point.

2. Context is used where the priority is engagement with the context, and the maths is used to help understand the context. This approach is better for solving problems in vocational contexts than in preparing for GCSE Maths.

- Authentic contexts are taken directly from real life, using, for example, news articles, bills or construction plans.
- Modified contexts are a version of an authentic context, simplified to make the context easier to access a particular problem.

Both approaches can be used in the FE setting, but students always need to be clear about the reason why a particular type of context has been chosen. They should be told clearly whether the outcome of what they are learning is mainly focused on the mathematical output or on understanding the context.

Putting contexts into practice

Adapting for different classes

When contextualising maths, you need to use your knowledge of your class and your students. A real or realistic context for students in one class will not be suitable for others. The following shows the type of contexts to use depending on the students in your class.

- For a class made up of students from the same subject area – you can use a vocational context familiar to them all. Even if students change to another career pathway, they should have enough interest in their current study programme to engage with related material.
- For a class made up of students from mixed subject areas – this class may be better suited to a more personal context, such as finance. Topics of interest would be financial situations they may currently be experiencing, such as buying driving lessons, or that they will expect to encounter in the next few years, such as working out salary and tax, or housing and utility costs. Alternatively, hot topics in the media could be used. However the context should always clearly align with the mathematical structure being taught.

It is important to use contexts relevant to post-16 students

Developing resources

The MEI's **Guide to developing contextualised teaching and learning resources**[21] can support you in developing contextualised resources for use in post-16 settings. The table shows some suggestions from the guide.

Different uses of contexts	What makes a good context	Examples of suitable contexts
• Consolidating skills • Learning new skills • Appreciating the usefulness of maths • Developing problem-solving skills • Engaging students	• Flexible and inclusive, so it meets the needs of all students • Allows for connections between different areas of maths (Note: good contexts can differ in an exam preparation course versus a vocational course focused on workplace preparation.)	Number • Costing materials in construction • Costing hair and beauty treatments • Personal finance Ratio and proportion • Staff-to-children ratios in childcare • Currency exchanges in leisure and tourism • Recipes in hospitality and catering

You may also find it useful to read through *Appendix 2* of MMU and Nuffield's **Investigating the impact of a Realistic Mathematics Education approach on achievement and attitudes in Post-16 GCSE resit classes**[7] for examples of teaching materials that show the RME approach to using contexts that map to mathematical structures.

If you are making your own resources then construction could be a suitable context to explore different types of problems, such as use of scale in building plans

Key principle 2: Staying in context

What does 'staying in context' mean?

Staying in context means working on maths in context for longer to help develop understanding of the maths. Students stay in context while they develop their mathematical understanding. This means that rather than using contexts just to set the scene, contexts are used throughout the lesson. This is successful and effective when contexts are deliberately designed to have the same fundamental structure as the mathematical structure.[17]

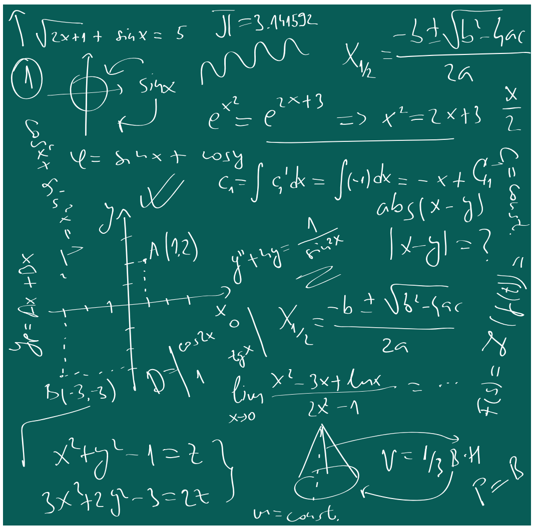
What the research shows

Some students struggle with maths because:

- it appears to them as a collection of random rules and procedures
- activities that are supposed to help them learn do not engage them in real mathematical thinking[22]
- often context may be used to introduce a topic but is then dropped and students move on too quickly to working with abstract mathematical ideas.[17]

It is much more effective to use real contexts as a route into maths and as a means of developing students' understanding. When students work in context, rather than in the abstract, they are not just 'doing maths', they are using maths to solve problems.

The MEI **Introduction to Realistic Mathematics Education and how it facilitates learning mathematics** states that students 'do not need to resort to memorising rules and procedures which have no meaning for them. "Maths" and "context" are not separated.'[17]



Maths can often feel like a random set of rules for students

A main feature of the RME approach is that all the maths is developed through contexts. If this happens, alongside a total commitment to enable students to 'make sense' of the subject matter, students are not only more motivated but make better progress.

In the Netherlands, where RME is used in 95% of schools from pre-school to late secondary, maths attainment has been consistently strong over recent years.[17] This has been measured in PISA and Trends in International Mathematics and Science Study (TIMMS).

> **Experience shows that, through staying connected with the context, students are able to continue to make sense of what they are doing.**
>
> MEI[17]

Putting it into practice

Instead of introducing a context, dropping it, then reintroducing it, a 'contextualised' rather than a 'typical' approach[17] can be used, keeping the same context throughout the lesson.

In a contextualised approach, students:

- explore the maths in context with a sequence of realistic problems
- stay in context and use maths to solve a problem, making sense of what they are doing
- move to working with mathematical models based on that context[17]
- gain enough insight into the maths to apply mathematical models in other contexts.

In a typical approach[17]:

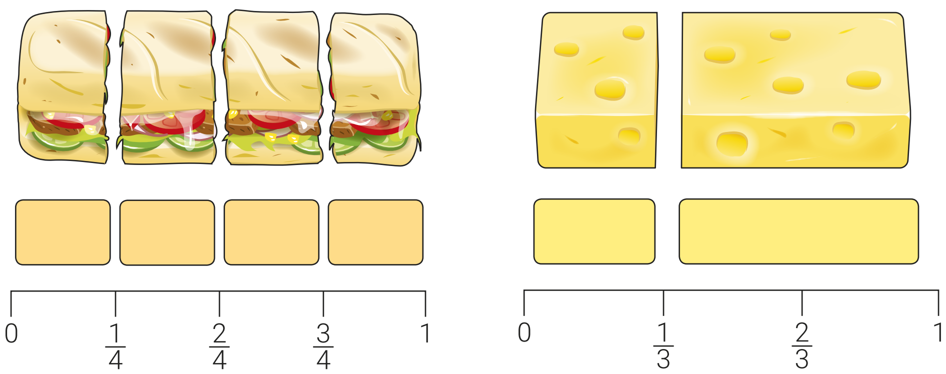
- a context is used for motivation and 'access' and then is dropped
- the maths is taught and practised out of context, giving students the impression that it is unrealistic
- contexts are then reintroduced and the maths is practised within these contexts.

Mathematical models

Models can be used to bridge the gap between the context and the formal maths. You should allow models to emerge from a context.[17]

A model may be just a representation, for example, a picture of the context. Later, the model can become a more sophisticated mathematical tool such as a number line or ratio table.

Students begin to recognise that the same model can be used in a variety of situations and to structure solutions to many kinds of problem.



Here you can see how models can be used to progress from the context to formal maths.

Key principle 3: Active learning

From passive receivers to active participants

Active learning occurs when students engage in a range of different activities that require active collaboration with others to develop conceptual understanding.

Although contextualisation is about choosing the right context and then staying in that context to embed the learning, it is also about encouraging a different type of student behaviour in the classroom. It is not just about how the students are taught; it is about how they learn and their relationship with maths.

This requires a move from the passive approach, in which the teacher simply demonstrates the maths methods and students practise rules and procedures, to a more active, problem-solving approach.

What this means for students

Students should be encouraged to show a different kind of behaviour in the classroom by engaging directly in problem-solving processes. Effective problem-solving in contextual scenarios requires students to engage with the context, the mathematical content and their own competencies.[7]

Students should…[23]	This will help students to…
• Think independently • Explain their strategies • Question each other's approaches	• Learn to communicate their thinking • Use **metacognition** strategies to deconstruct, monitor and direct their own learning • Identify strengths and weaknesses in the way they learn to become better students • Reduce their reliance on memory

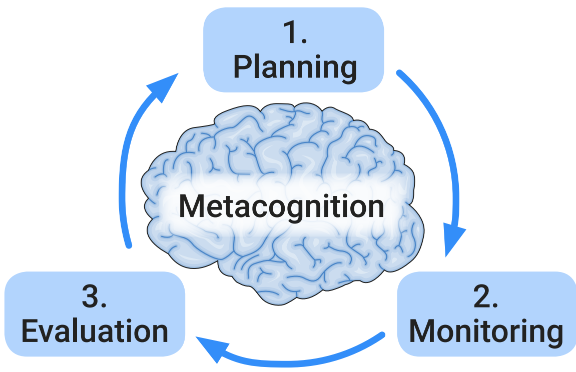
Key term

Metacognition is the ability to independently plan, monitor and evaluate one's own thinking and learning.[23]

Metacognition

Students can learn to use metacognition strategies through comprehensive worked examples which model elements of problem-solving practice involving planning, monitoring and evaluating.[16] They learn how to:

- interpret and filter relevant information (answer the question 'What is useful here?')
- access relevant mathematical content (answer the question 'What do I need to use?').



Metacognitive regulation cycle

What this means for teachers

A new classroom culture and different kinds of classroom interaction can help change the nature and pace of the work, allowing students to effectively problem-solve contextualised maths questions. Some strategies you could use are:

- Facilitate whole-class discussions and collaborative group work, sharing and evaluating ideas. Students can share their own life experiences, which may help others make sense of the contexts being used.
- Provide opportunities to share and challenge models, strategies and thinking. When a context that is familiar to students is used in a question, they may already have some informal strategies for solving the problem. You can then help students develop them into formal procedures for solving maths problems.
- Use careful **questioning** techniques, including **turn-taking** and ensuring longer **wait time**.
- Use strategies to develop students' comprehension, interpretation and metacognitive skills. The absence of formal maths at the start of a question encourages students to interpret the context they are given.
- Teach students to use and compare their existing knowledge with different approaches to solving problems.
- Encourage students to take responsibility for, and play an active role in, their own learning.

Key term

Questioning skills probe students' understanding of a concept. This means moving away from the normal pattern of closed questions and one-word answers which can feature in teacher–student interactions.

Turn-taking allows students more control over the flow of class discussions; the teacher does not always have to decide who speaks next.[11]

Extending **wait time** allows students more time to think and respond and increases the likelihood of students explaining and reasoning.

It is important to think of new ways you can interact with your students

Key principles in practice

The intention here is to exemplify the key principles of contextualisation for post-16 maths teaching. Here is a GCSE question that could be used when teaching in this way. We then look at how the question can be used to demonstrate each key principle.

Here is the list of ingredients for making 30 biscuits.

Ingredients for 30 biscuits

225 g butter
110 g caster sugar
275 g plain flour
75 g chocolate chips

Lucas has the following ingredients.

900 g butter
1000 g caster sugar
1000 g plain flour
225 g chocolate chips

What is the greatest number of biscuits Lucas can make?

You must show your working. **(3 marks)**

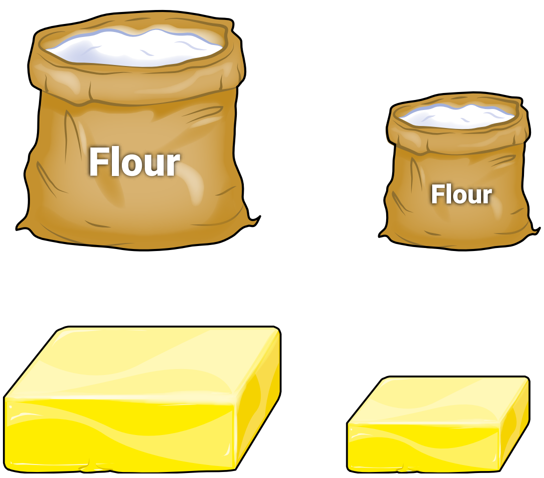
1MA1/2F, June 2018, Q17

Key principle 1: Choosing contexts carefully

To solve the question, students need to draw on their knowledge and understanding of cooking and of proportion. The underlying mathematical structure is one of direct proportion. Check that students understand that changing the proportions of the ingredients would produce unsatisfactory biscuits.

The key principle here is that the context and question focus student thinking on direct proportion, an important concept in maths. That is, '$y = kx$' (although when working with students it may not be advisable to refer to this relationship using algebra). There is a constant multiplier to get from the number of biscuits to grams of butter.

For example, for every 30 biscuits, 225 g of butter is needed. For one biscuit you need 225 ÷ 30 = 7.5 g of butter. You can work out the constant multiplier for each of the different ingredients. This is the mathematical structure to highlight. When first working with students on such problems, it may be useful to use a question that is set in a context likely to have more meaning to the group of students, even though many students are familiar with such questions based on recipes.



It is important to think about the context of the question

This question can be replicated in other contexts by thinking of a scenario where a finite amount of resources are used during a given time period. For example, in the context of the travel and tourism sector, the question could focus on the resources needed to service rooms in a hotel. You could simplify the example by reducing the number of different items needed to service the room. You would need to think carefully about which numbers to use to ensure students focus on understanding proportion, rather than being distracted by complicated multiplication.

> A hotel needs the following to service 10 rooms.
>
> 20 bath towels
> 10 hand towels
> 50 pillowcases
> 20 bed sheets
>
> The hotel has the following amounts of items available to use:
>
> 300 bath towels
> 140 hand towels
> 470 pillowcases
> 210 bed sheets
>
> How many rooms can be serviced using the available resources?
>
> You must show your working.

Key principle 2: Staying in context

When presented with the original question, some students may want to start working out an answer right away. You can remove the last part of the question, leaving just the two lists of ingredients, to ensure students focus on the underlying mathematical structure.

Mathematical structure

To begin the process of examining the mathematical structure of proportion, ask students questions that allow them to draw on their understanding of the context to help make sense of the maths.

Context questions

- If you want to bake 60 biscuits what do you need to do to work out how much you need of each of the ingredients? Why is that?
- What if you want to bake 90 biscuits? What do you need to do to work out how much you need of each of the ingredients?
- What if you wanted to only bake 10 biscuits?
- What if you wanted to only bake one biscuit?
- Now, what if you wanted to only bake five biscuits? What do you need to do to work out how much you need of each of the ingredients?
- If you had 75 g of butter how many biscuits could you bake? How do you work that out?

You need to spend time in context so that students think about mapping the maths to the real situation. Keep using the language of the context and encourage students to refer back to the real situation to help them do the calculations. Drawing out contrasts can also help in this. In real life you might add chocolate chips, but you would not do this in a maths question.

Non-context questions

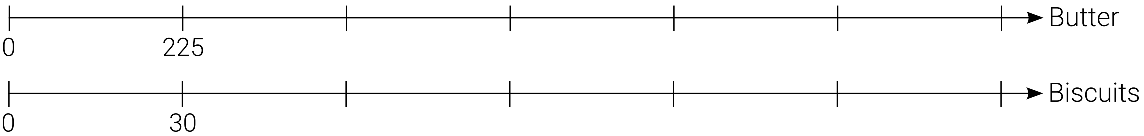
Some non-context specific questions that will help a student think about the underlying maths include:

- What questions do you have about this information?
- What type of maths skills might be involved in solving a question using this information?
- How could maths be useful to you in this context?
- What possible questions could you ask using this information? (You need to hide the actual question to make this meaningful.)

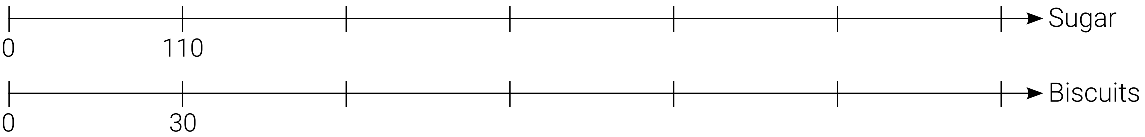
Number lines

Number lines can be used to help answer the question and can encourage students to think about the underlying structure of the question. Additionally, such representations can help you explore students' thinking and make connections across different approaches.

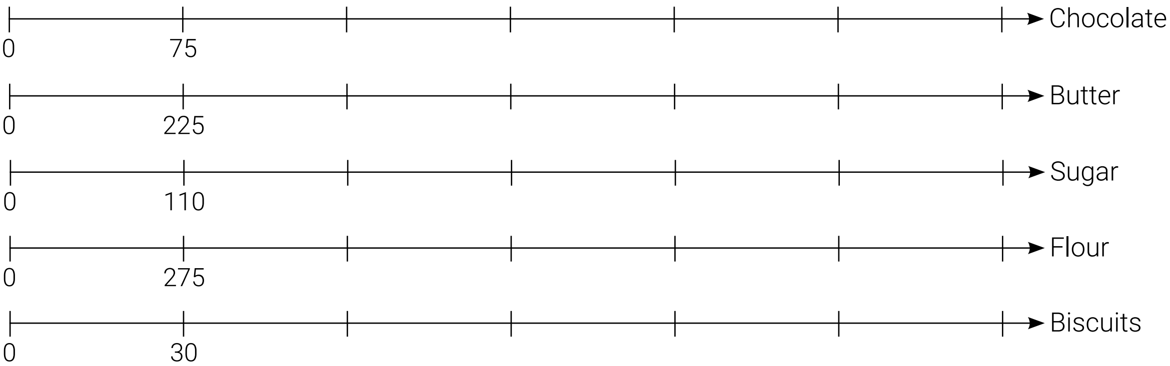
Start by asking students about the situation represented by a double number line diagram that shows the relationship between 30 biscuits and the amount of butter required by the recipe. The previous questions can be used in conjunction with the double number line representation.



You may wish to shift the focus of discussions to the relationship between the number of biscuits and another ingredient – sugar in this example.



At some point you may wish to introduce a diagram that brings multiple number lines into a single diagram.



Number lines like these can be used for many types of proportion question.

The student-friendly mark scheme below for the ingredients question suggests a different form of solution. This method arrives at the correct answer and is possibly quicker, but the number line is a technique that brings out the mathematical structure in a more replicable way across a variety of question types and context-based problems.

(Total 3 marks)

Working or answer an examiner might expect to see	Mark	Notes
900 ÷ 225 = 4 1000 ÷ 110 = 9.091 (to 3 decimal places) 1000 ÷ 275 = 3.636 (to 3 decimal places) 225 ÷ 75 = 3	P1	This mark is given for a process to find the number of batches for at least three of the ingredients listed
3 × 30	P1	This mark is given for a complete process to find the maximum number of biscuits
90	A1	This mark is given for the correct answer only supported by correct working

Key principle 3: Active learning

This key principle encourages the development of varied teaching approaches so that lessons involve students in a range of activities. Learning can be enhanced when students interact with each other and collaboratively work towards improving their knowledge together. Effective collaboration has to be planned for, and students have to be carefully guided and supported. You should share your expectations at the outset and give clear ground rules about how students should work. For example, if you ask students to share their way of working with a partner, the partner should be told that they will need to listen sufficiently to repeat the explanation back to them to check for understanding.

Collaboration

Here are some possible ways of engaging students with tasks that support collaboration.

Working in pairs

Ask students to work in pairs. Ask one of the pair to draw a diagram to show how Lucas could calculate the amount of butter needed for 40 biscuits, then explain their thinking to their partner. The other partner should now use the first student's way of thinking to calculate the ingredients for 75 biscuits and echo this back to the first student. Students could then be asked to swap roles so that the original second student now first works on calculating the amount of sugar for 20 biscuits.

Student-led discussion

Ask students the following questions, which can lead to a more open, student-led discussion about the underlying maths concepts.

- How would you draw a diagram for 36 biscuits?
  - Students have to think about a way of moving from the number 30 to the number 36. Given time, most will be able to think about dividing by 5 and multiplying by 6.
- What property of the number 30 allows you to find the ingredients for 36, 40 or 45 biscuits?
  - This question is similar to the previous question, but a little more abstract. It really makes students think about the properties of the number 30 that make it possible to convert easily to 40 or 45 or 36.
    - For 40: 10 is a factor of 30 and 40 is a multiple of 10.
    - For 45: 15 is a factor of 30 and 45 is a multiple of 15.
    - For 36: 6 is a factor of 30 and 36 is a multiple of 36.

Group discussion

Encourage discussion between groups. Some possible strategies include the following.

- Have the pairs or groups exchange their solutions and peer-assess them.
  - This puts students in a position of having to follow another student's working. This should help deepen their own understanding and also highlight to them what makes a solution easier or harder to follow.
- Have students create a mark scheme for the question they are marking and say how many marks they would award their answer.
  - This develops the previous idea a step further. Now students will have to think about how they would score different answers and what exactly they are looking for in an answer.
  - An extension of this activity would be to have the group create an answer that would receive, say, 2 out of 3 marks. Now they will have to think about common misconceptions and where students are likely to go wrong in answering a question.

Groups can exchange solutions and working out

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