CENTRES FOR EXCELLENCE IN MATHS

Mathematics in context

Do Ratio Tables help students make sense of maths problems?

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Working in partnership with the Education and Training Foundation to deliver this programme.

About CfEM

Centres for Excellence in Maths (CfEM) is a five-year national improvement programme aimed at delivering sustained improvements in maths outcomes for 16–19-year-olds, up to Level 2, in post-16 settings.

Funded by the Department for Education and delivered by the Education and Training Foundation, the programme is exploring what works for teachers and students, embedding related CPD and good practice, and building networks of maths professionals in colleges.

Summary

Over the past years, the mathematics education has seen lengthy and somewhat unproductive debates whether or not to teach for proficiency or conceptual understanding (Frykholm,2013). In reality, it is important to have them both.

It has to be said that ratio table allows learners to take a lead in finding a strategy to solution that is resonate to their strengths and mathematical thinking. We observed that some students took more than others with ratio table to solve a certain problem, thus allowing students to use more than one strategy is a vital part of using this approach when teaching.

Our approach to the AR into how and why ratio tables work took us on a journey where students were integral part of everything we did. We tested their prior knowledge and understanding to find that they have mainly had been exposed algorithmic proficiency education. We then delivered lessons championing contextualisation, use of ratio tables and helping them to make sense of exactly the same concepts they were taught before, but in a different and more efficient way. We have seen students start to think not only about “how” but “why”, and think about mathematical relationships.

We found that students just did not relate with mathematical problems and they just did operations with numbers, nothing more. Having taught students various strategies on using ratio tables, we have seen improvement in students uplift in making connections, and solving range of mathematical problems using ratio tables where they would not have used them previously.

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Background

Our Study

Following a review of the literature in the area of Realistic Mathematical Education (RME), this study proposes to investigate the following question:

What impact does Realistic Mathematical Education has on students improving their grades from grade 2 to 3, and grade 3 to 4?

The design of the research process will be guided by the following objectives:

- How does encouraging learners to develop informal strategies for solving problems help their understanding?
- How does building on their strategies increase a more thoughtful engagement with novel problems?
- How does different representation contribute in learner understanding?

There were 4 teachers who worked on this project, two from Harlow and other two from the Northampton College. Both colleges' staff have a significant experience in working together on the CfEM projects, which has made the collaboration amongst them seamless.

The Challenges

In post 16 education, we have students who come to us having previously failed to obtain a GCSE pass at the end of their secondary school education. This creates a challenge for us, as FE resit teachers, to ultimately teach the same content the students were taught in the past. To teach the students same thing, but expect the different results, one must look at to be creative and try different approaches to make sure these students understand the concepts this time.

The pressure of showing the students' progress as expected can often get in the way of the of using context in mathematics teaching. This pressure can grow even more if the students are to be put forward for the Nov resits, forcing the delivery of material with much quicker pace.

The aim of the project

The various research showed that students lack of understanding mathematical concepts often comes from the fact that they fail to see them in real life context. We have chosen the Realistic Mathematics Education (RME) strategies in order to fill the gaps in students lack of understanding of said concepts. The RME approach, we had hoped, would help our learners to make sense of things when relating them to real life scenarios, and ultimately result in improved grades across the colleges.

From the extensive research that was conducted by the researchers at Manchester Metropolitan University (MMU) and the recommendations on how to incorporate the RME strategies into the delivery, we have decided to mainly focus on using Ratio Tables as our intervention strategy.

Why focus specifically on using Ratio Tables?

- It seemed an excellent opportunity to find out how our learners actually think when presented with a mathematical problem (“Say what you see” and “what else do you know?”)
- A perfect toolkit for raising learner expectations to demand that the maths should make sense, rather than just doing something with the numbers
- It presented a much more interesting & interactive experience for learners and teachers alike, as well as using a dialogic approach to develop learners’ mathematical reasoning & communication skills

Literature Review

What is the RME approach?

Realistic Mathematics Education (RME) is a teaching and learning theory in mathematics education, which was originally introduced in Hans Freudenthal's interpretation of mathematics as a human activity (Freudenthal, 1973; Gravemeijer, 1994) and has been developed in the Netherlands over the last 40 years. RME approach focusses on the role of common sense and informal knowledge to help the development of students' mathematical understanding and thinking. It 'builds deep and long-term mathematical understanding by working from contexts that make sense to students'(https://rme.org.uk/what-is-rme/about-rme/).
Barnes (2005) explains why she selected the Realistic Mathematics Education (RME) approach as a theoretical framework for an intervention to improve the understanding of low attaining grade eight students in with regards to place value fractions and decimals. Barnes refers to the work of Abel (1983), Reusser (2000) and Baroody and Hume (1991) that low performance or attainment in mathematics is something that can be treated. In most cases it is not an incurable condition but something that develops as a result of the type of instruction learners receive and the teaching-learning environment within which they experience mathematics.

Barnes highlights the work of Haylock (1991) because it discusses factors associated with low attainers drawing on classroom-based research and proposes a strategy for teaching learners in this regard which includes the development of understanding as opposed to the learning of routines and procedures, the importance of tending to language development in teaching mathematics and the need to identify purposeful activities in meaningful contexts. This reinforces the need for learners to have a greater involvement in the learning process - in other words for learners to be more active.

Following on from her review of the literature Barnes identifies five aspects to include in the instructional approach of her intervention:

- More focus on relational and conceptual understanding as opposed to learning by rote and memorisation
- Creating meaningful learning contexts that actively involve learners
- Greater emphasis on problem solving and less emphasis on computation and arithmetic skills
- The importance of social interaction in the learning process I group work reciprocal teaching and games
- The importance of language development and discussion with and between learners in teaching mathematics.

As a teaching and learning pedagogy, RME has the following distinctive features:

1) Use of context. The word 'realistic', as explained by Marja van den Heuvel-Panhuizen from the Fruedenthal Institute, is 'not just the connection with the real world, but is related to the emphasis that RME puts on offering the students problem situations which they can imagine'. Therefore, the 'contexts' are not necessarily real-life situations, puzzles, manipulatives, and even formal mathematics can all provide

suitable contexts, as long as they're meaningful for students to make sense of the problems.
2) Use of models. Models bridge the gap between students' informal knowledge and the formal procedures.
3) Multiple strategies and formalisation. Instead of teachers imposing a 'standard method' or algorithm to students, the contexts in RME are chosen to draw out many different strategies for students to compare and reflect on.

Barnes literature review in relation to low attainers highlights that a lot of the teaching and learning in this domain has tended towards the mechanistic and structuralist trends. This has led to the dominance of what Skemp (1971) would call instrumental understanding rather than relation understanding. This focuses on symbolising formalising and generalising. As low attainers often struggle with these, they may have experienced repeated failure with continued emphasis on this method of teaching and learning.

Realistic mathematics education provides more of a focus on relational and conceptual understanding as opposed to rote learning. This is facilitated through the use of meaningful learning contexts which can be from everyday situations or 'imagined' reality. One of the general principles of progressive mathematization is that of "interactivity". Treffers (1987) suggests that when learners are confronted with the constructions and productions of their peers this can stimulate them to shorten their learning path, to help themselves up on the procedures of others, to become aware of the drawbacks or advantages of their own productions, and that copying others work slavishly will not aid their own progress.

RME approach in the UK mathematics education

Since 2004, Manchester Metropolitan University (MMU) has developed and trialled RME based materials at various levels: initially with KS3 pupils in 2004-2007, using a US-developed RME-based curriculum 'Mathematics in Context', later with KS4 pupils in 2007-2010, particularly those studying towards Foundation tier GCSE Mathematics, with Mathematics Education and Industry, the team produced a textbook *Making Sense of Maths*, published by Hodder.
In 2012-2017, RME-based materials were trialled in post-16 GCSE resit classes (Hough, Solomon, Gough and Dickinson, 2017). This study showed significant improvement from pre- to post- test, sustained at delayed-test alongside improved ability to explain/justify solutions. The team followed this up with an intervention study funded by the Nuffield Foundation, which has contributed to knowledge about the context and needs of learners in the post-16 resit classes.

The Efficiency of RME approach

RME is internationally recognised and has been adopted by a large number of countries all over the world (De Lange, 1996).

The efficiency of RME approach has been evidenced firstly in the Netherlands itself, where RME is used by over 80% of schools, as the Netherlands is considered one of the highest achieving countries in the world in mathematics according to the TIMSS report in the recent years (TIMSS 1999, 2007, 2010; PISA 2000, 2006, 2009, 2015). In addition, in the US, where RME based curriculum 'Mathematics in Context' is used, an association between RME and higher achievement has also been identified (Romberg and Shafer, 2005). In

England, MMU's previous implementation of RME in England has also found that the students who are exposed to RME-based materials are more likely to solve the problem, and to be able to explain their methods (Searle and Barmby, 2012).

However, RME has not yet been subject to a randomised controlled trial in the UK, particularly in the post-16 resit contexts. In light of this, as part of the CfEM research project, my action research will contribute to the trial and investigation of the effectiveness of RME approach in post-16 GCSE resit students, in terms of improving their understanding of mathematical concepts and problem-solving strategies. I would like to see the results for myself in my students. Furthermore, RME approach could be the push students need to change their grade 3 to grade 4.

It has become very clear to us from the literature review that varying teaching styles increases the difficulty of learning and understanding mathematics. According to Taat, Abdullah and Talip (2012), teachers need to use strategies that contribute to a deeper understanding of learners. Sabandar (2010) suggested that there is need for teachers to create challenging settings and problems to encourage students to learn more than they used to. As Mathematics is mainly problem solving-oriented, mathematics teachers must provide open, realistic problems with multiple probable answers (2005). In realistic mathematical learning that uses open problems, students use their problem-solving methods and understand the methods used by others. This ability is important because mathematics is used in almost every aspect of life.

Methods

The AR project consisted of three cycles.

- Cycle1(Oct 2021)- Pre-test questions were distributed to students. The questions were designed with the potential use of ratio tables for one of the possible solutions. Students were also asked to comment on how confident they had felt about their answers. Students also were encouraged to explain when unable to solve the problems

- Cycle2(Oct21-Jan 22)- Planning and delivering sessions focused on applications of ratio tables in various GCSE maths topics
- Cycle 3- Identical post-intervention questions given to students

Following the post-test questions, “Teacher as researcher” interviews of selected students based upon on individual responses to post-intervention questions. The interviews were conducted face to face and recordings of these interviews were stored securely on college’s own network, protected by password and only shared to staff involved in the research.

Results and Discussion

Across both colleges, four members of staff worked with approximately 50 students actively, as it was observed that the issue with student attendance, switching classes early in the year and other student personal circumstances have somewhat impacted this study.

In summary, what have we learnt from our AR?

- Many students were just simply doing something with the numbers
- Little evidence of learners attempting to make sense of maths
- Using Ratio Tables has helped learners to structure their mathematical reasoning and build confidence
- Use of a familiar context has helped to create a more dialogic and inclusive learning environment

The evidence showed that students struggled to get started with the maths questions, identifying what they had to do and how was a big challenge. Using the ratio tables has been a great tool to get students “unstuck” and get them started on solutions. It also helped some students who were rather anxious about things they thought they did not know how to solve, become less worried and by discussion, and flexibility of approach to the ratio tables allowed, they were able to form an appropriate path to ultimately solving the problem.

There were many examples in the pre-test scripts where students encountered problems with division. In many cases, students were able to identify a division, and re-write the sum using the standard ‘bus stop’ notation but could proceed no further. In other examples, students opted to find percentages such as 1% of 3300 by working out 3300÷100, using the standard division method, seemingly unaware of how inappropriate this is as a method for dividing by 100. Others applied the algorithm to finding (1)/8 of £600, when informal approaches linked to repeated halving may have proved much easier to perform. It seemed that students struggled to recognise the number of relationships involved. Instead, they opted for a formal procedure as soon as they understood that a division operation was required.

Some students talked about not being able to remember how to do a particular question or feeling that they had never been able to do it. It became evident that many students were just doing something of the numbers given, without attempting to make sense of a problem. At times, misapplication of procedures led to unrealistic answers, but students appeared oblivious to the unsuitability of their answers. Their ability to perform basic operations with numbers is a major barrier to success and a lack of fluency with times tables is still an obstacle for some. The concept of division is particularly challenging with many students showing that they cannot connect with the standard formal algorithm for dividing two numbers and yet they will have met this rule repeatedly over many years of schooling. The need for these students to work with different approaches to learning mathematics is very apparent.

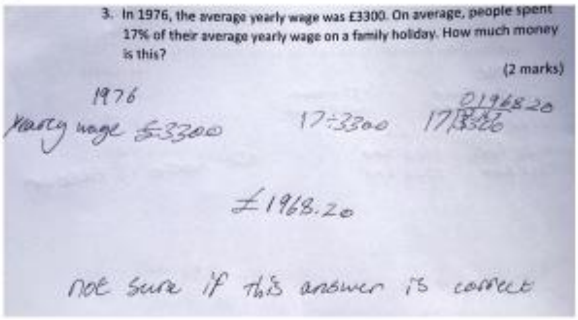
Excerpt from a student interview:

Teacher: "How confident do you feel about that answer?"

Student: "100%"

Teacher: "How can you be sure?"

Student: "The Bus Stop method. It never lets me down!"



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The researcher in question has reflects further, quote "No matter how many different ways I prompted the student avoided making sense of his answer in terms of the context.

He was fixated on performing an algorithm as that was what he had been trained to do. His method of checking was to complete a reverse calculation."

Another reflection from a different researcher at Northampton College highlighted how important this approach was for her, especially when helping a student with specific learning differences, she quotes: "When ratio table was used, most learners was able to see the proportional relationship between percentages and the amounts. It was particularly popular among dyslexic learners. One of the students explicitly said she liked this method because she is a visual learner."

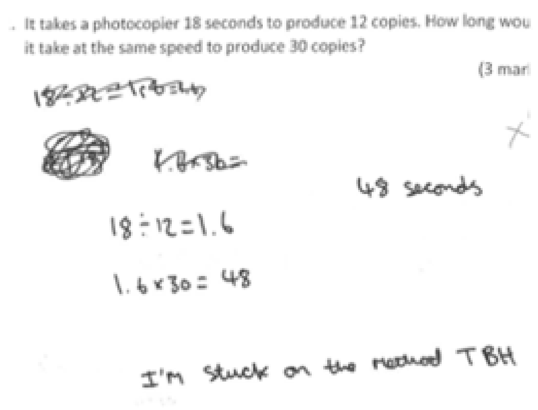
Strategies for recharging learners in common sense thinking

Using context, in order to support our learners to develop mathematical thinking, we have used following strategies:

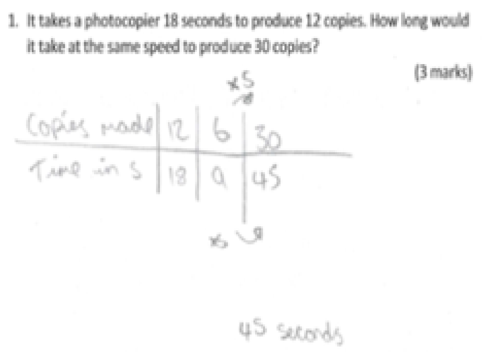
- Drawing a Ratio Table
- "Say what you see?"
- "What else do you know?"
- Go back to the context

Below, there are some examples of our pre and post test student answers:

Pre-Intervention



Post-Intervention



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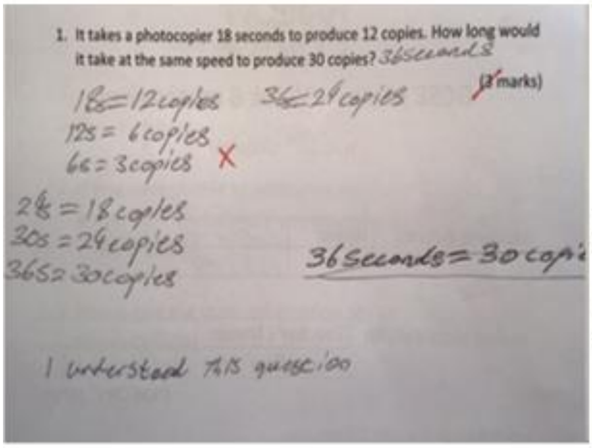
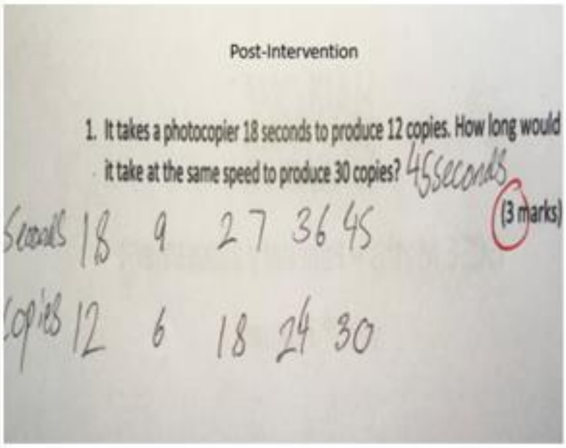
Pre-Intervention



Post-Intervention



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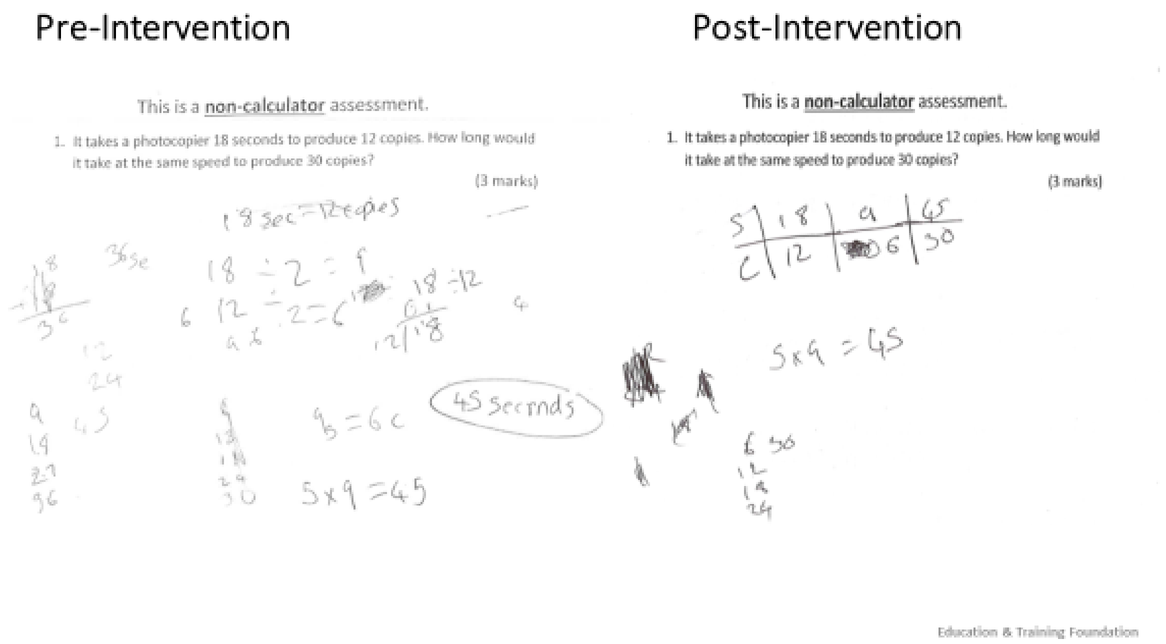

Students seem to be getting better at using the ratio tables, and more importantly in identifying where and how to use them. We insisted that they labelled the tables accordingly, but they had the freedom to choose which way to construct the table and so on.

It was refreshing to hear a student say during the interview “ “ I can see clearly what I need to do next”

Conclusions and Recommendations

Limitations

Although we have seen that the ratio tables do work, help learners with multiplicative reasoning, we know that nothing is perfect. One of the biggest drawbacks can be said of using this approach, is that students have previously been taught to use a different method, so it can be seen as being counter-productive to undo prior learning. We have seen the cases where more “able” students have outright dismissed the idea of using ratio tables with certain, less complex questions.

Conclusions (from limited dataset)

In conclusion, we can say that students, in general, find using ratio tables very useful in certain circumstances. We observed an uplift in proportional reasoning for students, especially with those of lower attainment and learning differences. Since the ratio tables rely heavily on visualisation and give students more freedom to come to the desired answer, it was welcomed and often a go-to tool for most students.

We’ve also observed that students rather quickly forgot this method, thus it is advised to spend a significant amount of time and put emphasis on such an approach to ensure some success.

Teachers involved in this AR have reported that they would most certainly continue using the ratio tables in future but ensure to introduce them well early into the semester to their students, pointing at the usefulness of this approach in a multitude of GCSE maths topics.

Recommendations

We have learnt that learners are unlikely to be familiar with using ratio tables and in general, with this style of maths teaching, using discussion encouraging students to return to the context is a useful pedagogical device for helping them debate, justify, and make sense of ‘shortcuts’ and ‘rules’. In a nutshell, we recommend that you:

- Start with a familiar context and model the use of a ratio table
- Capture & value each learners’ contribution, even if incorrect
- Be patient but persistent - create time for thinking, not just answer getting
- Do not rush to the introduction of formal procedures
- Always go back to the context

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