Lesson plan

Geometric reasoning

Contents

1. Rationale

Students at this level usually have some knowledge of angles and this lesson provides an opportunity to **establish what students already know** about properties of angles and their understanding of parallel lines (Key Principle 2). Students are encouraged to **make connections** (Key Principle 3) between different properties of angles in order to develop an understanding of geometric reasoning. The in-depth focus on reasoning in this lesson supports students in **developing both their fluency and understanding** (Key Principle 4) as they learn to recognise when and how to apply angle properties to a range of problems.

2. GCSE curriculum

Geometry and measures

G3 apply the properties of angles at a point, angles at a point on a straight line, … understand and use alternate and corresponding angles on parallel lines; … use the sum of angles in a triangle

3. Lesson objectives

- Understand what it means for lines to be parallel
- Calculate missing angles
- Use angles to determine whether lines are parallel
- Develop fluency and understanding when reasoning with angles

4. Starting points

The lesson assumes that students understand angle as a measure of turn and that they have some knowledge about the properties of angles.

5. Research questions

Pedagogic focus

In which ways does the teacher develop the lesson to value and build on what the students already know?

Maths focus

How do students demonstrate their knowledge and understanding?

6. Lesson structure

Activity	Time (min)	Description/Prompt	Materials
Introduction	15	Establish what students already know about angles by asking them to produce a poster. Discuss students' posters, highlighting key information needed for the lesson.	A3 paper Coloured pens Slides 2–7
Explore/ Discuss 1	15	Introduce problems involving parallel lines and triangles. Ask students to identify the information needed to be able to answer the questions posed.	'What do we need to know?' handout (optional) Slides 8–11
Explore 2	25	Ask students to work in pairs on problems involving angles and parallel lines. Emphasise the use of angle properties to support reasoning.	'Geometric reasoning' handout Slides 12–15
Discuss 2	15	Check that students have answered the questions posed correctly and explore different approaches.	Slides 16–19
Review	10	Provide an opportunity for students to review their posters and make any necessary additions. Consolidate the things that students need to remember for geometric reasoning.	Slides 20–21
Practice questions/ Discuss 3	10	Ask students to work individually to answer some practice exam questions.	'Practice questions' handout Slides 22–26

7. Teacher guidance

Introduction

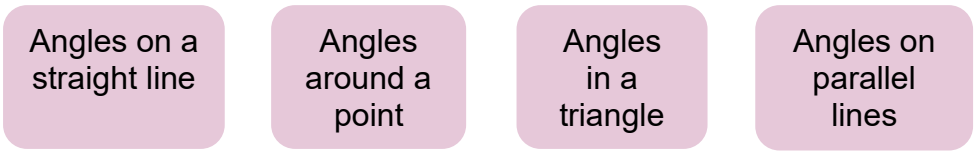
Aim	To establish students' existing knowledge
Materials	A3 paper, coloured pens
Slides	Slides 2–7
Time	15 minutes

Establishing students' existing knowledge is an important aspect of the mastery approach (Key Principle 2). The focus at the start of the lesson is identifying what students already know about angles and providing an opportunity for them to display their existing knowledge by making a poster.

What students might do and what you might do

Slide 2 Tell students that they will begin the lesson by reviewing what they know about angles.

Distribute A3 paper and some coloured pens (with tips that are not too thick). Ask students to work in pairs to produce a poster to show what they know about angles. Tell them that they may use the ideas on the slide as a starting point.



Allow students to work on their posters and try not to intervene. The aim of this activity is to provide you with information about what students appear to know already and to get them thinking about geometry. By **establishing what students already know, this existing knowledge can be built upon** (Key Principle 2).

As students work, identify a couple of students to share their posters with the rest of the class.

Discuss students' posters. Ask them to add anything that is missing to their poster during the discussion.

Slides 3–6 Use these slides to support a discussion of students' posters.

Slide 3 When discussing the properties of angles in triangles, ask students when the three angles in a triangle would be equal. Establish that an equilateral triangle has three equal sides and three equal angles that are each 60°, while an isosceles triangle has two sides of equal length and two equal angles.

You may want to ask students what other types of triangles they know (e.g. right angled and scalene triangles) and establish their properties. Emphasise that regardless of the type of triangle, the angles always add up to 180°.

Slide 4 Check students' understanding of what 'parallel' means and check that they know that arrows are used to denote parallel lines. Establish that when the lines are parallel, corresponding angles are equal. This slide is designed to emphasise the **mathematical structure** (Key Principle 1) of parallel lines on a transversal: essentially, the second line can be thought of as a copy of the first. Ask students what else we know about the angles if the lines are parallel. They may mention alternate angles and/or co-interior angles, for example.

Slide 5 Emphasise that alternate angles and corresponding angles are equal only when the lines are parallel. If the angles are not equal, we know that the lines are not parallel.

Contrast alternate and corresponding angles with co-interior angles, which add up to 180° only when the lines are parallel.

Slide 6 Before bringing the discussion to a close, check students' understanding of what 'perpendicular' means. Remind students of the way that a 90° angle is

represented and highlight the importance of not assuming that two lines are perpendicular just because the angle looks like a 90° angle.

Slide 7 Remind students of the angle properties that have been discussed and give them the opportunity to add to their poster, if they haven't already done so during the discussion. You may like to draw some labelled diagrams of angles on parallel lines on the board for students to copy onto their posters, to refer to later in the lesson.

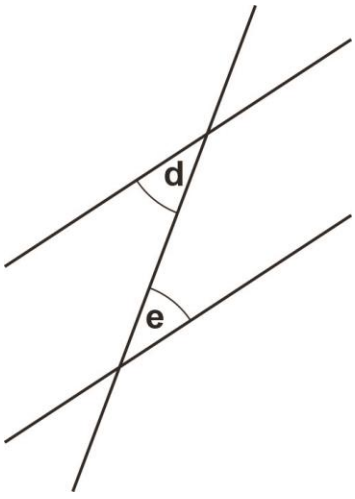
Explore/Discuss 1

Aim	To develop geometric reasoning
Materials	'What do we need to know?' handout (optional)
Slides	Slides 8–11
Time	15 minutes

In this next part of the lesson, the relationship between parallel lines and angles is explored further. Students are encouraged to think about the information needed to establish whether a pair of lines is parallel or whether angles are equal, and to consider the properties of angles in a triangle. By posing a question where students do not need to determine the answer, but are instead focussing on identifying the information required to be able to answer the question, **students' confidence can be built** (Key Principle 5).

What students might do and what you might do

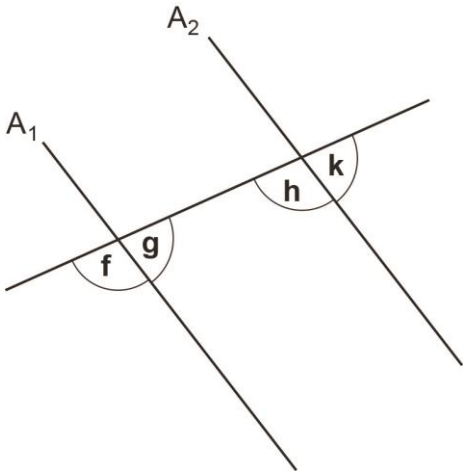
Slide 8 Tell students that Petra wants to know whether angles ***d*** and ***e*** are equal:



Ask students what we need to know to determine whether these angles are equal.

Establish that angles ***d*** and ***e*** are alternate angles and that if the lines are parallel, alternate angles will be equal. If they know that the lines are parallel, they can conclude that angles ***d*** and ***e*** are equal. If they know that the lines are not parallel, they can conclude that angles ***d*** and ***e*** are not equal.

Slide 9 Tell students that Raheela wants to know whether lines A_1 and A_2 are parallel:



Ask students what we need to know to be able to determine whether or not the lines are parallel. Discuss different possible approaches using either corresponding angles or co-interior angles. Check that students recognise that if they know that corresponding angles ***f*** and ***h*** are equal, they can conclude that lines A_1 and A_2 are parallel. Alternatively, if we know that corresponding angles ***g*** and ***k*** are equal, then we can also conclude that lines A_1 and A_2 are parallel. Highlight that if we know that angle ***f*** is equal to angle ***h*** then we also know that angle ***g*** is equal to angle ***k***.

Another approach is to work with co-interior angles instead. If we know ***g*** + ***h*** = 180˚, we can conclude that lines A_1 and A_2 are parallel.

Slide 10 Before moving on, remind students that:

- to show lines are parallel, they need to find a pair of equal alternate angles or corresponding angles, or a pair of supplementary co-interior angles
- if they know that two lines are parallel, then they know that the alternate and corresponding pairs of angles are equal and the co-interior angles are supplementary.

Slide 11 Introduce four different scenarios for students to identify the information needed to be able to answer the question posed. You may want to print out the 'What do we need to know?' handout and ask students to work on it in pairs. Alternatively, you could carry out the activity as a whole class.

Questions that students could ask include:

Q1: Is angle ***m*** equal to angle ***n***?
Q2: Are angles ***p***, ***q*** and ***r*** each 60˚?
Q3: Is CE = DE?
Q4: Is line F_1 parallel to line F_2?

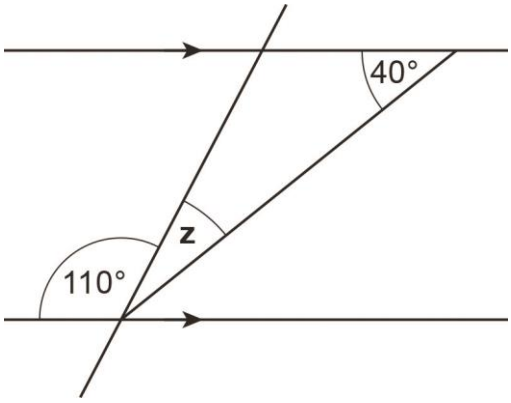
Explore 2

Aim	To develop fluency and understanding when reasoning with angles
Materials	'Geometric reasoning' handout
Slides	Slides 12–15
Time	25 minutes

In this section of the lesson, the focus is on reasoning questions where some information is given. A question is posed and students have to decide whether the answer is 'Yes', 'No' or 'Can't tell', giving reasons for their choice. Providing students with an opportunity to apply their previous understanding of angle properties with their newly developed understanding of reasoning helps to **develop students' fluency and understanding**, a key focus of the mastery approach (Key Principle 4).

What students might do and what you might do

Slide 12 Introduce a new problem and tell students that we need to determine whether or not angle ***z*** is equal to 30°.



Explain that one of the three boxes: 'Yes', 'No' or 'Can't tell' needs to be ticked/ crossed.

Give students a few minutes to discuss in pairs.

This question is provided to model what students will do in the pair work to follow.

Slide 13 Show students Petra's and Raheela's responses and ask them to explain what Petra and Raheela have done wrong. Petra assumes the diagram is drawn accurately and concludes from measuring, that angle ***z*** does not equal 30˚. Raheela uses the fact that the lines are parallel in her reasoning but has not correctly identified co-interior angles.

Slide 14 Tell students that Petra produces another response to the question. She shades the angles she wants to talk about. She provides some reasons and concludes that angle ***z*** is 30˚ and crosses the 'Yes' box.

Confirm that angle ***z*** is equal to 30˚ and emphasise that Petra's approach is just one way of showing this. Check that all approaches used by students have been explored. It is important to **value and make sense of students' different ways of working** to increase students' self-belief (Key Principle 5).

Slide 15 Distribute to each pair of students a copy of the 'Geometric reasoning' handout. Tell students that this activity is similar to the one they have just looked at. They should decide whether the answer to each question is 'Yes', 'No' or 'Can't tell' and tick/cross the appropriate box. They should write their reason(s) in the 'Reasoning' column.

When you judge that students have answered enough of the questions, go through possible responses, emphasising alternative valid approaches.

Discuss 2

Aim	To review different approaches to solving problems
Slides	Slides 16–19
Time	15 minutes

This section of the lesson provides an opportunity to discuss students' work and explore different possible approaches. It is important that students have their approaches validated and feel that they have something to contribute to the discussion so as to **build student confidence** (Key Principle 5).

Slides 16–19 These slides can be used to support a discussion of students' work.

Slide 16 When exploring different approaches, emphasise the benefits of being efficient. Rather than alternate angles, co-interior angles could be used to identify the 130° angle and the property of angles on a straight could then be used to determine the 50° angle shown in triangle PQR. This is an equally valid approach but is less efficient than working with alternate angles as shown.

Slide 17 Encourage students to consider a different way of showing that the lines are not parallel. For example, can they identify vertically opposite angles?

Slide 18 When thinking about different approaches, check that students understand that they need to show that lines K_1 and K_2 are parallel. Some students may try to show that lines J_1 and J_2 are parallel (which we know to be true) by mistake.

Slide 19 Isosceles triangles are often drawn with the equal angles at the base and the base of the triangle parallel to the bottom of the page or screen. In this case, the equal angles are at U and T. Check that students know which two sides are equal and perhaps ask them how they would indicate this. Check also that students recognise that the third angle in triangle TUV must be different to the two 65° angles as the only way that the three angles can be equal is if they are all 60° (i.e. where the triangle is equilateral).

Review

Aim	To review learning and understanding of geometric reasoning
Slides	Slides 20–21
Time	10 minutes

In this section of the lesson students revisit their posters. Providing an opportunity for them to review and add to their posters allows **students to deepen their understanding** while **building on their existing knowledge**, an important part of the mastery approach (Key Principle 2).

What students might do and what you might do

Slide 20 Ask students to return to their posters and add anything else that they feel is missing. Hold a class discussion of the posters, focusing on geometric reasoning.

Slide 21 Bring the discussion to an end by consolidating what students need to remember for geometric reasoning.

Practice questions/Discuss 3

Aim	Students apply their knowledge to practice questions
Materials	'Practice questions' handout
Slides	Slides 22–26
Time	10 minutes

In this part of the lesson, students apply what they have learned in the lesson to some practice questions.

What students might do and what you might do

Slide 22 Distribute to each student a copy of the 'Practice questions' handout. There are two questions, each with two parts. You may want to choose just one question or parts of a question for your class.

Give students a couple of minutes to work individually on the questions. Check that students are familiar with the notation used for describing angles.

When students have had sufficient time to complete the questions, discuss their work.

Slides 23–26 Use these slides to support a discussion of students' work.

Slide 23 When discussing part (a), emphasise the importance of not assuming an angle is a right angle because it looks like one. Highlight that the question states that lines AB and BC are perpendicular, so we know that angle ABC is 90°. x is therefore equal to 40°, as 90° – 25° – 25° = 40°.

Slide 24 Check that students have identified an angle that is 125° and that their reasoning is correct. They could have selected b or d. You may want to ask them to colour in all the angles that are equal to 125°.

When answering part (ii), students may use the fact that angles around a point add up to 360° (360° – 125° = 235°). Encourage students to identify the values of angles *a*, *b* and *c* using the properties of angles on parallel lines as well, to show that $a + b + c = 235°$.

Slide 25 Check that students recognise how the two sides of the triangle with equal length relate to the two equal angles. It is important that students explain why Mary is wrong and do not just state the correct size of angle x. (54°).

Slide 26 Establish that William's reason for angle *EGH* = 57° is wrong, as he has confused corresponding and alternate angles.