Lesson plan

Using frequencies and probabilities

Contents

1. Rationale

Students at this level are likely to be familiar with the probability scale and to know how to calculate theoretical probabilities. They usually understand that the relative frequency of an event can differ from its theoretical probability, but can sometimes focus on procedures rather than developing their understanding of the 'why'. In this lesson frequencies are explored and represented on a frequency tree diagram, and from this example, a probability model is developed. The shift from the actual frequency to the theoretical probability **develops students' understanding of mathematical structure** (Key Principle 1). Students work with decimals and fractions within these models, encouraging them to **make connections** (Key Principle 3) with other areas of mathematics.

Carefully designed solutions are presented to students within the lesson to expose common misconceptions and provide opportunities to **establish what students already know** (Key Principle 2), as well as **promoting a collaborative community** (Key Principle 5), where students are encouraged to contribute and share their own ways of working.

2. GCSE curriculum

Probability

P1 record, describe and analyse the frequency of outcomes … using tables and frequency trees

P8 calculate the probability of independent … combined events, including using tree diagrams and other representations, and know the underlying assumptions

3. Lesson objectives

- Interpret and construct frequency tree diagrams
- Use approximate values to produce a probability model
- Calculate probabilities using probability tree diagrams
- Use representations to reveal mathematical structure

4. Starting points

The lesson assumes that students have worked with both theoretical and experimental probabilities and have some fluency with converting between fractions, decimals and percentages.

5. Research questions

Pedagogic focus

In which ways do the activities of the lesson help students to believe that everyone can succeed?

Maths focus

How do the collaborative activities in the lesson help to develop students' confidence?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	10	Introduce the context of developing self-administered tests to detect whether or not someone has a particular disease and trialling them to see how successful they are at giving a correct diagnosis. Ask students to create a table of the results of an initial trial and make connections to a frequency tree representation.	‘Cards’ and ‘Lab trial results’ handout (A3) Calculators Slides 2–6
Explore/ Discuss 1	10	Tell students that an official trial involving more participants is carried out. Ask students to complete a frequency tree diagram for the results and explore how to represent the frequencies using a probability tree diagram.	Mini whiteboards Calculators Slides 7–11
Explore 2	15	Introduce scaling up for a bigger population. Ask students to work in pairs to complete the frequency tree and probability tree diagrams on the handout and to use the probabilities to scale up for 10 000 people.	‘Scaling up’ handout Calculators Slides 12–13
Discuss 2	10	Discuss the missing values and explore common misunderstandings. Emphasise the mathematical structure of a probability tree diagram.	Slides 14–15
Explore/ Discuss 3	10	Introduce the new context of sporting injuries and explore multiplying along the branches of a probability tree diagram.	Calculators Slides 16–18
Explore 4	15	Ask students to work in pairs to complete missing values on the probability tree diagrams for two different types of injury and use the probabilities to identify how many people we would expect to have two injuries in a year.	‘Using probabilities’ handout Calculators Slide 19
Discuss 4	10	Hold a class discussion. Ask students to explain their thinking and check their understanding.	Calculators Mini whiteboards Slides 20–23
Practice questions/ Discuss 5	10	Ask students to complete two practice questions and discuss their thinking.	‘Practice questions’ handout Calculators Slides 24–25

7. Teacher guidance

Introduction

Aim	To introduce the context and frequency tree representation
Materials	‘Cards’, ‘Lab trial results’ handout (A3) and calculators
Slides	Slides 2–6
Time	10 minutes

Encouraging the belief that all students have something to contribute is an important part of the mastery approach (Key Principle 5). At the start of the lesson each student is given a card with a result from a lab trial and asked to place it in a results table.

What students might do and what you might do

Slide 2 Tell students that Starla and Toby work in a medical science department developing tests to detect whether or not someone has a disease. They have a blood test for a specific disease, but they want to develop tests that can be self-administered. The tests are trialled to see how successful they are at correctly identifying whether or not someone has a disease.

Slide 3 Starla and Toby have developed a test and trial it with the 20 people that work in their lab. Their colleagues have a blood test first, to confirm whether or not they have the disease.

Distribute to each student one of the ‘Cards’. If there are fewer than 20 students in the class give some students more than one. Briefly discuss what false positive and false negative mean. Explain to students that treating someone for a disease they don’t have (false positive), may, in some cases, have no ill effects, but someone with a disease not receiving treatment could pose more of a problem (false negative).

Slide 4 Use the ‘Lab trial results’ handout enlarged to A3 size. Ask students to place their cards in the right place in the table. Check that students understand where to place their cards.

Slide 5 Once all the cards have been placed ask students questions to help to complete the totals in the table. Refer to the completed table and ask students whether or not they think this is a good test. Emphasise that if the test was 100% accurate then everyone with the disease would get a positive test result and everyone without the disease would get a negative test result.

Slide 6 Ask students how else the data could be represented and introduce the frequency tree diagram. Ask students to identify which value goes in each oval. Compare the two representations.

Explore/Discuss 1

Aim	To explore creating probability models
Materials	Mini whiteboards, calculators
Slides	Slides 7–11
Time	10 minutes

In this part of the lesson, probability tree diagrams are introduced and the **mathematical structures** of frequency tree and probability tree diagrams are explored (Key Principle 1). Students' understanding of how to use probabilities to model frequencies is developed by **exploring links** with rounding and approximation (Key Principle 3). Discussing probability models that have mistakes encourages students to **share their understanding and develops their confidence** (Key Principle 5).

What students might do and what you might do

Slide 7 Tell students that having trialled the test on their colleagues, Starla and Toby carry out an official trial involving 1001 people. 202 people have the disease and 151 of them get a positive test result. 398 of those that don't have the disease get a positive test result. Ask students what the remaining values should be. Encourage them to use a calculator.

Emphasise that for a frequency tree diagram, the numbers at the ends of arms from the same starting point must add up to give the starting point value, so the missing values can be found by subtracting. Establish too that the numbers at the ends of each set of arms always add up to the total (e.g. 202 + 799 = 1001 and 151 + 51 + 398 + 401 = 1001).

Slide 8 Introduce the probability tree diagram. Ask students why 0.2 has been used, rather than 0.202, and explain that the frequencies are real data and often result in numbers that are not nice to work with. 202 is approximately 200 and 1001 is approximately 1000 so 200 ÷ 1000 = 0.2 gives a good approximation.

Explain that we use approximate values to represent the trial data in a probability tree diagram and use these values to make reasonable predictions of what might happen if the tests were used beyond the trial.

Ask students to work in pairs to complete the probability tree diagram for the official trial results, writing a decimal value on their mini whiteboards for each of the letters a to e.

Slide 9 Ask different students to share a value for letters a to e and establish the values that provide the best probability model. Emphasise that the probabilities on each pair of branches add up to one.

Slide 10 Check that students can explain how the values of 0.15, 0.05 and 0.4 have been found and highlight that on probability tree diagrams the probabilities on each pair of branches must always add up to one.

Slide 11 Show students the diagram and ask them if it is wrong. Technically it is not wrong, and the decision to round to one decimal place rather than two decimal places, as in the solution on Slide 9, is a choice.

Explore 2

Aim	To use scaling to make predictions for bigger populations
Materials	'Scaling up' handout and calculators
Slides	Slides 12–13
Time	15 minutes

In this section of the lesson, students use probabilities to make predictions about what would happen if the tests were used by a larger number of people. Students work in pairs and are encouraged to explain their thinking to each other to promote a **collaborative approach** (Key Principle 5).

What students might do and what you might do

Slide 12 Explain that we can use probability models to scale up and predict what might happen if we extend to a bigger population, for example, the whole country. Ask students to use the probabilities to make some predictions about what would happen if 10 000 people used the tests.

Ask students how many people are likely to have the disease ($0.2 \times 10\,000 = 2000$) and check that they understand where 2000 comes from.

Now ask students to work out what goes in the other ovals and write the numbers on their mini whiteboards.

Go through the answers with them (click for them to appear), allowing enough time for them to explain their methods. It is important to **value and make sense of students' different ways of working** to increase students' self-belief (Key Principle 5).

Emphasise that the numbers at the ends of arms from the same starting point should add up to the starting point value (so the 500 and 4000 values can also be found using subtraction).

Slide 13 Tell students that Starla and Toby have developed tests for two more diseases (A and B) and they carry out a trial for both tests.

Give each pair of students a copy of the 'Scaling up' handout. Ask students to work in pairs to complete the frequency tree and probability tree diagrams and then use the probabilities to scale up for 10 000 people. Encourage students to explain their thinking to each other as they complete the rows.

Discuss 2

Aim	To check student understanding
Slides	Slides 14–15
Time	10 minutes

This section of the lesson provides an opportunity to discuss students' work and to explore some possible misconceptions. It is important to **promote a collaborative community** where students are confident to share their ways of working (Key Principle 5).

What students might do and what you might do

Slide 14 Once students have had sufficient time on the task, hold a class discussion. Ask students to explain how they worked out the missing values in the probability tree diagram. Emphasise that the probabilities on each pair of branches add up to one.

Check that students can explain why 5000 people are likely to have the disease.

Tell students that someone states that 2000 people are likely to get a false negative. Ask them why the statement is incorrect (they have multiplied 10 000 by 0.2 rather than 5000 by 0.2) and what the correct number should be (1000).

Slide 15 Tell students that there are some mistakes in this probability tree diagram. Ask students how we can tell that some of the values are not correct just by looking at the diagram (the probabilities on each pair of branches don't add up to one).

Ask students to identify the incorrect values and discuss how they could have been obtained (by truncating decimal values obtained from dividing frequencies rather than rounding). Emphasise that when we know one of the values on a pair of branches we can subtract from 1 to find the other, rather than needing to go back to the data.

Check that students can explain why 7500 people are likely to not have the disease and 3000 people are likely to get a false positive.

Explore/Discuss 3

Aim	To calculate probabilities using probability tree diagrams
Materials	Calculators
Slides	Slides 16–18
Time	10 minutes

A new context is now introduced and students use probability tree diagrams to calculate the probabilities of combined events. **Building on prior learning** is an important part of the mastery approach (Key Principle 2) and in this part of the

lesson **students use and further develop their understanding** as they apply it to problem solving (Key Principle 4).

What students might do and what you might do

Slide 16 Introduce a new scenario. Tell students that concussion, where someone receives a blow or jolt to the head, is a common injury in playing hockey and repeated concussion injuries can be serious.

The probability of having an injury from playing hockey that results in concussion in any one season is 0.4. Each year there are two hockey seasons: a winter season, which runs from September to April (with a winter break from December to February) and a summer season, which runs from May to August.

Tell students that the missing values for the probability tree diagram are shown on the sticky notes. Ask them to explain why the values are either 0.4 or 0.6 (as the probability of having an injury from playing hockey that results in concussion *in any one season* is 0.4, so is the same for both the winter and the summer seasons). Reinforce structure and how the probabilities on each pair of branches add up to one. Ask different students to come to the board and drag a sticky note to the correct place in the tree diagram.

Slide 17 Ask students to calculate the probability of a player having an injury resulting in concussion in the winter and then also in the summer season.

After a couple of minutes check that students have identified the correct branches. Students may think that they need to add the two 0.4s to get 0.8, so it is important to address this.

If students are struggling to understand why we need to multiply, discuss an example involving 100 injured players. We would expect that 40 (0.4 × 100) of the 100 injuries in the winter season would result in concussion. We only work with these 40 injuries now, as we are interested in injuries that result in concussion in both seasons. We would expect (0.4 × 40 =) 16 of the 40 players who had a concussion injury in the winter season to have an injury resulting in concussion in the summer season so 16 out of 100, a probability of 0.16.

Establish that multiplying along the branches gives 0.4 × 0.4 = 0.16.

Slide 18 Ask students to apply the probability of a player having an injury that results in concussion in both the winter and the summer season to two different scenarios. Establish that there is a chance that 8 of the 50 injured members and 2400 of the 15 000 injured students will have two concussion injuries from playing hockey in any one year.

Explore 4

Aim	To use probability tree diagrams to calculate the probabilities of combined events
Materials	‘Using probabilities’ handout and calculators
Slides	Slide 19
Time	15 minutes

Students are encouraged to **work collaboratively** in this part of the lesson and share their understanding (Key Principle 5) as they complete probability trees and use them to calculate the expected number of people for a particular outcome. **As students apply their understanding to different contexts, their fluency and understanding of the key ideas** can be developed (Key Principle 4).

What students might do and what you might do

Slide 19 Give each pair of students a copy of the 'Using probabilities' handout. You might like to hold a brief discussion about the fractions in row D, depending on your students. For example, they may need reminding that $\frac{2}{3} + \frac{1}{3} = 1$, or that when fractions are multiplied, the numerators are multiplied and the denominators are multiplied. Some students may want to convert the fractions to decimals; consider whether you will discourage them from doing this.

Ask students to work in pairs to complete the probability tree diagrams for rows C and D and then use the probability model to complete the final column. Encourage students to explain their thinking to each other.

Discuss 4

Aim	To check students' understanding and expose common misconceptions
Materials	Calculators and mini whiteboards
Slides	Slides 20–23
Time	10 minutes

During this part of the lesson it is important to **identify students' existing knowledge** and check their understanding (Key Principle 2). Discussing pre-prepared solutions with possible mistakes can **encourage students to share their understanding and build confidence** (Key Principle 5).

What students might do and what you might do

Slide 20 Check students' understanding of multiplying along the branches to determine the probability of having two injuries involving a broken bone during the year. Establish that we would expect 1800 (0.0225 × 80 000) of the 80 000 injured registered hockey players to sustain two injuries involving a broken bone.

Slide 21 Ask students to explain what mistake has been made and how they know. Reinforce to students that the probabilities on each pair of branches always add up to one.

Slide 22 Remind students that probability can be described using fractions (and percentages) as well as decimals. Discuss the missing values and check students' understanding.

Slide 23 Bring students' thinking together by asking them to identify whether these five statements are true or false. Ask them to write 'True' and 'False' on their mini whiteboards and note down the statement number under the correct heading.

When discussing statement 1 emphasise the probability scale and what it means when the probability of an event is equal to one.

Ask students to explain the difference between frequency tree and probability tree diagrams when discussing statement 2.

With statement 3 emphasise both the links between fractions, decimals and percentages and how connections can be made between the concepts of probability and ways of expressing a part of a whole.

When discussing statement 4 highlight that a frequency tree diagram is a way of representing data that can be displayed using a two-way table.

Statement 5 introduces a theoretical probability, which, while not discussed in this lesson, should be familiar to students. Emphasise that probability is not definite; it provides us with a model.

Practice questions/Discuss 5

Aim	Students apply their knowledge to an unfamiliar task
Materials	'Practice questions' handout and calculators
Slides	Slides 24–25
Time	10 minutes

In this part of the lesson students apply what they have learned in the lesson to some practice questions. Distribute to each student a copy of the 'Practice questions' handout. Give students a couple of minutes to work on the questions individually and then discuss their thinking.

What students might do and what you might do

Slide 24 As students work on the first practice question, encourage them to note down any calculations they use. Discuss students' thinking and check that the correct values ('22 and 23' and '4, 18, 7 and 16') have been completed.

Slide 25 Discuss students' thinking when completing the second practice question. Check students' understanding that in probability tree diagrams the probabilities on each pair of branches should always add up to one. Establish that the 0.25 should be 0.35 (as 1 – 0.65 = 0.35) and that the probabilities on the first set of branches for the second throw need to exchange places, as the probability that the dice will land on 4 is 0.65 (and that it will not land on 4 is 0.35).